

A Study on Cubic Fuzzy Competition Graph Labeling

M.Ragavi¹, T.Kannan².

¹Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu, India.

²Department of Mathematics, Sree Sevugan Annamalai college, Devakottai, affiliated with Alagappa University, Karaikudi, Tamilnadu, India.

Abstract

In this work, we study the notion of cubic fuzzy competition graphs as a hybrid framework for modeling competitive interactions under uncertainty. The proposed model combines the structure of competition graphs with cubic fuzzy information, where each arc or induced competition relation is described by a point membership and an interval-valued membership. This dual representation provides greater flexibility and precision than ordinary fuzzy competition graphs in capturing ambiguous relations. We define the cubic fuzzy competition graph associated with a cubic fuzzy digraph and discuss with example.

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Cubic Fuzzy Graph, Competition Graph $C(G)$, Cubic Fuzzy Competition Labeling Graph $(\mathfrak{C}_f C)$

1 Introduction

Zadeh [1] developed fuzzy set theory and, by extension, fuzzy logic in his ground-breaking paper "Fuzzy Sets," which was published in 1965. The goal of Zadeh's study was to construct a theory that might address the ambiguity and imprecision of specific classes or sets in human thought, especially in the areas of abstraction, pattern recognition, and information exchange. According to this theory, an element's grade of membership in a subset of a universal set should have a value in the closed interval $[0,1]$ of real numbers. Computer sciences, artificial intelligence, decision analysis, information sciences, system sciences, control engineering, expert systems, pattern recognition, management sciences, operations research, and robotics

are among the fields where Zadeh's concepts have found use. Topology, abstract algebra, geometry, graph theory, and analysis have all included concepts from fuzzy set theory. Additionally, in 1975, he extended fuzzy sets to interval-valued fuzzy sets, where a particular element is not required to have a certain membership. The notion of fuzzy graphs was first presented by Kauffmann in 1973[2, 3], and Rosenfeld addressed it in 1975[4]. Rosenfeld also took into account the fuzzy interactions between fuzzy sets and created the structure of fuzzy graphs, which yielded analogues of several graph theoretical ideas and labeling[5]. Bhattacharya[6, 7] conducted additional research on fuzzy graphs. Akram and Dudek gave the idea of interval valued fuzzy graphs in 2011 where they used interval membership for an element in the vertex set. Akram further extended the idea of interval valued fuzzy graphs to Interval-valued fuzzy line graphs in 2012. More detail of fuzzy graphs, we refer the reader to[8, 9, 10]. In 1986, Atanassov use the notion of membership and non-membership of an element in a set X and gave the idea of intuitionistic fuzzy sets. He extended this idea to intuitionistic fuzzy graphs and for more detail in this direction, we refer the reader to[11, 12, 13, 14]. Akram and Davvaz[15] introduced the notion of strong intuitionistic fuzzy graphs and investigated some of their properties. They discussed some propositions of self complementary and self weak complementary strong intuitionistic fuzzy graphs. Smarandache introduced the neutrosophic set[16, 17], which is an extension of the intuitionistic fuzzy set, which represents an intuitionistic fuzzy structure in real life. Neutrosophic is defined by the memberships of appearance (A), indeterminate (I), and non-appearance (NA), with sums ranging from 0 to 3, which are independent. Kandasamy[18] created the concept of the neutrosophic graph, and his work gives forth a new dimension of graph theory. Neutrosophic fuzzy graphs are more effective, adaptable, and compatible with specific real-world problems than fuzzy graphs. After that, many researchers used the idea of neutrosophic sets in different directions. The idea of neutrosophic graphs is provided by Kandasamy et al. in the book title as Neutrosophic graphs, where they introduce idea of neutrosophic graphs. This study reveals that these neutrosophic graphs give a new dimension to graph theory. An important feature of this book is that it contains over 200 neutrosophic graphs to provide better understandings of these concepts. Akram[19] and others discussed different aspects of neutrosophic graphs. Further Jun et al.[20, 21] gave the idea of cubic set and it was characterized by interval valued fuzzy set and fuzzy set, which is more general tool to capture uncertainty and vagueness, while fuzzy set deals with single value membership and interval valued fuzzy set ranges the membership in the form of interval. The hybrid platform provided by the cubic set is the main advantage, in that it contains more information than a fuzzy set and interval valued fuzzy set. By using this concept, we can solve different problems arising in several areas and can pick finest choice by means of cubic sets in various decision making problems. Recently, Rashid et al[22]. introduced the notion of cubic graphs where they introduced many new types of graphs and provided their application. In the realm of Uncertain Graph Theory, the concept of a Competition Graph has

emerged as significant. A competition graph[23, 24, 25, 26] represents interactions where directed edges in a digraph indicate common neighbors, connecting vertices that "compete" for the same neighbor. Notable examples include Fuzzy Competition Graphs. Each arc or induced competition relation can be defined by a point membership and an interval-valued membership in the proposed model, which blends the structure of competition graphs with cubic fuzzy information. Compared to standard fuzzy competition graphs, this dual representation offers more flexibility and accuracy in capturing ambiguous associations. We define a cubic fuzzy digraph's corresponding cubic fuzzy competition graph. As an application, the model is used to analyze predator-prey relationships, where the strength of competition between species sharing common food resources can be measured more realistically. The framework is also suitable for other domains such as traffic flow, social competition, and decision-making problems, where uncertainty and partial information play a significant role. In this study, we focus on simple finite directed graphs and examine the special case in which every pair of distinct vertices competes for exactly one common neighbor. Section 2 of this paper comprises the fundamental definitions cubic fuzzy competition labeling graph. In section 3, we define cubic fuzzy competition labeling graph and some related example and theorem. Sections 4 include conclusion.

2 Preliminaries

Let U be the universal set. A fuzzy set in U is represented by $R = \{ (x, \mu_R(x)) | \mu_R(x) > 0, x \in U \}$, where the function $\mu_R: U \rightarrow [0, 1]$ is the appearance of element x in the fuzzy set S . A fuzzy graph $G = (\mu, \gamma)$ is a pair of functions, where μ is a fuzzy subset of $\mathfrak{N}$, and γ is a symmetric fuzzy relation on μ .

Definition 1. A graph $G: (\mu, \gamma)$ is said to be fuzzy labeling graph, if $\mu: \mathfrak{N} \rightarrow [0, 1]$ and $\gamma: \mathfrak{N} \times \mathfrak{N} \rightarrow [0, 1]$, μ, γ are bijective and $\gamma_A(mn) \leq \min(\mu_A(m), \mu_A(n))$ for all $m, n \in \mathfrak{N}$.

Definition 2. A cubic graph is a graph in which every vertex has degree 3; equivalently, it is a 3-regular graph or trivalent graph. Formally, for a graph for a graph $G = (\mathfrak{N}, \mathfrak{E})$, G is cubic if $\deg(n) = 3$ for every $n \in \mathfrak{N}$.

Definition 3. For a graph $D = (\mathfrak{N}, \mathfrak{E})$, G is cubic if $\deg(n) = 3$ for every $n \in \mathfrak{N}$. Let $D = (\mathfrak{N}, \mathfrak{E})$ be a simple graph. A cubic fuzzy graph $(\mathfrak{C}_f)$ on G is a pair $\check{G} = (V, L)$, where V is a cubic fuzzy set on $\mathfrak{N}$ and L is a cubic fuzzy set on $\mathfrak{E}$. For each $n \in \mathfrak{N}$ and each edge $e \in \mathfrak{E}$, the membership is given by $V(n) = (\lambda_V(n), [\alpha_V(n), \beta_V(n)])$ $L(e) = (\lambda_L(e), [\alpha_L(e), \beta_L(e)])$ where, $\lambda_V(n), \lambda_L(E) \in [0, 1]$ are the point fuzzy membership degrees and $[\alpha_V(n), \beta_V(n)], [\alpha_L(e), \beta_L(e)] \subseteq [0, 1]$ are interval-valued fuzzy memberships. The vertices satisfy the usual cubic fuzzy consistency conditions, $\alpha_V(n) \leq \lambda_V(n) \leq \beta_V(n)$, $\alpha_L(e) \leq \lambda_L(e) \leq \beta_L(e)$ and for each edge $e = mn \in \mathfrak{E}$, $\lambda_L(mn) \leq \min(\lambda_L(m), \lambda_L(n))$, $\alpha_L(mn) \leq \min(\alpha_L(m), \alpha_L(n))$, $\beta_L(mn) \leq \min(\beta_L(m), \beta_L(n))$.

Definition 4. Let $G = (\mathfrak{N}, \mathfrak{E})$ be a digraph. The competition graph $C(G)$ is the graph with $\mathfrak{N}(C(G)) = \mathfrak{N}(G)$, $mn \in \mathfrak{E}(C(G))$ if and only if there exists $p \in \mathfrak{N}(G)$, $p \neq m, n$ such that $m \rightarrow p$ and $n \rightarrow p$ are arcs of G .

3 Cubic Fuzzy Labeling Competition Graph

$(\mathfrak{C}_f C)$

Definition 5. Let $\check{D} = (\mathfrak{N}, \mathfrak{E}, V, L)$ be a cubic fuzzy digraph where $\mathfrak{N}$ is vertex set, $\mathfrak{E} \subseteq \mathfrak{N} \times \mathfrak{N}$, for each vertex $m \in \mathfrak{N}$ and define its neighborhood by $\mathfrak{N}^+(m) = \{o \in N : (m, o) \in \mathfrak{E}\}$ and assigned a bijective cubic fuzzy value $V(n) = (\lambda_V(n), [\alpha_V(n), \beta_V(n)])$, $L(m, o) = (\lambda_L(m, o), [\alpha_L(m, o), \beta_L(m, o)])$ where, $\lambda_L(m, o) \in [0, 1]$ are the point fuzzy membership degrees and $[\alpha_L(m, o), \beta_L(m, o)] \subseteq [0, 1]$ are interval-valued fuzzy memberships and the vertices satisfy the usual cubic fuzzy consistency conditions, $\alpha_V(n) \leq \lambda_V(n) \leq \beta_V(n)$, $\alpha_L(e) \leq \lambda_L(e) \leq \beta_L(e)$ and for each $e = (m, o) \in \mathfrak{E}$, $\lambda_L(m, o) \leq \min(\lambda_V(m), \lambda_V(o))$, $\alpha_L(m, o) \leq \min(\alpha_V(m), \alpha_V(o))$, $\beta_L(m, o) \leq \min(\beta_V(m), \beta_V(o))$. A cubic fuzzy labeling graph is said to be cubic fuzzy labeling competition graph. If for any two distinct vertices $n, m \in \mathfrak{N}$, define their common one neighborhood by $\mathfrak{N}^+(m, n) = \mathfrak{N}^+(m) \cap \mathfrak{N}^+(n)$, $\mathfrak{N}^+(m, n) \neq \emptyset$, $\mathfrak{N}^+(m, n) = \{o \in N : (m, o) \in \mathfrak{E}, (n, o) \in \mathfrak{E}\}$ and define their cubic fuzzy competition strength is $\mathfrak{C}_f C(\check{D}) = (\lambda_{\mathfrak{C}_f C}(m, n), [\alpha_{\mathfrak{C}_f C}(m, n), \beta_{\mathfrak{C}_f C}(m, n)])$ where $\lambda_{\mathfrak{C}_f C}(m, n) = \min\{\lambda_L(m, o), \lambda_L(n, o), \lambda_V(m), \lambda_V(n)\}$, $\alpha_{\mathfrak{C}_f C}(m, n) = \min\{\alpha_L(m, o), \alpha_L(n, o), \alpha_V(m), \alpha_V(n)\}$, $\beta_{\mathfrak{C}_f C}(m, n) = \min\{\beta_L(m, o), \beta_L(n, o), \beta_V(m), \beta_V(n)\}$, then the cubic fuzzy labeling competition graph $(\mathfrak{C}_f C)$ is the undirected graph with vertex set $V(\mathfrak{C}_f C)$, such that $\{m, n\} \in \mathfrak{E}_\theta$ if only if $\lambda_{\mathfrak{C}_f C}(m, n) > \theta$, for some threshold $\theta \in [0, 1]$.

Theorem 1. Every cubic fuzzy labeling of the directed graph induces a corresponding cubic fuzzy labeling on its competition graph.

Proof. Let $D = (\mathfrak{N}, \mathfrak{E})$ be a simple and finite digraph. Define a cubic fuzzy labeling graph. Let $\check{D} = (\mathfrak{N}, \mathfrak{E}, V, L)$, where V is a cubic fuzzy set on $\mathfrak{N}$ and L is a cubic fuzzy set on $\mathfrak{E}$. For each $m \in \mathfrak{N}$, define its neighborhood by $\mathfrak{N}^+(m) = \{o \in N : (m, o) \in \mathfrak{E}\}$ and each arc $e \in \mathfrak{E}$ and assigned a cubic fuzzy membership value $V(n) = (\lambda_V(n), [\alpha_V(n), \beta_V(n)])$, $L(m, o) = (\lambda_L(m, o), [\alpha_L(m, o), \beta_L(m, o)])$ where, $\lambda_L(m, o) \in [0, 1]$ are the point fuzzy membership degrees and $[\alpha_L(m, o), \beta_L(m, o)] \subseteq [0, 1]$ are interval-valued fuzzy memberships and the vertices satisfy the usual cubic fuzzy consistency conditions: $\alpha_V(n) \leq \lambda_V(n) \leq \beta_V(n)$, $\alpha_L(e) \leq \lambda_L(e) \leq \beta_L(e)$ and for each $e = (m, o) \in \mathfrak{E}$, $\lambda_L(m, o) \leq \min(\lambda_V(m), \lambda_V(o))$, $\alpha_L(m, o) \leq \min(\alpha_V(m), \alpha_V(o))$, $\beta_L(m, o) \leq \min(\beta_V(m), \beta_V(o))$. We get a cubic fuzzy labeling competition graph "If every pair of distinct vertices is competitive to the exactly one neighborhood, then their induced cubic fuzzy competition value is defined by the

following the definition 5. Hence, every cubic fuzzy labeling of the directed graph induces a corresponding cubic fuzzy labeling on its competition graph. $\square$

Example:

Take $\mathfrak{N} = \{P, Q, R, S\}$ and $\mathfrak{E} = \{(P, S), (Q, S), (R, S)\}$,

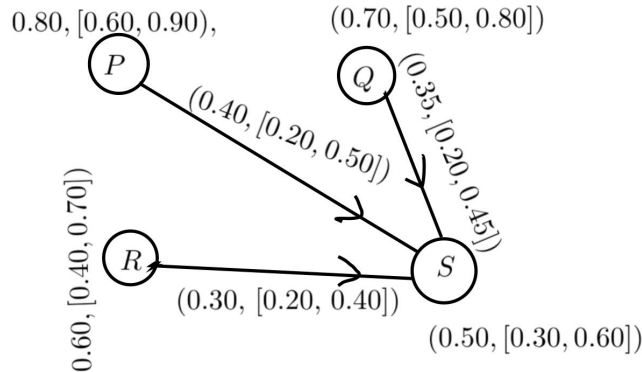


Figure 1: Cubic Fuzzy Competition Labeling Graph

$P \rightarrow S, Q \rightarrow S, R \rightarrow S$.

Only one common neighborhood $\mathfrak{N}^+(P) = \{S\}$, $\mathfrak{N}^+(Q) = \{S\}$, $\mathfrak{N}^+(R) = \{S\}$, $\mathfrak{N}^+(S) = \{\emptyset\}$. Therefore, $\mathfrak{N}^+(P, Q) = \{S\}$, $\mathfrak{N}^+(P, R) = \{S\}$, $\mathfrak{N}^+(Q, R) = \{S\}$.

Let us take vertex membership and edge membership as

$V(P) = (0.80, [0.60, 0.90])$, $V(Q) = (0.70, [0.50, 0.80])$, $V(R) = (0.60, [0.40, 0.70])$, $V(S) = (0.50, [0.30, 0.60])$

$L(P, S) = (0.40, [0.20, 0.50])$, $L(Q, S) = (0.35, [0.20, 0.45])$, $L(R, S) = (0.30, [0.20, 0.40])$.

competition between P and Q :

since, S is only a common neighborhood;

$$\lambda_{\mathfrak{C}_f C}(P, Q) = \min(0.40, 0.35, 0.80, 0.70) = 0.35,$$

$$\alpha_{\mathfrak{C}_f C}(P, Q) = \min(0.20, 0.20, 0.60, 0.50) = 0.20,$$

$$\beta_{\mathfrak{C}_f C}(P, Q) = \min(0.50, 0.45, 0.90, 0.80) = 0.45$$

$$(\mathfrak{C}_f C)(P, Q) = (0.35, [0.20, 0.45])$$

competition between P and R :

$$\lambda_{\mathfrak{C}_f C}(P, R) = \min(0.40, 0.30, 0.80, 0.60) = 0.30,$$

$$\alpha_{\mathfrak{C}_f C}(P, R) = \min(0.20, 0.20, 0.60, 0.40) = 0.20,$$

$$\beta_{\mathfrak{C}_f C}(P, R) = \min(0.50, 0.40, 0.90, 0.70) = 0.40$$

$$(\mathfrak{C}_f C)(P, R) = (0.30, [0.20, 0.40])$$

competition between Q and R :

$$\begin{aligned}\lambda_{\mathfrak{C}_f C}(Q, R) &= \min(0.35, 0.30, 0.70, 0.60) = 0.30, \\ \alpha_{\mathfrak{C}_f C}(Q, R) &= \min(0.20, 0.20, 0.50, 0.40) = 0.20, \\ \beta_{\mathfrak{C}_f C}(Q, R) &= \min(0.45, 0.40, 0.80, 0.70) = 0.40\end{aligned}$$

$$V_{(\mathfrak{C}_f C)}(Q, R) = (0.30, [0.20, 0.40])$$

choose the threshold $\theta = 0.25$

$$\begin{aligned}\lambda_{\mathfrak{C}_f C}(P, Q) &= 0.35 > 0.25, \\ \lambda_{\mathfrak{C}_f C}(P, R) &= 0.3 > 0.25, \\ \lambda_{\mathfrak{C}_f C}(Q, R) &= 0.3 > 0.25, \\ \text{therefore } E_\theta &= (P, Q), (P, R), (Q, R).\end{aligned}$$

4 Conclusion

In this work, we introduced the concept of a cubic fuzzy competition graph as an extended framework for representing competitive relationships in uncertain environments. By combining the structure of competition graphs with cubic fuzzy information, the proposed model simultaneously incorporates a point-valued membership and an interval-valued membership for each arc or induced competition relation. This dual representation enables the model to express both the estimated strength of a competitive interaction and the range of possible variation associated with incomplete, imprecise, or conflicting information. In addition to discuss the ecological networks, the proposed model can be applied to transportation systems, social networks, economic competition, communication networks, and multicriteria decision-making problems. Future research may investigate cubic fuzzy k-competition graphs, domination parameters, connectivity measures, matrix representations, and spectral properties of cubic fuzzy competition graphs. Further extensions involving intuitionistic, or neutrosophic cubic information may also provide useful tools for modeling increasingly complex uncertain systems.

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