

Fixed point results for $(\alpha-\psi)$ - contraction mapping on neutrosophic double controlled metric spaces

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Abstract:

The concept of $(\alpha-\psi)$ -neutrosophic contractive mappings on neutrosophic double controlled metric spaces is introduced in this study as a way to extend neutrosophic metric spaces. Triangle inequalities involve the two non-comparable functions ρ and ϕ for this reason.

We present some results for contraction mappings with non-trivial examples. Ultimately, our theoretical work has been utilized to prove the existenc solution of integral integral type operators.

Keywords: $(\alpha-\psi)$ -contraction; α -admissible mapping; Neutrosophic double controlled metric space; Integral equations; Fixed point.

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1. Introduction

The fixed-point theory is based on two fundamental ideas: metric spaces and the Banach contraction principle. Spaciousness attracts hundreds of researchers with its axiomatic interpretation of metric space. Over the years, metric spaces have seen several generalizations. Zadeh [26], laid the foundation for fuzzy mathematics in 1965. This concept takes standard set theory to a higher level of analysis. After then, the idea received a number of proven improvements, and the reasoning has been used in a variety of scientific and engineering fields, including the study of approximation theory, linear systems and matrix theory. Several authors investigated at the theory with its topological aspects from their own angle and came to some important basic results that seem important when examining the idea in connection to different other generalized spaces. Kramosil and Michalek [10] initially introduced the concept of fuzzy metric space and then modified by George and Veeramani [4]. The concept of fuzzy b-metric space was initially introduced by Nadaban [14] and Kim et al. established a number of fixedpoint results within this framework. The idea of fuzzy double-controlled metric space was introduced by Saleem et al. [24], and they also provided an illustration of the Banach contraction principle. Azmi [1] applied $(\alpha-\psi)$ -contractive mappings to get certain fixed-point results on double-controlled metric type spaces. Samet et al. [21], established the concept of $(\alpha-\psi)$ -contractive mappings and the α admissible mappings on complete metric spaces with different fixed-point theorems. Park [16] defined the notion of Intuitionistic Fuzzy Metric Space with the help of continuous t-norms and continuous. In 1998, Smarandache[25] developed the ideas of neutrosophic logic and Neutrosophic Set. The idea of Neutrosophic Metric Space was established by Kirisci and Simsek[11] which deals with membership, non-membership and neutralness. The objective of this work is to expand some fixed point results for contraction mapping and to introduce $(\alpha-\psi)$ -contractive mapping in neutrosophic double controlled metric spaces using the approaches described in [5]. We conclude by an application of findings to integral equation which was motivated by the work done in [5].

2. Preliminaries

Definition 2.1[13] If the binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ enjoys the following axioms, it is named to be a continuous t-norm:

- (i) $\mathfrak{z} * \tilde{\zeta} = \tilde{\zeta} * \mathfrak{z}$ for all $\mathfrak{z}, \tilde{\zeta} \in [0, 1]$;
- (ii) $*$ is continuous;
- (iii) $\mathfrak{z} * 1 = \mathfrak{z}$, for all $\mathfrak{z} \in [0, 1]$;
- (iv) $*$ is associative;
- (v) If $\mathfrak{z} \leq \tilde{\zeta}$ and $\mathfrak{d} \leq \vartheta$, with $\mathfrak{z}, \tilde{\zeta}, \mathfrak{d}, \vartheta \in [0, 1]$, then $\mathfrak{z} * \mathfrak{d} \leq \tilde{\zeta} * \vartheta$.

Definition 2.2 [13] If the binary operation $\star: [0, 1] \times [0, 1] \rightarrow [0, 1]$ fulfils the following claims, it is referred to as a continuous t-conorm:

- (i) $\mathfrak{z} \star \tilde{\zeta} = \tilde{\zeta} \star \mathfrak{z}$ for all $\mathfrak{z}, \tilde{\zeta} \in [0, 1]$;
- (ii) $\star$ is continuous;
- (iii) $\mathfrak{z} \star 0 = 0$;
- (iv) $\star$ is associative;
- (v) If $\mathfrak{z} \leq \tilde{\zeta}$ and $\mathfrak{d} \leq \vartheta$, with $\mathfrak{z}, \tilde{\zeta}, \mathfrak{d}, \vartheta \in [0, 1]$, then $\mathfrak{z} \star \mathfrak{d} \leq \tilde{\zeta} \star \vartheta$.

Definition 2.3 Let $\Xi \neq \emptyset$ and $\ast, \star$ are continuous t-norm and continuous t-conorm and $\ell \geq 1$ be a given real number, $\tilde{\mathfrak{P}}, \tilde{\mathfrak{L}}$ and $\tilde{\mathfrak{Q}}$ are neutrosophic sets on $\Xi^2 \times (0, \infty)$ fulfilling the following assertions. Then $(\Xi, \tilde{\mathfrak{P}}, \tilde{\mathfrak{L}}, \tilde{\mathfrak{Q}}, \ast, \star)$ named a Neutrosophic b-Metric Space[NbMS]. For all $e, \dot{a}, w \in \Xi$ and $\tau, \tau' > 0$.

- (Nb1) $\tilde{\mathfrak{P}}(e, \dot{a}, \tau) + \tilde{\mathfrak{L}}(e, \dot{a}, \tau) + \tilde{\mathfrak{Q}}(e, \dot{a}, \tau) \leq 3$;
- (Nb2) $0 < \tilde{\mathfrak{P}}(e, \dot{a}, \tau)$;
- (Nb3) $\tilde{\mathfrak{P}}(e, \dot{a}, \tau) = 1$ for every $\tau > 0 \Leftrightarrow e = \dot{a}$;
- (Nb4) $\tilde{\mathfrak{P}}(e, \dot{a}, \tau) = \tilde{\mathfrak{P}}(\dot{a}, e, \tau)$;
- (Nb5) $\tilde{\mathfrak{P}}\left(e, \dot{a}, \frac{\tau}{\ell}\right) \ast \tilde{\mathfrak{P}}\left(\dot{a}, w, \frac{\tau'}{\ell}\right) \leq \tilde{\mathfrak{P}}(e, \dot{a}, \tau + \tau')$;
- (Nb6) $\tilde{\mathfrak{P}}(e, \dot{a}, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous;
- (Nb7) $\tilde{\mathfrak{L}}(e, \dot{a}, \tau) < 1$;
- (Nb8) $\tilde{\mathfrak{L}}(e, \dot{a}, \tau) = 0$, for all $\tau \in (0, \infty) \Leftrightarrow e = \dot{a}$;
- (Nb9) $\tilde{\mathfrak{L}}(e, \dot{a}, \tau) = \tilde{\mathfrak{L}}(\dot{a}, e, \tau)$;
- (Nb10) $\tilde{\mathfrak{L}}\left(e, \dot{a}, \frac{\tau}{\ell}\right) \star \tilde{\mathfrak{L}}\left(\dot{a}, w, \frac{\tau'}{\ell}\right) \geq \tilde{\mathfrak{L}}(e, w, \tau + \tau')$;
- (Nb11) $\tilde{\mathfrak{L}}(e, \dot{a}, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous;
- (Nb12) $\tilde{\mathfrak{Q}}(e, \dot{a}, \tau)$;
- (Nb13) $\tilde{\mathfrak{Q}}(e, \dot{a}, \tau) = 0$, for all $\tau \in (0, \infty) \Leftrightarrow e = \dot{a}$;
- (Nb14) $\tilde{\mathfrak{Q}}(e, \dot{a}, \tau) = \tilde{\mathfrak{Q}}(\dot{a}, e, \tau)$;
- (Nb15) $\tilde{\mathfrak{Q}}\left(e, \dot{a}, \frac{\tau}{\ell}\right) \star \tilde{\mathfrak{Q}}\left(\dot{a}, w, \frac{\tau'}{\ell}\right) \geq \tilde{\mathfrak{Q}}(e, w, \tau + \tau')$;
- (Nb16) $\tilde{\mathfrak{Q}}(e, \dot{a}, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous.

Definition 2.4

Consider two non-comparable functions $\rho, \bar{\Gamma} : \Xi^2 \rightarrow [1, \infty)$. Let Ξ be a nonvoid set, and $\ast, \star$ are continuous t-norm and continuous t-conorm, $\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{L}}_{\rho, \bar{\Gamma}}$ and $\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}$ are neutrosophic sets on $\Xi^2 \times (0, \infty)$ fulfilling the following assertions. Then $(\Xi, \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{L}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, \ast, \star)$ named a Neutrosophic Double controlled metric space [NDCMS]. For every $e, \dot{a}, \dot{b} \in \Xi$ and $\tau, \tau' > 0$.

- (1) $\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau) + \tilde{\mathfrak{L}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau) + \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau) \leq 3$;
- (2) $0 < \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau)$;
- (3) $\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau) = 1$ for every $\tau > 0 \Leftrightarrow e = \dot{a}$;
- (4) $\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau) = \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(\dot{a}, e, \tau)$;
- (5) $\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) \ast \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau'}{\bar{\Gamma}(e, \dot{a})}\right) \leq \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau + \tau')$;

- (6) $\tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}(e, \dot{a}, \cdot)$ is continuous from $(0, \infty)$ to $[0, 1]$;
- (7) $\tilde{\mathfrak{X}}_{\rho,\bar{\Gamma}}(e, \dot{a}, \tau) < 1$;
- (8) $\tilde{\mathfrak{X}}_{\rho,\bar{\Gamma}}(e, \dot{a}, \tau) = 0$, for all $\tau \in (0, \infty) \Leftrightarrow e = \dot{a}$;
- (9) $\tilde{\mathfrak{X}}_{\rho,\bar{\Gamma}}(e, \dot{a}, \tau) = \tilde{\mathfrak{X}}_{\rho,\bar{\Gamma}}(\dot{a}, e, \tau)$;
- (10) $\tilde{\mathfrak{X}}_{\rho,\bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) \star \tilde{\mathfrak{X}}_{\rho,\bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau'}{\bar{\Gamma}(e, \dot{a})}\right) \geq \tilde{\mathfrak{X}}_{\rho,\bar{\Gamma}}(e, \dot{b}, \tau + \tau')$;
- (11) $\tilde{\mathfrak{X}}_{\rho,\bar{\Gamma}}(e, \dot{a}, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous;
- (12) $\tilde{\mathfrak{Q}}_{\rho,\bar{\Gamma}}(e, \dot{a}, \tau)$;
- (13) $\tilde{\mathfrak{Q}}_{\rho,\bar{\Gamma}}(e, \dot{a}, \tau) = 0$, for all $\tau \in (0, \infty) \Leftrightarrow e = \dot{a}$;
- (14) $\tilde{\mathfrak{Q}}_{\rho,\bar{\Gamma}}(e, \dot{a}, \tau) = \tilde{\mathfrak{Q}}_{\rho,\bar{\Gamma}}(\dot{a}, e, \tau)$;
- (15) $\tilde{\mathfrak{Q}}_{\rho,\bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) \star \tilde{\mathfrak{Q}}_{\rho,\bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau'}{\bar{\Gamma}(e, \dot{a})}\right) \geq \tilde{\mathfrak{Q}}_{\rho,\bar{\Gamma}}(e, \dot{b}, \tau + \tau')$;
- (16) $\tilde{\mathfrak{Q}}_{\rho,\bar{\Gamma}}(e, \dot{a}, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous.

Remark 2.5

Taking $\rho(e, \dot{a}) = \bar{\Gamma}(e, \dot{a}) = \mathcal{L}$, one may see that the class of NDCMS spaces is a super space of class of neutrosophic b-metric spaces. Additionally, assuming $\mathcal{L} = 1$, the class of NbMS is effectively superior to neutrosophic metric spaces.

Example 2.6 Let $\Xi = [0, 1]$, and define $\rho, \bar{\Gamma} : \Xi^2 \rightarrow [1, \infty)$ by $\rho(e, \dot{a}) = 2(e + \dot{a})$ and $\bar{\Gamma}(e, \dot{a}) = 2(e^2 + \dot{a}^2 + 1)$. Let $\tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}(e, \dot{a}, \tau) = e^{\frac{-(e-\dot{a})^2}{\tau}}$, $\tilde{\mathfrak{X}}_{\rho,\bar{\Gamma}}(e, \dot{a}, \tau) = 1 - e^{\frac{-(e-\dot{a})^2}{\tau}}$ and $\tilde{\mathfrak{Q}}_{\rho,\bar{\Gamma}}(e, \dot{a}, \tau) = e^{\frac{(e-\dot{a})^2}{\tau}} - 1$, for all $e, \dot{a}, \dot{b} \in \Xi$ and $\tau, \tau' > 0$. Then $(\Xi, \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}, \tilde{\mathfrak{X}}_{\rho,\bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho,\bar{\Gamma}}, \star, \star)$ is an NDCMS with the triangular norm $e \star \dot{a} = e\dot{a}$ and triangular co-norm $e \star \dot{a} = \max\{e, \dot{a}\}$. We need check the triangle inequality only, because remaining conditions are straightforward.

$$\tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) = e^{\frac{-(e-\dot{a})^2}{\frac{\tau}{\rho(e, \dot{a})}}} = e^{\frac{-(e-\dot{a})^2}{2(e+\dot{a})}} \leq e^{\frac{-(e-\dot{a})^2}{\tau}} \leq e^{\frac{-(e-\dot{a})^2}{\tau+\tau'}}.$$

$$\text{Moreover, } \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau'}{\bar{\Gamma}(\dot{a}, \dot{b})}\right) = e^{\frac{-(\dot{a}-\dot{b})^2}{\frac{\tau'}{\bar{\Gamma}(\dot{a}, \dot{b})}}} = e^{\frac{-(\dot{a}-\dot{b})^2}{2(\dot{a}^2+\dot{b}^2+1)}} \leq e^{\frac{-(\dot{a}-\dot{b})^2}{\tau'}} \leq e^{\frac{-(\dot{a}-\dot{b})^2}{\tau+\tau'}}.$$

$$\text{Observe } \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}(e, \dot{b}, \tau + \tau') = e^{\frac{-(e-\dot{b})^2}{\tau+\tau'}} = e^{\frac{-(e-\dot{a}+\dot{a}-\dot{b})^2}{\tau+\tau'}} \geq e^{\frac{-(e-\dot{a})^2}{\tau}} e^{\frac{-(\dot{a}-\dot{b})^2}{\tau+\tau'}} \geq \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) \star \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau'}{\bar{\Gamma}(\dot{a}, \dot{b})}\right).$$

$$\tilde{\mathfrak{X}}_{\rho,\bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) = e^{\frac{-(e-\dot{a})^2}{\frac{\tau}{\rho(e, \dot{a})}}} = e^{\frac{-(e-\dot{a})^2}{2(e+\dot{a})}} \leq e^{\frac{-(e-\dot{a})^2}{\tau}} \leq e^{\frac{-(e-\dot{a})^2}{\tau+\tau'}}.$$

$$\text{Moreover, } \tilde{\mathfrak{X}}_{\rho,\bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau'}{\bar{\Gamma}(\dot{a}, \dot{b})}\right) = 1 - e^{\frac{-(\dot{a}-\dot{b})^2}{\frac{\tau'}{\bar{\Gamma}(\dot{a}, \dot{b})}}} = 1 - e^{\frac{-(\dot{a}-\dot{b})^2}{2(\dot{a}^2+\dot{b}^2+1)}} \leq 1 - e^{\frac{-(\dot{a}-\dot{b})^2}{\tau'}} \leq 1 - e^{\frac{-(\dot{a}-\dot{b})^2}{\tau+\tau'}}.$$

$$\text{Observe } \tilde{\mathfrak{X}}_{\rho,\bar{\Gamma}}(e, \dot{b}, \tau + \tau') = 1 - e^{\frac{-(e-\dot{b})^2}{\tau+\tau'}} = 1 - e^{\frac{-(e-\dot{a}+\dot{a}-\dot{b})^2}{\tau+\tau'}} \leq \max\{1 - e^{\frac{-(e-\dot{a})^2}{\tau}}, 1 - e^{\frac{-(\dot{a}-\dot{b})^2}{\tau+\tau'}}\} \geq \tilde{\mathfrak{X}}_{\rho,\bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) \star \tilde{\mathfrak{X}}_{\rho,\bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau'}{\bar{\Gamma}(\dot{a}, \dot{b})}\right).$$

$$\tilde{\mathfrak{Q}}_{\rho,\bar{\Gamma}}\left(\dot{a}, \dot{w}, \frac{\tau'}{\bar{\Gamma}(\dot{a}, \dot{b})}\right) = e^{\frac{(\dot{a}-\dot{b})^2}{\frac{\tau'}{\bar{\Gamma}(\dot{a}, \dot{b})}}} - 1 = e^{\frac{(\dot{a}-\dot{b})^2}{2(\dot{a}^2+\dot{b}^2+1)}} - 1 \leq e^{\frac{(\dot{a}-\dot{b})^2}{\tau'}} - 1 \leq 1 - e^{\frac{-(\dot{a}-\dot{b})^2}{\tau+\tau'}}.$$

$$\text{Observe } \tilde{\mathfrak{Q}}_{\rho,\bar{\Gamma}}(e, \dot{b}, \tau + \tau') = e^{\frac{(e-\dot{b})^2}{\tau+\tau'}} - 1 = e^{\frac{(e-\dot{a}+\dot{a}-\dot{b})^2}{\tau+\tau'}} - 1 \leq \max\{e^{\frac{(e-\dot{a})^2}{\tau}} - 1, e^{\frac{(\dot{a}-\dot{b})^2}{\tau+\tau'}} - 1\} \geq \tilde{\mathfrak{Q}}_{\rho,\bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) \star \tilde{\mathfrak{Q}}_{\rho,\bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau'}{\bar{\Gamma}(\dot{a}, \dot{b})}\right). \text{ Thus, } (\Xi, \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}, \tilde{\mathfrak{X}}_{\rho,\bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho,\bar{\Gamma}}, \star, \star) \text{ is a NDCMS, which is not a NbMS.}$$

Example 2.7 Let $\Xi = [0,1]$ and let $\ell = 4$. Define $\tilde{\mathfrak{P}}(e, \dot{a}, \tau) = e^{\frac{-(e-\dot{a})^2}{\tau}}$, $\tilde{\mathfrak{Q}}(e, \dot{a}, \tau) = 1 - e^{\frac{-(e-\dot{a})^2}{\tau}}$ and $\tilde{\mathfrak{Q}}(e, \dot{a}, \tau) = e^{\frac{-(e-\dot{a})^2}{\tau}} - 1$, for all $e, \dot{a}, \dot{b} \in \Xi$ and $\tau > 0$. Then $(\Xi, \tilde{\mathfrak{P}}, \tilde{\mathfrak{Q}}, \tilde{\mathfrak{Q}}, *, *, \ell)$ is an NbMS that is not a neutrosophic metric space. We need only prove (Nb5), (Nb10) and (Nb15), because the remaining conditions are straightforward.

$$\frac{(e-\dot{b})^2}{\tau+\tau'} \leq 4 \frac{(e-\dot{a})^2}{\tau+\tau'} + 4 \frac{(\dot{a}-\dot{b})^2}{\tau+\tau'} \leq \frac{(e-\dot{a})^2}{\frac{\tau}{4}} + \frac{(\dot{a}-\dot{b})^2}{\frac{\tau'}{4}}.$$

$$\text{Hence, } \tilde{\mathfrak{P}}(e, \dot{b}, \tau + \tau') = e^{\frac{-(e-\dot{b})^2}{\tau+\tau'}} \geq e^{\frac{-(e-\dot{a})^2}{\tau}} e^{\frac{-(\dot{a}-\dot{b})^2}{\tau'}} \geq \tilde{\mathfrak{P}}(e, \dot{a}, \frac{\tau}{4}) * \tilde{\mathfrak{P}}(\dot{a}, \dot{b}, \frac{\tau'}{4}).$$

This shows that $(\Xi, \tilde{\mathfrak{P}}, \tilde{\mathfrak{Q}}, \tilde{\mathfrak{Q}}, *, *, \ell)$ is a NbMS that is not a neutrosophic metric space.

Definition 2.8 Let $(\Xi, \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, *, *)$ be a NDCMS. Then the following:

1. If $\lim_{i \rightarrow \infty} \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e_i, e, \tau) = 1$, $\lim_{i \rightarrow \infty} \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e_i, e, \tau) = 0$ and $\lim_{i \rightarrow \infty} \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e_i, e, \tau) = 0$ then the sequence $\{e_i\}$ converges to e belongs to Ξ
2. A sequence $\{e_i\}$ is called Cauchy, $\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e_i, e_{i+j}, \tau) = 1$, $\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e_i, e_{i+j}, \tau) = 0$ and $\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e_i, e_{i+j}, \tau) = 0$, for each $i, j \in \mathbb{N}$ and $\tau > 0$.
3. If every Cauchy sequence is convergent in $(\Xi, \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, *, *)$, then it is named complete NDCMS.
4. The open ball $\mathcal{B}(e_0, r, \tau)$ with centre e_0 and radius $r \in (0,1)$ and $\tau > 0$ is defined as follows:

$$\mathcal{B}(e_0, r, \tau) = \{e \in \Xi: \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e_i, e, \tau) > 1 - r, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e_i, e, \tau) < r \text{ and } \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e_i, e, \tau) < r\}.$$

Definition 2.9 Let $(\Xi, \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, *, *)$ be a NDCMS. $(\Xi, \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, *, *)$ is referred to be triangular if it fulfils the condition that is stated below :

$$\left(\frac{1}{\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau)} - 1\right) \leq \left(\frac{1}{\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau)} - 1\right) + \left(\frac{1}{\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(\dot{a}, \dot{b}, \tau)} - 1\right), \quad (1)$$

$$\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau) \geq \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau) + \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(\dot{a}, \dot{b}, \tau) \text{ and} \quad (2)$$

$$\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau) \geq \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau) + \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(\dot{a}, \dot{b}, \tau). \quad (3)$$

Lemma 2.10 Let $\{e_i\}$ be a Cauchy sequence in NDCMS $(\Xi, \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, *, *)$ such that $e_i \neq e_j$ whenever $i, j \in \mathbb{N}$. Then, limit point of the sequence $\{e_i\}$ is unique.

Main Results

Definition 3.1 Let $(\Xi, \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, *, *)$ be a NDCMS. If there exists $\alpha: \Xi^2 \times (0, \infty) \rightarrow [0, \infty)$, such that for all $\tau > 0$, $e, \dot{a} \in \Xi$, $\alpha(e, \dot{a}, \tau) \geq 1 \Rightarrow \alpha(Je, J\dot{a}, \tau) \geq 1$. (4) Then we say that $J: \Xi \rightarrow \Xi$ is α -admissible.

Let $\mathfrak{F}$ denote the family of all right continuous function $\Psi: [0, \infty) \rightarrow [0, \infty)$ with $\Psi(r) < r$ for all $r > 0$.

Remark 3.2 For any $\Psi \in \mathfrak{F}$, then $\lim_{n \rightarrow \infty} \Psi^n(r) = 0$, for all $r > 0$, where Ψ^n is the n -th iterate of Ψ .

Definition 3.3 Let $(\Xi, \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, *, *)$ be a NDCMS, and we say that $J: \Xi \rightarrow \Xi$ is a NDCMS, and we say $J: \Xi \rightarrow \Xi$ is an $(\alpha - \Psi)$ -Neutrosophic contractive mapping if there exists two functions $\Psi \in \mathfrak{F}$ and $\alpha: \Xi^2 \times (0, \infty) \rightarrow [0, \infty)$ such that $\alpha(e, \dot{a}, \tau) \left(\frac{1}{\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(Je, J\dot{b}, \tau)} - 1\right) \leq \Psi\left(\frac{1}{\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau)} - 1\right)$ (5)

$$\alpha(e, \dot{a}, \tau) \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(Je, J\dot{b}, \tau) \leq \Psi(\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau)) \quad (6)$$

$$\alpha(e, \dot{a}, \tau) \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(Je, J\dot{b}, \tau) \leq \Psi(\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau)) \quad (7)$$

for all $e, \dot{a}, \dot{b} \in \Xi$ and $\tau > 0$.

Theorem 3.4 Let $(\Xi, \mathfrak{F}_{\rho, \bar{\Gamma}}, \mathfrak{L}_{\rho, \bar{\Gamma}}, \mathfrak{Q}_{\rho, \bar{\Gamma}}, *, \star)$ be a complete NDCMS, $\rho, \bar{\Gamma} : \Xi^2 \rightarrow [1, \frac{1}{\delta})$ are two non-comparable functions, where $0 < \delta < 1$. Let $\mathcal{J} : \Xi \rightarrow \Xi$ be a $(\alpha - \Psi)$ - Neutrosophic contractive mapping, for some $\Psi \in \mathfrak{F}$, fulfills the requirements listed below:

1. $\mathcal{J}$ is α - admissible,
2. There exists $e_0 \in \Xi$ such that $\alpha(e_0, \mathcal{J}e_0, \tau) \geq 1$ for all $\tau > 0$,
3. $\mathcal{J}$ is continuous,
4. For any $e \in \Xi$, both $\lim_{i \rightarrow \infty} \rho(e_i, e)$, and $\lim_{n \rightarrow \infty} \bar{\Gamma}(e_i, e)$ exists and are finite, where the sequence $\{e_i\}$ is defined as $e_i = \mathcal{J}^n e_0$, for some $e_0 \in \Xi$.

(8)

Then, $\mathcal{J}$ admits a fixed point.

Proof. Let $e_0 \in \Xi$ so that $\alpha(e_0, \mathcal{J}e_0, \tau) \geq 1$ for all $\tau > 0$, and we have a sequence $\{e_i\}$ in Ξ with $e_i = \mathcal{J}^n e_0$, for all $i \in \mathbb{N}$.

Note that if $e_j = e_{j+1}$ for some $j \in \mathbb{N}$, then this implies that $\mathcal{J}^m e_0$ is a fixed point of the mapping $\mathcal{J}$. Thus, without loss of generality, we may assume that $e_i \neq e_{i+1}$ for every $i \in \mathbb{N}$.

According to the hypothesis, $\alpha(e_0, e_1, \tau) = \alpha(e_0, \mathcal{J}e_0, \tau) \geq 1$, as $\mathcal{J}$ is α - admissible, this implies that $\alpha(\mathcal{J}e_0, \mathcal{J}e_1, \tau) = \alpha(e_1, e_2, \tau) \geq 1$. By induction, we get

$$\alpha(e_i, e_{i+1}, \tau) \geq 1, \text{ for every } i \in \mathbb{N}, \text{ and for every } \tau > 0. \quad (9)$$

Thus, using equation (5), (6), (7) and (9), we obtain

$$\begin{aligned} \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(e_i, e_{i+1}, \tau)} - 1 \right) &= \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(\mathcal{J}e_{i-1}, \mathcal{J}e_i, \tau)} - 1 \right) \leq \alpha(e_{i-1}, e_i, \tau) \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(\mathcal{J}e_{i-1}, \mathcal{J}e_i, \tau)} - 1 \right) \\ &\leq \Psi \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(e_{i-1}, e_i, \tau)} - 1 \right), \text{ repeating the process.} \\ &\leq \Psi \left(\Psi \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(e_{i-1}, e_i, \tau)} - 1 \right) \right) = \Psi^2 \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(e_{i-2}, e_{i-1}, \tau)} - 1 \right) \\ &\leq \dots \leq \Psi^n \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(e_0, e_1, \tau)} - 1 \right). \end{aligned} \quad (10)$$

Utilizing the fact that $\lim_{n \rightarrow \infty} \Psi^n(r) = 0$ and $r = \frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(e_0, e_1, \tau)} - 1$. Letting $i \rightarrow \infty$ in equation (10).

$$\text{we get } \lim_{i \rightarrow \infty} \mathfrak{F}_{\rho, \bar{\Gamma}}(e_0, e_1, \tau) = 1 \text{ for all } \tau > 0. \quad (11)$$

$$\begin{aligned} \mathfrak{L}_{\rho, \bar{\Gamma}}(e_i, e_{i+1}, \tau) &= \mathfrak{L}_{\rho, \bar{\Gamma}}(\mathcal{J}e_{i-1}, \mathcal{J}e_i, \tau) \leq \alpha(e_{i-1}, e_i, \tau) \mathfrak{L}_{\rho, \bar{\Gamma}}(\mathcal{J}e_{i-1}, \mathcal{J}e_i, \tau) \\ &\leq \Psi(\mathfrak{L}_{\rho, \bar{\Gamma}}(e_{i-1}, e_i, \tau)), \text{ repeating the process.} \\ &\leq \Psi(\Psi(\mathfrak{L}_{\rho, \bar{\Gamma}}(e_{i-1}, e_i, \tau))) = \Psi^2(\mathfrak{L}_{\rho, \bar{\Gamma}}(e_{i-2}, e_{i-1}, \tau)) \\ &\leq \Psi^n(\mathfrak{L}_{\rho, \bar{\Gamma}}(e_0, e_1, \tau)). \end{aligned} \quad (12)$$

Using the fact that $\lim_{n \rightarrow \infty} \Psi^n(r) = 0$ and $r = \mathfrak{L}_{\rho, \bar{\Gamma}}(e_0, e_1, \tau)$. Letting $i \rightarrow \infty$ in equation (12).

$$\text{we obtain } \lim_{i \rightarrow \infty} \mathfrak{L}_{\rho, \bar{\Gamma}}(e_0, e_1, \tau) = 0 \text{ for all } \tau > 0. \quad (13)$$

$$\mathfrak{Q}_{\rho, \bar{\Gamma}}(e_i, e_{i+1}, \tau) = \mathfrak{Q}_{\rho, \bar{\Gamma}}(\mathcal{J}e_{i-1}, \mathcal{J}e_i, \tau) \leq \alpha(e_{i-1}, e_i, \tau) \mathfrak{Q}_{\rho, \bar{\Gamma}}(\mathcal{J}e_{i-1}, \mathcal{J}e_i, \tau)$$

$$\begin{aligned} &\leq \Psi(\tilde{\mathcal{Q}}_{\rho,\bar{\Gamma}}(e_{i-1}, e_i, \tau)), \text{ repeating the process.} \\ &\leq \Psi(\Psi(\tilde{\mathcal{Q}}_{\rho,\bar{\Gamma}}(e_{i-1}, e_i, \tau))) = \Psi^2(\tilde{\mathcal{Q}}_{\rho,\bar{\Gamma}}(e_{i-2}, e_{i-1}, \tau)) \\ &\leq \Psi^n(\tilde{\mathcal{Q}}_{\rho,\bar{\Gamma}}(e_0, e_1, \tau)). \end{aligned} \quad (14)$$

Using the fact that $\lim_{n \rightarrow \infty} \Psi^n(r) = 0$ and $r = \tilde{\mathcal{Q}}_{\rho,\bar{\Gamma}}(e_0, e_1, \tau)$. Letting $i \rightarrow \infty$ in equation (14).

$$\text{we obtain } \lim_{i \rightarrow \infty} \tilde{\mathcal{Q}}_{\rho,\bar{\Gamma}}(e_0, e_1, \tau) = 0 \text{ for every } \tau > 0. \quad (15)$$

For any $i, j \in \mathbb{N}$, with $j > i$, then

$$\begin{aligned} \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}(e_i, e_j, \tau) &\geq \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(e_i, e_{i+1}, \frac{\tau}{\rho(e_i, e_{i+1})}\right) * \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(e_{i+1}, e_j, \frac{\tau}{\bar{\Gamma}(e_{i+1}, e_j)}\right) \\ &\geq \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(e_i, e_{i+1}, \frac{\tau}{\rho(e_i, e_{i+1})}\right) * \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(e_{i+1}, e_{i+2}, \frac{\tau}{\rho(e_{i+1}, e_{i+2})\bar{\Gamma}(e_{i+1}, e_j)}\right) \\ &\quad * \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(e_{i+2}, e_j, \frac{\tau}{\bar{\Gamma}(e_{i+1}, e_j)\bar{\Gamma}(e_{i+2}, e_j)}\right) \\ &\geq \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(e_i, e_{i+1}, \frac{\tau}{\bar{\Gamma}(e_i, e_{i+1})}\right) * \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(e_{i+1}, e_{i+2}, \frac{\tau}{\rho(e_{i+1}, e_{i+2})\bar{\Gamma}(e_{i+1}, e_j)}\right) \\ &\quad * \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(e_{i+2}, e_{i+3}, \frac{\tau}{\rho(e_{i+2}, e_{i+3})\bar{\Gamma}(e_{i+1}, e_j)}\right) * \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(e_{i+3}, e_j, \frac{\tau}{\bar{\Gamma}(e_{i+1}, e_j)\bar{\Gamma}(e_{i+2}, e_j)\bar{\Gamma}(e_{i+3}, e_j)}\right) \\ \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}(e_i, e_j, \tau) &\geq \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(e_i, e_{i+1}, \frac{\tau}{2\rho(e_i, e_{i+1})}\right) ** \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(e_{i+2}, e_{i+3}, \frac{\tau}{2^3\rho(e_{i+1}, e_{i+2})\bar{\Gamma}(e_{i+1}, e_j)}\right) \dots * \\ &\quad \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(e_{i-2}, e_{j-1}, \frac{\tau}{2^{j-1}\rho(e_{j-2}, e_{j-1})\bar{\Gamma}(e_{j-2}, e_j)\bar{\Gamma}(e_{j-3}, e_j)\dots\bar{\Gamma}(e_{j+1}, e_j)}\right) * \\ &\quad \tilde{\mathfrak{P}}_{\rho,\bar{\Gamma}}\left(e_{j-1}, e_j, \frac{\tau}{2^{j-1}\bar{\Gamma}(e_{j-1}, e_j)\bar{\Gamma}(e_{j-2}, e_j)\bar{\Gamma}(e_{j-3}, e_j)\dots\bar{\Gamma}(e_{j+1}, e_j)}\right). \quad (16) \\ \tilde{\mathfrak{L}}_{\rho,\bar{\Gamma}}(e_i, e_j, \tau) &\leq \tilde{\mathfrak{L}}_{\rho,\bar{\Gamma}}\left(e_i, e_{i+1}, \frac{\tau}{\rho(e_i, e_{i+1})}\right) \star \tilde{\mathfrak{L}}_{\rho,\bar{\Gamma}}\left(e_{i+1}, e_j, \frac{\tau}{\bar{\Gamma}(e_{i+1}, e_j)}\right) \\ &\leq \tilde{\mathfrak{L}}_{\rho,\bar{\Gamma}}\left(e_i, e_{i+1}, \frac{\tau}{\rho(e_i, e_{i+1})}\right) \star \tilde{\mathfrak{L}}_{\rho,\bar{\Gamma}}\left(e_{i+1}, e_{i+2}, \frac{\tau}{\rho(e_{i+1}, e_{i+2})\bar{\Gamma}(e_{i+1}, e_j)}\right) \\ &\quad \star \tilde{\mathfrak{L}}_{\rho,\bar{\Gamma}}\left(e_{i+2}, e_j, \frac{\tau}{\bar{\Gamma}(e_{i+1}, e_j)\bar{\Gamma}(e_{i+2}, e_j)}\right) \\ &\leq \tilde{\mathfrak{L}}_{\rho,\bar{\Gamma}}\left(e_i, e_{i+1}, \frac{\tau}{\rho(e_i, e_{i+1})}\right) \star \tilde{\mathfrak{L}}_{\rho,\bar{\Gamma}}\left(e_{i+1}, e_{i+2}, \frac{\tau}{\rho(e_{i+1}, e_{i+2})\bar{\Gamma}(e_{i+1}, e_j)}\right) \\ &\quad \star \tilde{\mathfrak{L}}_{\rho,\bar{\Gamma}}\left(e_{i+2}, e_{i+3}, \frac{\tau}{\rho(e_{i+2}, e_{i+3})\bar{\Gamma}(e_{i+1}, e_j)}\right) \star \\ &\quad \tilde{\mathfrak{L}}_{\rho,\bar{\Gamma}}\left(e_{i+3}, e_j, \frac{\tau}{\bar{\Gamma}(e_{i+1}, e_j)\bar{\Gamma}(e_{i+2}, e_j)\bar{\Gamma}(e_{i+3}, e_j)}\right). \end{aligned}$$

$$\begin{aligned} \tilde{\mathfrak{L}}_{\rho, \bar{\Gamma}}(e_i, e_j, \tau) &\leq \tilde{\mathfrak{L}}_{\rho, \bar{\Gamma}}\left(e_i, e_{i+1}, \frac{\tau}{2\rho(e_i, e_{i+1})}\right) \\ &\star \tilde{\mathfrak{L}}_{\rho, \bar{\Gamma}}\left(e_{i+1}, e_{i+2}, \frac{\tau}{2^2\rho(e_{i+1}, e_{i+2})\bar{\Gamma}(e_{i+1}, e_j)}\right) \star \tilde{\mathfrak{L}}_{\rho, \bar{\Gamma}}\left(e_{i+2}, e_{i+3}, \frac{\tau}{2^3\rho(e_{i+1}, e_{i+2})\bar{\Gamma}(e_{i+1}, e_j)}\right) \dots \star \\ &\tilde{\mathfrak{L}}_{\rho, \bar{\Gamma}}\left(e_{j-2}, e_{j-1}, \frac{\tau}{2^{j-1}\rho(e_{j-2}, e_{j-1})\bar{\Gamma}(e_{j-2}, e_j)\bar{\Gamma}(e_{j-3}, e_j) \dots \bar{\Gamma}(e_{i+1}, e_j)}\right) \star \\ &\tilde{\mathfrak{L}}_{\rho, \bar{\Gamma}}\left(e_{j-1}, e_j, \frac{\tau}{2^{j-1}\bar{\Gamma}(e_{j-1}, e_j)\bar{\Gamma}(e_{j-2}, e_j)\bar{\Gamma}(e_{j-3}, e_j) \dots \bar{\Gamma}(e_{i+1}, e_j)}\right) \end{aligned} \quad (17)$$

$$\begin{aligned} \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e_i, e_j, \tau) &\leq \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e_i, e_{i+1}, \frac{\frac{\tau}{2}}{\rho(e_i, e_{i+1})}\right) \star \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e_{i+1}, e_j, \frac{\frac{\tau}{2}}{\bar{\Gamma}(e_{i+1}, e_j)}\right) \\ &\leq \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e_i, e_{i+1}, \frac{\frac{\tau}{2}}{\rho(e_i, e_{i+1})}\right) \star \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e_{i+1}, e_{i+2}, \frac{\frac{\tau}{2^2}}{\rho(e_{i+1}, e_{i+2})\bar{\Gamma}(e_{i+1}, e_j)}\right) \\ &\star \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e_{i+2}, e_j, \frac{\frac{\tau}{2^2}}{\bar{\Gamma}(e_{i+1}, e_j)\bar{\Gamma}(e_{i+2}, e_j)}\right) \\ &\leq \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e_i, e_{i+1}, \frac{\frac{\tau}{2}}{\rho(e_i, e_{i+1})}\right) \star \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e_{i+1}, e_{i+2}, \frac{\frac{\tau}{2^2}}{\rho(e_{i+1}, e_{i+2})\bar{\Gamma}(e_{i+1}, e_j)}\right) \\ &\star \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e_{i+2}, e_{i+3}, \frac{\frac{\tau}{2^3}}{\rho(e_{i+2}, e_{i+3})\bar{\Gamma}(e_{i+1}, e_j)}\right) \star \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e_{i+3}, e_j, \frac{\frac{\tau}{2^3}}{\bar{\Gamma}(e_{i+1}, e_j)\bar{\Gamma}(e_{i+2}, e_j)\bar{\Gamma}(e_{i+3}, e_j)}\right). \\ \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e_i, e_j, \tau) &\leq \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e_i, e_{i+1}, \frac{\tau}{2\rho(e_i, e_{i+1})}\right). \\ &\star \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e_{i+1}, e_{i+2}, \frac{\tau}{2^2\rho(e_{i+1}, e_{i+2})\bar{\Gamma}(e_{i+1}, e_j)}\right) \star \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e_{i+2}, e_{i+3}, \frac{\tau}{2^3\rho(e_{i+1}, e_{i+2})\bar{\Gamma}(e_{i+1}, e_j)}\right) \dots \star \\ &\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e_{j-2}, e_{j-1}, \frac{\tau}{2^{j-1}\rho(e_{j-2}, e_{j-1})\bar{\Gamma}(e_{j-2}, e_j)\bar{\Gamma}(e_{j-3}, e_j) \dots \bar{\Gamma}(e_{i+1}, e_j)}\right) \star \\ &\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e_{j-1}, e_j, \frac{\tau}{2^{j-1}\bar{\Gamma}(e_{j-1}, e_j)\bar{\Gamma}(e_{j-2}, e_j)\bar{\Gamma}(e_{j-3}, e_j) \dots \bar{\Gamma}(e_{i+1}, e_j)}\right). \end{aligned} \quad (18)$$

Taking the limit as $i \rightarrow \infty$ in (16), (17), (18) and using equation (11), (13), (15) and (8), we obtain $\lim_{i \rightarrow \infty} \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e_i, e_j, \tau) = 1 \star 1 \star \dots \star 1 = 1$, $\lim_{i \rightarrow \infty} \tilde{\mathfrak{L}}_{\rho, \bar{\Gamma}}(e_i, e_j, \tau) = 0 \star 0 \star \dots \star 0 = 0$ and $\lim_{i \rightarrow \infty} \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e_i, e_j, \tau) = 0 \star 0 \star \dots \star 0 = 0$. (19)

This implies that the sequence $\{e_i\}$ is a Cauchy sequence in Ξ , as Ξ is a complete NDCMS, so there exists some $e^* \in \Xi$ such that $e_i \rightarrow e^*$, i.e.,

$$\lim_{i \rightarrow \infty} \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e_i, e^*, \tau) = 1, \lim_{i \rightarrow \infty} \tilde{\mathfrak{L}}_{\rho, \bar{\Gamma}}(e_i, e^*, \tau) = 0 \text{ and } \lim_{i \rightarrow \infty} \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e_i, e^*, \tau) = 0 \quad (20)$$

The continuity of $\mathcal{J}$ implies that $\mathcal{J}(e_i) \rightarrow \mathcal{J}(e^*)$, i.e., $\lim_{i \rightarrow \infty} \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(\mathcal{J}(e_i), \mathcal{J}(e^*), \tau) = 1$, $\lim_{i \rightarrow \infty} \tilde{\mathfrak{L}}_{\rho, \bar{\Gamma}}(\mathcal{J}(e_i), \mathcal{J}(e^*), \tau) = 0$ and $\lim_{i \rightarrow \infty} \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(\mathcal{J}(e_i), \mathcal{J}(e^*), \tau) = 0$ for all $\tau > 0$. Thus, we have

$$\lim_{i \rightarrow \infty} \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e_{i+1}, J(e^*), \tau) = \lim_{i \rightarrow \infty} \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(J(e_i), J(e^*), \tau) = 1, \quad \lim_{i \rightarrow \infty} \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e_{i+1}, J(e^*), \tau) = \lim_{i \rightarrow \infty} \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(J(e_i), J(e^*), \tau) = 0, \quad \text{for all } \tau > 0. \quad (21)$$

This yields $J(e_i) \rightarrow J(e^*)$, and by Lemma(2.10) we obtain $J(e^*) = e^*$, so e^* is an invariant point of J .

Corollary 3.5 Let $(\Xi, \tilde{\mathfrak{P}}, \tilde{\mathfrak{Q}}, \star, \ell)$ be a neutrosophic metric space which is also complete. Let $J : \Xi \rightarrow \Xi$ be a $(\alpha - \Psi)$ - Neutrosophic contractive mapping, for some $\Psi \in \mathfrak{F}$, fulfils the condition stated below:

1. J is continuous.
2. There exists $e_0 \in \Xi$ such that $\alpha(e_0, J(e_0), \tau) \geq 1$ for every $\tau > 0$,
3. J is α - admissible,

Then, J admits a fixed point.

Proof. By applying $\rho(e, \dot{a}) = \bar{\Gamma}(e, \dot{a}) = 1$ in Theorem 3.4 and doing the proof. J fulfils all the conditions of Theorem 3.4. Hence, it has a fixed point.

Example 3.6 Let $\Xi = [0, 1]$, define the control functions, $\rho, \bar{\Gamma} : \Xi^2 \rightarrow [1, \infty)$ as $\rho(e, \dot{a}) = e + \dot{a} + 1$ and $\bar{\Gamma}(e, \dot{a}) = e^2 + \dot{a}^2 + 1$. Define the neutrosophic sets $\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}$ and $\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}$ by

$\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau) = e^{\frac{-|e-\dot{a}|}{\tau}}$, $\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau) = 1 - e^{\frac{-|e-\dot{a}|}{\tau}}$ and $\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau) = e^{\frac{|e-\dot{a}|}{\tau}} - 1$. Then one can easily show that $(\Xi, \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, \star, \ell)$ is a complete NDCMS, and we will check the triangle inequality only.

$$\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) = e^{\frac{-|e-\dot{a}|}{\tau}} = e^{\frac{-\rho(e, \dot{a})|e-\dot{a}|}{\tau}} \leq e^{\frac{-|e-\dot{a}|}{\tau}} \leq e^{\frac{-|e-\dot{a}|}{\tau+\tau'}}.$$

$$\text{Similarly, } \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau}{\rho(\dot{a}, \dot{b})}\right) = e^{\frac{-|\dot{a}-\dot{b}|}{\tau}} = e^{\frac{-\rho(\dot{a}, \dot{b})|\dot{a}-\dot{b}|}{\tau}} \leq e^{\frac{-|\dot{a}-\dot{b}|}{\tau}} \leq e^{\frac{-|\dot{a}-\dot{b}|}{\tau+\tau'}}. \text{ Hence, for } \tau, \tau' > 0,$$

$$\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) \star \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau}{\rho(\dot{a}, \dot{b})}\right) \leq e^{\frac{-|e-\dot{a}|}{\tau+\tau'}} \cdot e^{\frac{-|\dot{a}-\dot{b}|}{\tau+\tau'}} \leq e^{\frac{-|e-\dot{b}|}{\tau+\tau'}} \leq \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau + \tau').$$

$$\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) = 1 - e^{\frac{-|e-\dot{a}|}{\tau}} = 1 - e^{\frac{-\rho(e, \dot{a})|e-\dot{a}|}{\tau}} \geq 1 - e^{\frac{-|e-\dot{a}|}{\tau}} \geq 1 - e^{\frac{-|e-\dot{a}|}{\tau+\tau'}}.$$

$$\text{Similarly, } \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau}{\rho(\dot{a}, \dot{b})}\right) = 1 - e^{\frac{-|\dot{a}-\dot{b}|}{\tau}} = 1 - e^{\frac{-\rho(\dot{a}, \dot{b})|\dot{a}-\dot{b}|}{\tau}} \geq 1 - e^{\frac{-|\dot{a}-\dot{b}|}{\tau}} \geq 1 - e^{\frac{-|\dot{a}-\dot{b}|}{\tau+\tau'}}. \text{ Hence, for } \tau, \tau' > 0,$$

$$\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) \star \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau}{\rho(\dot{a}, \dot{b})}\right) \geq 1 - e^{\frac{-|e-\dot{a}|}{\tau+\tau'}} \cdot 1 - e^{\frac{-|\dot{a}-\dot{b}|}{\tau+\tau'}} \geq 1 - e^{\frac{-|e-\dot{b}|}{\tau+\tau'}} \geq \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau + \tau') \text{ and}$$

$$\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) = e^{\frac{|e-\dot{a}|}{\tau}} - 1 = e^{\frac{\rho(e, \dot{a})|e-\dot{a}|}{\tau}} - 1 \geq e^{\frac{|e-\dot{a}|}{\tau}} - 1 \geq e^{\frac{|e-\dot{a}|}{\tau+\tau'}} - 1.$$

$$\text{Similarly, } \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau}{\rho(\dot{a}, \dot{b})}\right) = e^{\frac{|\dot{a}-\dot{b}|}{\tau}} - 1 = e^{\frac{\rho(\dot{a}, \dot{b})|\dot{a}-\dot{b}|}{\tau}} - 1 \geq e^{\frac{|\dot{a}-\dot{b}|}{\tau}} - 1 \geq e^{\frac{|\dot{a}-\dot{b}|}{\tau+\tau'}} - 1. \text{ Hence, for } \tau, \tau' > 0,$$

$$\begin{aligned} \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) \star \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau}{\rho(\dot{a}, \dot{b})}\right) &\geq e^{\frac{|e-\dot{a}|}{\tau+\tau'}} - 1 \cdot e^{\frac{|\dot{a}-\dot{b}|}{\tau+\tau'}} - 1 \geq e^{\frac{|e-\dot{b}|}{\tau+\tau'}} - 1 \\ &\geq \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau + \tau') \end{aligned}$$

Let $J : \Xi \rightarrow \Xi$, $\alpha : \Xi^2 \times (0, \infty) \rightarrow [0, \infty)$ and $\Psi : [0, \infty) \rightarrow [0, \infty)$ be defined as

$$\mathcal{J}(e, \dot{a}, \tau) = \begin{cases} 1 & \text{if } 0 < e, \dot{a} \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

It is simple to demonstrate that $\mathcal{J}$ is α -admissible and continuous, since for $e, \dot{a}, \dot{b} \in \Xi$, with $\alpha(e, \dot{a}, \tau) \geq 1$, then $\alpha(\mathcal{J}e, \mathcal{J}\dot{a}, \tau) = \alpha(1 - \frac{e}{4}, 1 - \frac{\dot{a}}{4}, \tau) \geq 1$.

We must demonstrate that equation (5), (6), (7) holds in order to demonstrate that $\mathcal{J}$ is a $(\alpha - \Psi)$ -neutrosophic contractive mapping.

$$\alpha(e, \dot{a}, \tau) \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(\mathcal{J}e, \mathcal{J}\dot{b}, \tau)} - 1 \right) \leq \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(\mathcal{J}e, \mathcal{J}\dot{b}, \tau)} - 1 \right) = \left(\frac{1}{e^{\frac{-|\mathcal{J}e - \mathcal{J}\dot{b}|}{\tau}}} - 1 \right) = \left(\frac{1}{e^{\frac{-|e - \dot{b}|}{\tau}}} - 1 \right) \leq \frac{e^{\frac{|e - \dot{b}|}{\tau}} - 1}{2} = \Psi \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau)} - 1 \right).$$

$$\alpha(e, \dot{a}, \tau) \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(\mathcal{J}e, \mathcal{J}\dot{b}, \tau) \leq \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(\mathcal{J}e, \mathcal{J}\dot{b}, \tau) = 1 - e^{\frac{-|\mathcal{J}e - \mathcal{J}\dot{b}|}{\tau}} \leq 1 - e^{\frac{-|e - \dot{b}|}{\tau}} = \Psi(\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau))$$

$$\alpha(e, \dot{a}, \tau) \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(\mathcal{J}e, \mathcal{J}\dot{b}, \tau) \leq \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(\mathcal{J}e, \mathcal{J}\dot{b}, \tau) = e^{\frac{|\mathcal{J}e - \mathcal{J}\dot{b}|}{\tau}} - 1 \leq e^{\frac{|e - \dot{b}|}{\tau}} - 1 = \Psi(\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau)).$$

Thus, $\mathcal{J}$ fulfils the assumptions of Theorem 3.4, so we get a fixed point $e^* = \frac{4}{5}$, such that $\mathcal{J}\left(\frac{4}{5}\right) = \frac{4}{5}$.

Theorem 3.7 Let $(\Xi, \mathfrak{F}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, *, \star)$ be a triangular complete NDCMS, $\rho, \bar{\Gamma} : \Xi^2 \rightarrow [1, \frac{1}{\delta})$ are two non-comparable function (for some $\delta \in (0, 1)$). Let $\mathcal{J} : \Xi \rightarrow \Xi$ be a $(\alpha - \Psi)$ -Neutrosophic contractive mapping, for some $\Psi \in \mathfrak{F}$, fulfils the requirements listed below:

1. $\mathcal{J}$ is α -admissible,
2. There exists $e_0 \in \Xi$ such that $\alpha(e_0, \mathcal{J}e_0, \tau) \geq 1$ for all $\tau > 0$,
3. If $\{e_i\}$ is a sequence in Ξ such that $\alpha(e_i, e_{i+1}, \tau) \geq 1$ for every $i \in \mathbb{N}$ and $e_i \rightarrow e$ as $i \rightarrow \infty$, then $\alpha(e_i, e, \tau) \geq 1$ for all $i \in \mathbb{N}$
4. For any $e \in \Xi$, both $\lim_{i \rightarrow \infty} \rho(e_i, e)$, and $\lim_{i \rightarrow \infty} \bar{\Gamma}(e_i, e)$ exists and are finite, where the sequence $\{e_i\}$ is defined as $e_i = \mathcal{J}^n e_0$, for some $e_0 \in \Xi$.

(22)

Then, $\mathcal{J}$ admits a fixed point.

Proof.

According to the proof of the Theorem 3.4, we get that $\{e_i\}$ is a Cauchy sequence in a complete NDCMS $(\Xi, \mathfrak{F}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, *, \star)$. This implies there exists some $e_0 \in \Xi$ such that $e_i \rightarrow e^*$ as $i \rightarrow \infty$. Thus, by assumption (3),

$$\text{We get } \alpha(e_i, e^*, \tau) \geq 1 \text{ for all } i \in \mathbb{N} \text{ and for all } \tau > 0. \quad (23)$$

Using the fact that $\mathfrak{F}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}$ are triangular and by (22) and equation (5), (6), (7), we have

$$\begin{aligned} \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(\mathcal{J}e^*, e^*, \tau)} - 1 \right) &= \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(\mathcal{J}e^*, \mathcal{J}e_i, \tau)} - 1 \right) + \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(\mathcal{J}e_i, e^*, \tau)} - 1 \right) \\ &\leq \alpha(e_i, e^*, \tau) \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(\mathcal{J}e_i, \mathcal{J}e^*, \tau)} - 1 \right) + \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(\mathcal{J}e_i, e^*, \tau)} - 1 \right) \\ &\leq \Psi \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(e_i, e^*, \tau)} - 1 \right) + \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(\mathcal{J}e_i, e^*, \tau)} - 1 \right), \text{ because } \Psi(r) < r \\ &\leq \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(e_i, e^*, \tau)} - 1 \right) + \left(\frac{1}{\mathfrak{F}_{\rho, \bar{\Gamma}}(\mathcal{J}e_i, e^*, \tau)} - 1 \right) \end{aligned} \quad (24)$$

$$\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(\mathcal{J}e^*, e^*, \tau) = \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(\mathcal{J}e^*, \mathcal{J}e_i, \tau) + \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(\mathcal{J}e_i, e^*, \tau)$$

$$\begin{aligned} &\geq \alpha(e_i, e^*, \tau) \tilde{\mathfrak{F}}_{\rho, \bar{\Gamma}}(Je_i, Je^*, \tau) + \tilde{\mathfrak{F}}_{\rho, \bar{\Gamma}}(Je_{i+1}, e^*, \tau) \\ &\geq \Psi(\tilde{\mathfrak{F}}_{\rho, \bar{\Gamma}}(e_i, e^*, \tau)) + \tilde{\mathfrak{F}}_{\rho, \bar{\Gamma}}(Je_{i+1}, e^*, \tau), \text{ because } \Psi(r) < r \\ &\geq \tilde{\mathfrak{F}}_{\rho, \bar{\Gamma}}(e_i, e^*, \tau) + \tilde{\mathfrak{F}}_{\rho, \bar{\Gamma}}(Je_{i+1}, e^*, \tau) \end{aligned} \quad (25)$$

$$\begin{aligned} \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(Je^*, e^*, \tau) &= \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(Je^*, Je_i, \tau) + \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(Je_{i+1}, e^*, \tau) \\ &\geq \alpha(e_i, e^*, \tau) \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(Je_i, Je^*, \tau) + \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(Je_{i+1}, e^*, \tau) \\ &\geq \Psi(\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e_i, e^*, \tau)) + \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(Je_{i+1}, e^*, \tau), \text{ because } \Psi(r) < r \\ &\geq \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e_i, e^*, \tau) + \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(Je_{i+1}, e^*, \tau) \end{aligned} \quad (26)$$

Letting $i \rightarrow \infty$ in equation(24), (25) and (26), we obtain $\lim_{i \rightarrow \infty} \mathfrak{F}_{\rho, \bar{\Gamma}}(Je^*, e^*, \tau) = 1$, $\lim_{i \rightarrow \infty} \tilde{\mathfrak{F}}_{\rho, \bar{\Gamma}}(Je^*, e^*, \tau) = 0$ and $\lim_{i \rightarrow \infty} \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(Je^*, e^*, \tau) = 0$ for all $\tau > 0$, that is, $J(e^*) = e^*$, so J has a fixed point.

Example 3.8 Consider $\Xi = [0, \infty)$ and let the control functions $\rho, \bar{\Gamma} : \Xi^2 \rightarrow [1, \infty)$ be defined as $\rho(e, \dot{a}) = \begin{cases} \max\{e, \dot{a}\} + 1 & \text{if } e, \dot{a} \in [0, 1] \\ 1 & \text{otherwise} \end{cases}$ and

$$\bar{\Gamma}(e, \dot{a}) = \begin{cases} \max\{e, \dot{a}\} + 2 & \text{if } e, \dot{a} \in [1, 2] \\ 1 & \text{otherwise} \end{cases}. \text{ The neutrosophic set } \mathfrak{F}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{F}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}} \text{ is defined by } \mathfrak{F}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau) = \frac{\tau}{\tau + |\rho(e, \dot{a})|e - \dot{a}|}, \tilde{\mathfrak{F}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau) = \frac{|e - \dot{a}|}{\tau + |e - \dot{a}|} \text{ and } \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau) = \frac{|e - \dot{a}|}{\tau} \quad (27)$$

Then it is simple to demonstrate that $(\Xi, \mathfrak{F}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{F}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, *, \star)$ is a complete NDCMS which is triangular, and we will check triangle inequality only.

$$\mathfrak{F}_{\rho, \bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) = \frac{\tau}{\tau + \rho(e, \dot{a})|e - \dot{a}|} \leq \frac{\tau}{\tau + |e - \dot{a}|} \leq \frac{\tau + \tau'}{\tau + \tau' + |e - \dot{a}|}.$$

$$\text{Similarly, } \mathfrak{F}_{\rho, \bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau'}{\bar{\Gamma}(\dot{a}, \dot{b})}\right) = \frac{\tau'}{\tau' + \bar{\Gamma}(\dot{a}, \dot{b})|\dot{a} - \dot{b}|} \leq \frac{\tau'}{\tau' + |\dot{a} - \dot{b}|} \leq \frac{\tau + \tau'}{\tau + \tau' + |\dot{a} - \dot{b}|}.$$

$$\begin{aligned} \text{Thus, } \mathfrak{F}_{\rho, \bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) * \mathfrak{F}_{\rho, \bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau'}{\bar{\Gamma}(\dot{a}, \dot{b})}\right) &\leq \left(\frac{\tau + \tau'}{\tau + \tau' + |e - \dot{a}|}\right) \left(\frac{\tau + \tau'}{\tau + \tau' + |\dot{a} - \dot{b}|}\right) \\ &\leq \frac{\tau + \tau'}{\tau + \tau' + |e - \dot{b}|} = \mathfrak{F}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau + \tau'), \end{aligned} \quad (28)$$

$$\tilde{\mathfrak{F}}_{\rho, \bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) = \frac{\rho(e, \dot{a})|e - \dot{a}|}{\tau + \rho(e, \dot{a})|e - \dot{a}|} \geq \frac{|e - \dot{a}|}{\tau + |e - \dot{a}|} \geq \frac{|e - \dot{a}|}{\tau + \tau' + |e - \dot{a}|}.$$

$$\text{Similarly, } \tilde{\mathfrak{F}}_{\rho, \bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau'}{\bar{\Gamma}(\dot{a}, \dot{b})}\right) = \frac{\bar{\Gamma}(\dot{a}, \dot{b})|\dot{a} - \dot{b}|}{\tau' + \bar{\Gamma}(\dot{a}, \dot{b})|\dot{a} - \dot{b}|} \geq \frac{|\dot{a} - \dot{b}|}{\tau' + |\dot{a} - \dot{b}|} \geq \frac{|\dot{a} - \dot{b}|}{\tau + \tau' + |\dot{a} - \dot{b}|}.$$

$$\begin{aligned} \text{Thus, } \tilde{\mathfrak{F}}_{\rho, \bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) \star \tilde{\mathfrak{F}}_{\rho, \bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau'}{\bar{\Gamma}(\dot{a}, \dot{b})}\right) &\geq \left(\frac{|e - \dot{a}|}{\tau + \tau' + |e - \dot{a}|}\right) \left(\frac{|\dot{a} - \dot{b}|}{\tau + \tau' + |\dot{a} - \dot{b}|}\right) \\ &\geq \frac{|e - \dot{b}|}{\tau + \tau' + |e - \dot{b}|} = \tilde{\mathfrak{F}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau + \tau') \text{ and} \end{aligned} \quad (29)$$

$$\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) = \frac{\rho(e, \dot{a})|e - \dot{a}|}{\tau} \geq \frac{|e - \dot{a}|}{\tau} \geq \frac{|e - \dot{a}|}{\tau + \tau'}.$$

$$\text{Similarly, } \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau'}{\bar{\Gamma}(\dot{a}, \dot{b})}\right) = \frac{\bar{\Gamma}(\dot{a}, \dot{b})|\dot{a} - \dot{b}|}{\tau'} \geq \frac{|\dot{a} - \dot{b}|}{\tau'} \geq \frac{|\dot{a} - \dot{b}|}{\tau + \tau'}.$$

$$\begin{aligned} \text{Thus, } \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(e, \dot{a}, \frac{\tau}{\rho(e, \dot{a})}\right) \star \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(\dot{a}, \dot{b}, \frac{\tau'}{\bar{\Gamma}(e, \dot{a})}\right) &\geq \left(\frac{|e-\dot{a}|}{\tau+\tau'}\right)\left(\frac{|\dot{a}-\dot{b}|}{\tau+\tau'}\right) \\ &\geq \frac{|e-\dot{b}|}{\tau+\tau'} = \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau + \tau'). \end{aligned} \quad (30)$$

Note that Equation (28), (29) and (30) follows the below inequality:

$$\left(\frac{1}{1+|e-\dot{a}|}\right)\left(\frac{1}{1+|\dot{a}-\dot{b}|}\right) = \frac{1}{1+|e-\dot{a}|+|\dot{a}-\dot{b}|+|e-\dot{a}||\dot{a}-\dot{b}|} \leq \frac{1}{1+|e-\dot{a}|+|\dot{a}-\dot{b}|} \leq \frac{1}{1+|e-\dot{b}|}, \text{ then replace 1 by } \tau + \tau'.$$

Then $\mathcal{J} : \Xi \rightarrow \Xi$ and $\alpha : \Xi^2 \times (0, \infty) \rightarrow [0, \infty)$, be defined as $\mathcal{J}(e) = \begin{cases} \frac{e^2}{4} & \text{if } e \in [0, 1] \\ 2 & \text{otherwise} \end{cases}$ and

$\alpha(e, \dot{a}, \tau) = \begin{cases} 1 & e, \dot{a} \in [0, 1], \tau > 0 \\ 0 & \text{otherwise} \end{cases}$. Let $\Psi(r) = \frac{r}{2}$ for $r \geq 0$. To show that $\mathcal{J}$ is α -admissible, for any $e, \dot{a} \in [0, 1]$; hence, both $\mathcal{J}(e), \mathcal{J}(\dot{a}) \in [0, 1]$ which implies that $\alpha(\mathcal{J}e, \mathcal{J}\dot{a}, \tau) \geq 1$, for every $\tau > 0$. In order to establish $\mathcal{J}$ is $(\alpha - \Psi)$ -neutrosophic contractive mapping, we have to demonstrate the equation (5), (6) and (7) holds. For $1 < e, \dot{a}$, the case is obvious, thus, we consider the case $e, \dot{a} \in [0, 1]$.

$$\begin{aligned} \alpha(e, \dot{a}, \tau) \left(\frac{1}{\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(\mathcal{J}e, \mathcal{J}\dot{a}, \tau)} - 1 \right) &\leq \left(\frac{1}{\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(\mathcal{J}e, \mathcal{J}\dot{a}, \tau)} - 1 \right) = \left(\frac{1}{\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}\left(\frac{e^2}{4}, \frac{\dot{a}^2}{4}, \tau\right)} - 1 \right) = \frac{1}{4\tau} |e^2 - \dot{a}^2| \\ &\leq \frac{1}{2} \frac{|e-\dot{a}|}{\tau} = \Psi\left(\frac{1}{\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau)} - 1\right). \end{aligned}$$

$$\begin{aligned} \alpha(e, \dot{a}, \tau) \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(\mathcal{J}e, \mathcal{J}\dot{b}, \tau) &\leq \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau) \leq \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(\frac{e^2}{4}, \frac{\dot{a}^2}{4}, \tau\right) = \frac{|e^2 - \dot{a}^2|}{4\tau + |e^2 - \dot{a}^2|} \\ &\leq \frac{1}{2} \frac{|e-\dot{a}|}{|e-\dot{a}| + \tau} = \Psi(\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau)), \end{aligned}$$

$\alpha(e, \dot{a}, \tau) \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(\mathcal{J}e, \mathcal{J}\dot{b}, \tau) \leq \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau) \leq \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}\left(\frac{e^2}{4}, \frac{\dot{a}^2}{4}, \tau\right) = \frac{1}{2} \frac{|e-\dot{a}|}{\tau} = \Psi(\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{b}, \tau))$. If $\{e_i\}$ is any sequence in Ξ such that $\alpha(e_i, e_{i+1}, \tau) \geq 1$ for all $i \in \mathbb{N}$, and $e_i \rightarrow e$ as $i \rightarrow \infty$, then $\{e_i\} \subset [0, 1]$. Hence, we have $e \in [0, 1]$ which implies that $\alpha(e_i, e, \tau) \geq 1$ for every $i \in \mathbb{N}$. Hence, all the assumption of Theorem 3.8 are fulfilled, therefore, $\mathcal{J}$ has an invariant points, which are $e = 0$ and $e = 2$.

4. Application

Let $\Xi = C([0, I], \mathbb{R})$ be the space of all continuous real-valued functions defined on the interval $[0, I]$, for some $I > 0$. Define the control functions $\rho, \bar{\Gamma} : \Xi^2 \rightarrow [1, \infty)$ by $\rho(e, \dot{a}) = e + \dot{a} + 1$ and $\bar{\Gamma}(e, \dot{a}) = e^2 + \dot{a}^2 + 1$. Define the neutrosophic sets $\tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}$ and $\tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}$ by

$$\begin{aligned} \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau) &= e^{-\sup_{\kappa \in [0, 1]} \frac{|e(\kappa) - \dot{a}(\kappa)|}{\tau}}, \quad \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau) = 1 - e^{-\sup_{\kappa \in [0, 1]} \frac{|e(\kappa) - \dot{a}(\kappa)|}{\tau}} \quad \text{and} \quad \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}(e, \dot{a}, \tau) = \\ &e^{\sup_{\kappa \in [0, 1]} \frac{|e(\kappa) - \dot{a}(\kappa)|}{\tau}} - 1, \quad e, \dot{a} \in \Xi \text{ where } \tau > 0. \end{aligned} \quad (31)$$

Then $(\Xi, \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, *, \star)$ is a complete NDCM.

Theorem 4.1. Let $(\Xi, \tilde{\mathfrak{P}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, \tilde{\mathfrak{Q}}_{\rho, \bar{\Gamma}}, *, \star)$ be a complete NDCMS, as defined above. Let integral operator $\mathcal{J} : \Xi \rightarrow \Xi$ is defined by $\mathcal{J}e(\kappa) = q(\kappa) + \int_0^\kappa Y(\kappa, t, u(t))dt$, (32) where $q \in \Xi$, and $Y(\kappa, t, e(t)) : [0, I]^2 \rightarrow \mathbb{R}$ is a continuous function. If there exists a function $h : [0, I] \times [0, I] \rightarrow [0, \infty)$ such that for every $\kappa, t \in \Xi$, we have $h \in L^1([0, 1], \mathbb{R})$ satisfying the following:

$$I. \quad |Y(\kappa, t, e(t)) - Y(\kappa, t, \dot{a}(t))| \leq h(\kappa, t)|e(t) - \dot{a}(t)|$$

- II. The integral $\int_0^\kappa h(\kappa, t)dt$ is bounded, i.e., there exists some $\theta \in (0,1)$ such that $0 < \sup_{\kappa \in [0,1]} \int_0^\kappa h(\kappa, t)dt \leq \theta < 1$. Furthermore, this holds $e^{-\sup_{\kappa \in [0,1]} \frac{\theta|e(\kappa)-\dot{a}(\kappa)|}{\tau}} \geq 2e^{-\sup_{\kappa \in [0,1]} \frac{|e(\kappa)-\dot{a}(\kappa)|}{\tau}}$. Then, the integral equation (32) has a solution.

Proof. Define $\alpha: \Xi^2 \times (0, \infty) \rightarrow [0, \infty)$ by $\alpha(e, \dot{a}, \tau) = \begin{cases} \frac{1}{2} & \text{if } e = \dot{a} \\ \frac{1}{4} & \text{if } e = 0 \text{ or } \dot{a} = 0 \\ 0 & \text{otherwise} \end{cases}$ (33)

and $\Psi(\tau) = \frac{\tau}{2}$ for $e, \dot{a} \in \Xi$, consider the neutrosophic metrics

$$\begin{aligned} \tilde{\mathfrak{P}}_{\rho, \Gamma}(Je, J\dot{a}, \tau) &= e^{-\sup_{\kappa \in [0,1]} \frac{|Je(\kappa) - J\dot{a}(\kappa)|}{\tau}} \\ &\geq e^{-\sup_{\kappa \in [0,1]} \frac{\int_0^\kappa |Y(\kappa, t, e(t)) - Y(\kappa, t, \dot{a}(t))|}{\tau}} \geq e^{-\sup_{\kappa \in [0,1]} \frac{\int_0^\kappa h(\kappa, t)|e(t) - \dot{a}(t)|}{\tau}} \\ &\geq e^{-\sup_{\kappa \in [0,1]} \frac{|e(t) - \dot{a}(t)| \sup \int_0^\kappa h(\kappa, t)}{\tau}} \geq e^{-\sup_{\kappa \in [0,1]} \frac{\theta|e(t) - \dot{a}(t)|}{\tau}} \geq 2e^{-\sup_{\kappa \in [0,1]} \frac{|e(t) - \dot{a}(t)|}{\tau}} \\ &= 2(\tilde{\mathfrak{P}}_{\rho, \Gamma}(e, \dot{a}, \tau)), \end{aligned} \quad (34)$$

$$\begin{aligned} \tilde{\mathfrak{Q}}_{\rho, \Gamma}(Je, J\dot{a}, \tau) &= 1 - e^{-\sup_{\kappa \in [0,1]} \frac{|Je(\kappa) - J\dot{a}(\kappa)|}{\tau}} \\ &\leq 1 - e^{-\sup_{\kappa \in [0,1]} \frac{\int_0^\kappa |Y(\kappa, t, e(t)) - Y(\kappa, t, \dot{a}(t))|}{\tau}} \leq 1 - e^{-\sup_{\kappa \in [0,1]} \frac{\int_0^\kappa h(\kappa, t)|e(t) - \dot{a}(t)|}{\tau}} \\ &\leq 1 - e^{-\sup_{\kappa \in [0,1]} \frac{|e(t) - \dot{a}(t)| \sup \int_0^\kappa h(\kappa, t)}{\tau}} \leq 1 - e^{-\sup_{\kappa \in [0,1]} \frac{\theta|e(t) - \dot{a}(t)|}{\tau}} \leq 2(1 - e^{-\sup_{\kappa \in [0,1]} \frac{|e(t) - \dot{a}(t)|}{\tau}}) \\ &= 2(\tilde{\mathfrak{Q}}_{\rho, \Gamma}(e, \dot{a}, \tau)) \text{ and} \end{aligned} \quad (35)$$

$$\begin{aligned} \tilde{\mathfrak{Q}}_{\rho, \Gamma}(Je, J\dot{a}, \tau) &= e^{\sup_{\kappa \in [0,1]} \frac{|Je(\kappa) - J\dot{a}(\kappa)|}{\tau}} - 1 \\ &\leq e^{\sup_{\kappa \in [0,1]} \frac{\int_0^\kappa |Y(\kappa, t, e(t)) - Y(\kappa, t, \dot{a}(t))|}{\tau}} - 1 \leq e^{\sup_{\kappa \in [0,1]} \frac{\int_0^\kappa h(\kappa, t)|e(t) - \dot{a}(t)|}{\tau}} - 1 \\ &\leq e^{\sup_{\kappa \in [0,1]} \frac{|e(t) - \dot{a}(t)| \sup \int_0^\kappa h(\kappa, t)}{\tau}} - 1 \leq e^{\sup_{\kappa \in [0,1]} \frac{\theta|e(t) - \dot{a}(t)|}{\tau}} - 1 \leq 2(e^{\sup_{\kappa \in [0,1]} \frac{|e(t) - \dot{a}(t)|}{\tau}} - 1) \\ &= 2(\tilde{\mathfrak{Q}}_{\rho, \Gamma}(e, \dot{a}, \tau)). \end{aligned} \quad (36)$$

To show that that J is a $(\alpha - \Psi)$ -neutrosophic contractive mapping, we must demonstrate that equations (5), (6), and (7) are true. For $e, \dot{v} \in \Xi$, then by Equation (32), (33), (34) and (35) we have

$$\begin{aligned} \alpha(e, \dot{a}, \tau) \left(\frac{1}{\tilde{\mathfrak{P}}_{\rho, \Gamma}(Je, J\dot{a}, \tau)} - 1 \right) &\leq \left(\frac{1}{\tilde{\mathfrak{P}}_{\rho, \Gamma}(Je, J\dot{a}, \tau)} - 1 \right) \leq \left(\frac{1}{2\tilde{\mathfrak{P}}_{\rho, \Gamma}(e, \dot{a}, \tau)} - 1 \right) \\ &\leq \frac{1}{2} \left(\frac{1}{\tilde{\mathfrak{P}}_{\rho, \Gamma}(e, \dot{a}, \tau)} - 1 \right) = \Psi \left(\frac{1}{\tilde{\mathfrak{P}}_{\rho, \Gamma}(e, \dot{a}, \tau)} - 1 \right). \end{aligned}$$

$$\alpha(e, \dot{a}, \tau) \tilde{\mathfrak{Q}}_{\rho, \Gamma}(Je, J\dot{a}, \tau) \leq 2\tilde{\mathfrak{Q}}_{\rho, \Gamma}(e, \dot{a}, \tau) \leq \Psi(\tilde{\mathfrak{Q}}_{\rho, \Gamma}(e, \dot{a}, \tau)),$$

$$\alpha(e, \dot{a}, \tau) \tilde{\mathfrak{Q}}_{\rho, \Gamma}(Je, J\dot{a}, \tau) \leq \tilde{\mathfrak{Q}}_{\rho, \Gamma}(e, \dot{a}, \tau) \leq \Psi(\tilde{\mathfrak{Q}}_{\rho, \Gamma}(e, \dot{a}, \tau)).$$

Since all the requirements of Theorem 3.4 are fulfilled, hence the operator J has a fixed point $e^* \in C([0, I], \mathbb{R})$, which is a solution of the integral equation (31).

Conclusion In this paper we have defined $(\alpha-\psi)$ -neutrosophic contractive mappings on neutrosophic double controlled metric spaces and we have proved fixed point theorems on complex valued neutrosophic metric like space. We hope that the results proved in this paper will form new connection for those who are working in this field.

References.

1. Azmi, F.M. New fixed point results in double controlled metric type spaces with applications, AIMS. Mathematics 2022, 8, 1592–1609.
2. Banach, S. Sur les operations dans les ensembles et leur application aux equation sitegrales. Fundam. Math. 1922, 3, 133–181.
3. Di Bari, C.; Vetro, C. Fixed points, attractors and weak fuzzy contractive mappings in a fuzzy metric space. J. Fuzzy Math. 2005, 13, 973–982.
4. George, A.; Veeramani, P. On some results in fuzzy metric spaces. Fuzzy Sets Syst. 1994, 64, 395–399.
5. Gopal, D.; Vetro, C. Some new fixed point theorems in fuzzy metric spaces. Iran. J. Fuzzy Syst. 2014, 11, 95–107.
6. Haque, S.; Souayah, A.; Mlaiki, N.; Rizk, D. Double controlled Quasi Metric Like Spaces. Symmetry 2022, 14, 618.
7. Jeyaraman. M, Sowndrarajan. S. Some common fixed point theorems for $(\varphi - \psi)$ -weak contraction on intuitionistic Generalized fuzzy cone metric spaces. Malaya journal mathematic. 2019,1, 154-159.
8. Johnsy, Jeyaraman. Fixed point theorems for $(\psi - \varphi)$ -contractions in generalized neutrosophic metric spaces. Bulletin of Mathematical Analysis and Applications. 2024, 16 (1), 13-25.
9. Kamran, T.; Samreen, M.; UL Ain, Q. A Generalization of b-metric space and some fixed point theorems. Mathematics 2017, 5, 19. Symmetry 2023, 15, 716 12 of 12.
10. Kramosil, O.; Michalek, J. Fuzzy metric and statistical metric spaces. Kybernetika 1975, 11, 336–344.
11. M. Kirişci, N. Simsek, Neutrosophic metric spaces, Math. Sci., 14 (2020), 241–248.
12. Mlaiki, N.; Aydi, H.; Souayah, N.; Abdeljawad, T. controlled Metric Type Spaces and the Related Contraction Principle. Mathematics 2018, 6, 194.
13. Mlaiki, N.; Celik, U.; Tas, N.; O' zgu'r, N.Y.; Mukheimer, A. Wardowski Type contractives and the Fixed-Circle Problem on s-Metric Spaces. J. Math. 2018, 2018, 9.
14. Nadaban, S. Fuzzy b-metric spaces. Int. J. Comput. Commun. Control. 2016, 2, 273–281.
15. O' zgu'r, N.Y.; Tas, N.; Celik, U. New fixed-circle results on S-metric spaces. Bull. Math. Anal. Appl. 2017, 9, 10–23.
16. Park, J.H. Intuitionistic fuzzy metric spaces. Chaos Solitons Fractals 2004, 22, 10391046.
17. Poovaragavan, Jeyaraman. Common fixed point theorems in right complete dislocated quasi-G-fuzzy metric space. South East Asian Journal of Mathematics & Mathematical Sciences. 2022, 18 (1), 235-246.

18. Rathivel, Jeyaraman, Rahul Shukla. A Study of Fixed Point Results for Incompatible Mappings in Neutrosophic Double Controlled Metric Spaces with Application. *International Journal of Analysis and Applications*. 2025, 23, 96-96.
19. Rathivel, Inbam, Jeyaraman. Fixed Point Results in Neutrosophic n-Controlled Bipolar Metric Spaces. *Statistics, Optimization & Information Computing*. 2026.
20. Saleem, N.; Furqan, S.; Jarad, F. On extended b-Rectangular and Controlled Rectangular Fuzzy Metric-like Spaces with Application. *J. Funct. Spaces* 2022, 2022, 14.
21. Samet, B.; Vetro, C.; Vetro, P. Fixed points theorems for α - ψ -contractive type mappings. *Nonlinear Analysis. Theory Methods Appl.* 2012, 75, 2154–2165.
22. Saadati, R.; Park, J.H. On the intuitionistic fuzzy topological spaces. *Chaos Solitons Fractals* 2006, 27, 331–344.
23. Sezen, M.S. Controlled fuzzy metric spaces and some related fixed-point results. *Numer. Methods Partial. Differ. Equ.* 2021, 37, 583–593.
24. Saleem, N.; I, sik, H.; Furqan, S.; Park, C. Fuzzy double controlled metric spaces and related results. *J. Intell. Fuzzy Syst.* 2021, 40, 9977–9985.
25. F.Smarandache, *Neutrosophy: Neutrosophic probability, set and logic*, Rehoboth: American Research Press, 1998.
26. Zadeh, L.A. Fuzzy sets. *Inform. Control* 1965, 8, 338–353.