



Applications of αg^*S -Continuous Multifunction's

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ARTICLE INFO	ABSTRACT
Published Online: 19 August 2026 Corresponding Author: T. D. Rayanagoudar	The authors have introduced and studied the concept of αg^*s-continuous multifunction which are known as upper(lower) contra and almost-contra αg^*s-continuous multifunction and basic characterizations and fundamental properties related to contra and almost-contra upper and lower αg^*s-continuous multifunction's. Also, weakly αg^*s-continuous multifunctions are discussed and studied their basic properties.
KEYWORDS: αg^*s-closed set, αg^*s-open set, αg^*s-continuous multifunctions, weakly αg^*s-continuous multifunctions. AMS Subject Classification: 54C05, 54C08.	

1 INTRODUCTION

Many authors have continued their research and looked at many sorts of continuous functions and multifunctions, both stronger and weaker. It has been observed that multifunction's and continuity are essential ideas in general topology, set valued analysis, and other branches of mathematics. Continuous functions and continuous multifunction's are two of the most important and closely related mathematics notions.

The concept of upper and lower continuity for multifunctions was first studied and introduced by Berge [2] in the literature. After reading his study, several authors shifted their attention to examine several research on both strong and weak kinds of continuity. Pre-irresolute multi-valued functions were initially introduced by Mahmoad[12] in 1999. Neubrunn [14] was the first to suggest and investigate upper (lower) α -continuous multifunctions. Some characteristics of weakly α -continuous multifunctions were examined in [15].

This paper elaborates a new weaker form of upper αg^*s -continuous multifunctions and lower αg^*s -continuous multifunctions. Furthermore, a few preservation theorems and properties of an upper αg^*s -continuous (or lower αg^*s -continuous) multifunctions are studied and examined. Additionally, the idea of upper αg^*s -irresolute (or lower αg^*s -irresolute) multifunctions are presented and some of their characteristics are examined.

We recall that, a multifunction (M.F.) $\psi : (R, \tau) \rightarrow (S, \sigma)$, we will denote the upper and lower inverse of a set B in S by $\psi^+(B)$ and $\psi^-(B)$ respectively,

that is, $\psi^+(B) = \{r \in R : \psi(r) \subset B\}$ and $\psi^-(B) = \{r \in R : \psi(r)$

$\cap B \neq \emptyset\}$.

In particular, $\psi^-(A) = \{r \in R : s \in \Xi(r)\}$, holds for each $s \in S$ and for each $A \subset R, \Xi(A) = \cup r \in A \Xi(r)$. Then Ξ is said to be surjective, if $\psi(R) = S$ and injective if $r \neq s$, implies $\Xi(r) \cap \Xi(s) = \emptyset$.

Lemma 1.1[2]: For a M.F. $\psi : (R, \tau) \rightarrow (S, \sigma)$ we have the following:

(i) $G_{\psi^+}(A \times B) = A \cap \psi^+(B)$

(ii) $G_{\psi^+}(A \times B) = A \cap \psi^-(B)$.

For any subsets A of R , B of S , we have $G_{\psi} : R \rightarrow R \times S$ is defined as $G_{\psi}(r) = \{r\} \times \psi(r)$ holds for every $r \in R$.

Lemma 1.2[16]: Let $A, B \subseteq R$, then

(i) If $A \in \alpha g^*s\text{-}O(R)$ and $B \in O(R)$, then $A \cap B \in \alpha g^*s\text{-}O(R)$.

(ii) If $A \in \alpha g^*s\text{-}O(R)$ and $B \in \alpha g^*s\text{-}O(R)$, then $A \cap B \in \alpha g^*s\text{-}O(R)$.

Definition 1.3[2]: For a M. F. $\psi : R \rightarrow R$, the graph M.F. $G_{\psi} : R \rightarrow R \times S$ is defined as $G_{\psi}(r) = \{r\} \times \psi(r)$, for every $r \in R$.

Definition 1.4.[16] Let $A \subset R$. Then A is αg^*s -dense if $\alpha g^*s\text{-cl}(A) = R$.

2. UPPER AND LOWER CONTRA αg^*S -CONTINUOUS MULTIFUNCTION'S

Definition 2.1: A multifunction (M.F) $\psi : (R, \tau) \rightarrow (S, \sigma)$ is said to be

(i) **Upper αg^*s -continuous**, if for each point $r \in R, V \in O(S, \psi(r))$, there exist $U \in \alpha g^*s\text{-}O(R, r)$ with $\psi(U) \subseteq V$.

(ii) **Lower αg^*s -continuous**, if for each point $r \in R, V \in O(S, \psi(r))$ with $\psi(r) \cap V \neq \emptyset$, then there exists $U \in \alpha g^*s\text{-}O(R, r)$ with

$U \in \psi^{-1}(V)$.

Definition 2.2: A M.F. $\psi: (R, \tau) \rightarrow (S, \sigma)$ is said to be:

(i) upper contra αg^*s -continuous (u.c. $\alpha g^*s.C$) if for each point $r \in R$ and $V \in C(S, \psi(r))$, there exists $U \in \alpha g^*s-O(R, r)$ with $\psi(U) \subseteq V$.

(ii) lower contra αg^*s -continuous (l.c. $\alpha g^*s.c$) if for each $r \in R$, $V \in C(S)$ with $\psi(r) \cap V \neq \emptyset$, there exists $U \in \alpha g^*s-O(R, r)$ such that $U \subseteq \psi(V)$.

Example 2.3: Let $R = S = \{r_1, r_2, r_3\}$, $\tau = \{R, \phi, \{r_3\}\}$ and $\sigma = \{S, \phi, \{r_1\}, \{r_2\}, \{r_1, r_2\}, \{r_1, r_3\}\}$. Let $\psi: R \rightarrow S$ be an identity M.F. Consider the set $V = \{r_1, r_3\}$ and $U = \{r_1, r_3\}$. Then, ψ is u.c. $\alpha g^*s.C$ M.F.

Example 2.4: Let $R = S = \{r_1, r_2, r_3\}$, $\tau = \{R, \phi, \{r_1, r_2\}\}$ and $\sigma = \{S, \phi, \{r_1\}, \{r_2\}, \{r_1, r_2\}, \{r_1, r_3\}\}$. The set $V = \{r_2, r_3\}$ which is a subset S . Consider the point $r_2 \in R$, then there exists a αg^*s -open set $U = \{r_2\} \in R$ with $U \subseteq \psi^{-1}(V)$. Hence ψ is l.c. $\alpha g^*s.c$.

Theorem 2.5: The conditions listed below are equivalent for $\psi: R \rightarrow S$:

- (i) ψ is u.c. $\alpha g^*s.c$.
- (ii) for each $V \in C(S)$, $\psi^+(V) \in \alpha g^*s-O(R)$
- (iii) for all $V \in O(R)$, $\psi^-(V) \in \alpha g^*s-C(S)$
- (iv) for each $r \in R$ and $K \in C(S, \psi(r))$, there exists $U \in \alpha g^*s-O(R, r)$ with $t \in U$ then $\psi(t) \subseteq K$.

Proof: (i) $\Leftrightarrow$ (ii) Since ψ is u.c. $\alpha g^*s.c$, there exists $V \in C(S)$ with $r \in \psi^+(V)$, we have $U \in \alpha g^*s-O(R, r)$ such that $\psi(U) \subseteq V$. Hence $\psi^+(V) \in O(R)$.

In the same way, we may demonstrate the other component.

(ii) $\rightarrow$ (iii) for every $V \subseteq S$, we have $\psi^+(S - V) = R - \psi^+(V)$.

(iii) $\Leftrightarrow$ (iv) It's clear from the definition.

Theorem 2.6: A multifunction in the following statements are equivalent to a function ψ :

- (i) ψ is l.c. $\alpha g^*s.c$
- (ii) $\psi^{-1}(V) \in \alpha g^*s-O(R)$, for each $V \in C(S)$
- (iii) $\psi^{-1}(K) \in \alpha g^*s-C(R)$, for each $K \in O(S)$
- (iv) for each point $r \in R$ and each $K \in C(R)$ with $\psi(r) \cap K \neq \emptyset$, there exists $U \in \alpha g^*s-O(R, r)$ and $s \in U$, then $\psi(s) \subseteq K \neq \emptyset$.

Definition 2.7: A subset K in R is strongly S -closed (αg^*s -compact [16]) if every cover of K by closed (resp. αg^*s -open) sets of R has a finite subcover.

Theorem 2.8: Let ψ be an u.c. $\alpha g^*s.c$ subjective M.F. Then for each point r in R , $\psi(A)$ is strongly closed if A is αg^*s -compact relative to R .

Proof: Consider any cover of $\psi(A)$ by closed sets in S to be $\{A_i : i \in \alpha\}$. Then, there is a finite subset $\alpha(r)$ such that for any $\alpha(r)$, we have $\psi(r) \subseteq \bigcup \{A_i : i \in \alpha(r)\}$. Let $V(r) = \bigcup \{A_i : i \in \alpha(r)\}$ and so $\psi(r) \subseteq V(r)$. Then there exist $U(R) \in \alpha g^*s-O(R, r)$ with $\psi(U(r)) \subseteq V(r)$ as $\{U(R) : r \in A\}$ is cover of A by αg^*s -open sets. Therefore, there are a limited number of points, such as $r_1, r_2, r_3, \dots, r_n$ with $A \subseteq \bigcup \{U(r_i) : i = 1, 2, \dots, n\}$.

Theorem 2.9: Let $\psi: R \rightarrow S$ be a M.F. and V be an open set in R . If ψ is u.c. $\alpha g^*s.c$ (l.c. $\alpha g^*s.c$) then $\psi|_V : U \rightarrow S$ is u.c. $\alpha g^*s.c$ (l.c. $\alpha g^*s.c$).

Proof: Let V be any closed set in S with $r \in U$. By l.c.wgc.c of ψ , we have $r \in \psi|_V^{-1}(V)$. Then there exist a αg^*s -open set S in R containing r with $G \subseteq \psi^{-1}(V)$. Hence $r \in G \cap U$, with $G \cap U \in \psi^{-1}|_U(V)$. This meets the prerequisites for l.c. $\alpha g^*s.c$.

We can demonstrate the same for u.c. $\alpha g^*s.c$ M.F.

Theorem 2.10: The set of points r of the space R at which a M.F. $\psi: R \rightarrow S$ is not u.c. $\alpha g^*s.c$ (l.c. $\alpha g^*s.c$) is identical with the union of the αg^*s -frontiers of the upper (lower) inverse images of a closed sets containing (meeting) $\psi(r)$.

Proof: Let r be any point of R at which ψ is not u.c. $\alpha g^*s.c$. Consider the closed set V in S with $U \cap (R - \psi^+(V)) \neq \emptyset$ which is true for every αg^*s -open set V in R containing a point r . Since $r \in \psi^+(V)$ and so $r \in \alpha g^*s-cl(\psi^+(S))$. Thus $r \in \alpha g^*s-Fr(\psi^+(S))$.

However, consider any closed set in S with $r \in \alpha g^*s-Fr(\psi^+(V))$. Let us assume ψ is u.c. $\alpha g^*s.c$, then there exists an αg^*s -open set U in R with $\psi(U) \subseteq V$. Hence, $r \in U \in \alpha g^*s-int(\psi^+(V))$ which contradicts that $r \in \alpha g^*s-Fr(\psi^+(V))$. As such, ψ is not u.c. $\alpha g^*s.c$.

Likewise, we may demonstrate l.c. $\alpha g^*s.c$ M.F.

3. MULTIFUNCTION'S WITH αg^*S -CLOSED GRAPHS

Definition 3.1: A M.F. $\psi: R \rightarrow S$ is contra αg^*s -closed graph if for each $(r, s) \in (R \times S) - G(\psi)$, there exist αg^*s -open set U in R containing a point r and V be any closed set in S containing a point s such that $(U \times V) \cap G(\psi) = \emptyset$.

Theorem 3.2: Consider a T_2 -space S and $\psi: R \rightarrow S$ be u.c. $\alpha g^*s.c$. Then $\psi(r)$ is α -paracompact, for each point $r \in R$ where $G_{(\psi)}$ is αg^*s -closed.

Proof: Let us assume that $(r, s) \notin G(\psi)$ with $s \in \psi(r)$. As S is T_2 , there exist two disjoint open sets $V(s), W(s)$ in S . Then the family $\{V(s) : s \in \psi(s)\}$ is an open cover of the space $\psi(s)$. Hence, there is a locally finite open cover $\Delta = \{U_\beta : \beta \in I\}$ which refines the cover $\{V(s) : s \in \psi(s)\}$. Consequently, there is an open neighbourhood W_0 , where W_0 intersects only finitely members $U_{\beta_1}, U_{\beta_2}, \dots, U_{\beta_n}$ of Δ . Choose $s_1, s_2, \dots, s_n$ in $\psi(r)$ with $U_{\beta_i} \subseteq V(s_i)$, $1 \leq i \leq n$. Let $W = W_0 \cap (\bigcap_{i=1}^n W(n_i))$. So W is αg^*s -continuity the point r . So there exists an αg^*s -open set U_1 in R with $U_1 \subseteq \psi^+(\bigcup_{\beta \in I} V_\beta)$. Thus, $(U_1 \times W_1) \cap G(\psi) = \emptyset$ which shows that $G(\psi)$ is αg^*s -closed.

Theorem 3.3: Let R be connected space. Then $\psi: R \rightarrow S$ is u.c. $\alpha g^*s.c$ (l.c. $\alpha g^*s.c$) if the graph ψ is u.c. $\alpha g^*s.c$ (l.c. $\alpha g^*s.c$).

Proof: Let $r \in R$ and V be an open set in S . Then $R \times V$ is αg^*s -open in $R \times S$, with $G(r) \subset R \times V$. Then $U \in \alpha g^*s-O(R, r)$ with $G(\psi)(U) \subset M \times V$. By lemma 1.2, $U \subset G(\psi)^+(R \times V) = \psi^+(V)$ and $\psi(U) \subset V$, which shows that ψ is u.c. $\alpha g^*s.c$.

Likewise, we may demonstrate for l.c. $\alpha g^*s.c$ M.F.

4. UPPER AND LOWER ALMOST CONTRA αg^*S -CONTINUOUS MULTIFUNCTION’S:

Definition 4.1: A M.F. $\psi: R \rightarrow S$ is

- (i) upper almost αg^*S -continuous (u.a. αg^*S .c) if for each point r in R , an open set V in S , there exists αg^*S -open set U in R such that $U \subset \psi^+(\text{int}(\text{cl}(V)))$.
- (ii) lower almost αg^*S -continuous (l.a. αg^*S .c) if for each point r in R and an open set V in S with $\psi(r) \cap V \neq \emptyset$, there exists αg^*S -open set U in R containing r such that $U \subset \psi^-(\text{int}(\text{cl}(V)))$.
- (iii) upper almost contra αg^*S -continuous (u.a.c. αg^*S .c) if for each point r in R and regular closed set V in R with $r \in \psi^+(V)$, there exists αg^*S -open set U in R with $U \subseteq \psi^+(V)$.
- (iv) lower almost contra αg^*S -continuous (l.a.c. αg^*S .c) if for each point r in R and regular-closed set V in S with $r \in \psi^-(V)$, there exists αg^*S -open set U in S containing r such that $U \subset \psi^-(V)$.

Theorem 4.2: Every u.a.c. αg^*S .c M.F. is u.w. αg^*S .c.

Proof: Let r be a point of R and V be an open set in S with $\psi(r) \subset V$, that is $\text{cl}(V)$ is regular closed with $\psi(r) \subset \text{cl}(V)$. By u.a.c. αg^*S .c, there exists $U \in \alpha g^*S\text{-O}(R, r)$ with $U \subset \psi^-(\text{cl}(V))$. Thus ψ is u.w. αg^*S .c M.F.

Similarly, we may demonstrate for l.a.c. αg^*S .c M.F.

Example 4.3: Let $R = S = \{r_1, r_2, r_3\}$, $\tau = \{R, \emptyset, \{r_1\}, \{r_1, r_3\}\}$ and $\sigma = \{S, \emptyset, \{r_1\}, \{r_2\}, \{r_1, r_2\}, \{r_1, r_3\}\}$. Let $A = \{r_1, r_3\}$ and $U = \{r_1, r_2\}$. Let ψ be the identity function. Consider the point r_3 . Then, ψ is u.w. αg^*S .c M.F. but not u.a.c. αg^*S .c M.F. Since $\psi(U) = \{r_1, r_2\} \subset \text{cl}(A) = R$. Consider $V = \{r_2\}$ and $U = \{r_1, r_2\}$. Then $U \not\subseteq \psi^+(\text{int}(\text{cl}(A)))$.

Theorem 4.4: The properties listed below are true for a M.F. $\psi: R \rightarrow S$:

- (i) ψ is u.a.c. αg^*S .c.
- (ii) for every $A \subset \text{RC}(S)$, $\psi^+(A) \in \alpha g^*S\text{-O}(R)$.
- (iii) for every $U \in \text{RO}(S)$, $\psi^-(U) \in \alpha g^*S\text{-C}(R)$.
- (iv) for every $A \subset \text{O}(S)$, $\psi^-(\text{int}(\text{cl}(A))) \in \alpha g^*S\text{-C}(R)$.
- (v) for every $A \subset \text{C}(S)$, $\psi^+(\text{cl}(\text{int}(A))) \in \alpha g^*S\text{-O}(R)$.
- (vi) for each point r in R and semi-open set V in S with $\psi(r) \subset V$, there exists an αg^*S -open set U in R such that $\psi(U) \subseteq \text{cl}(V)$.
- (vii) for each $V \in \text{SO}(S)$, $\psi^+(V) \in \alpha g^*S\text{-int}(\psi^+(\text{cl}(V)))$.

Proof: (i) $\rightarrow$ (ii): Let A be any regular-closed set in S with $r \in \psi^+(A)$. By the definition, as ψ is u.a.c. αg^*S .c, there exists $U \in \alpha g^*S\text{-O}(R, r)$ such that $U \subseteq \psi^+(A)$ and hence $\psi^+(A) \in \alpha g^*S\text{-O}(R)$.

(ii) $\rightarrow$ (i) : The definition is followed.

(iii) $\Leftrightarrow$ (iv) and (iv) $\Leftrightarrow$ (v): As, $\psi^+(S - A) = R - \psi^-(A)$ holds for all set in S .

(iii) $\Leftrightarrow$ (iv): Consider an open set A in S . As $\text{int}(\text{cl}(A)) \in \text{RO}(S)$ and so $\psi^-(\text{int}(\text{cl}(A)))$ is αg^*S -closed in R .

The other consequence (v) $\rightarrow$ (ii) can also be proven.

(vi) $\rightarrow$ (vii): Consider semi-open set V in S and $r \in \psi^+(V)$. Hence, $\psi(r) \subset V$. By (vi), there exists an αg^*S -open set U in R such that $\psi(U) \subset \text{cl}(V)$, that is $r \in U \subset \psi^+(\text{cl}(V))$. Thus, r

$\in \alpha g^*S\text{-int}(\psi^+(\text{cl}(V)))$ and $\psi^+(V) \subset \alpha g^*S\text{-int}(\psi^+(\text{cl}(V)))$.

(vii) $\rightarrow$ (ii): Let A be any regular-closed in S . Then $\psi^+(A) \subset \alpha g^*S\text{-int}(\psi^+(\text{cl}(A)))$. So, $\psi^+(A)$ is αg^*S -open in R .

(ii) $\rightarrow$ (vi): Let V be any semi-open set in S with $\psi(r) \subset V$. As $\text{cl}(V)$ is regular-closed in S , there exists αg^*S -open set A in R with $A \subset \psi^+(\text{cl}(A))$. Thus, $\psi(A) \subset \text{cl}(V)$.

We can demonstrate the same results for l.a.c. αg^*S .c M.F. $\psi: R \rightarrow S$:

- (i) ψ is l.a.c. αg^*S .c
- (ii) for each regular closed set A in S , $\psi^-(A) \in \alpha g^*S\text{-O}(R)$.
- (iii) for each regular-open set U in S , $\psi^+(U) \in \alpha g^*S\text{-C}(R)$.
- (iv) for an open set A in S , $\psi^+(\text{int}(\text{cl}(A))) \in \alpha g^*S\text{-C}(R)$.
- (v) for each closed set A in S , $\psi^-(\text{cl}(\text{int}(A))) \in \alpha g^*S\text{-O}(R)$.
- (vi) for each semi-open set V in S with $\psi(r) \cap V \neq \emptyset$, there exists αg^*S -open set U in R with $\psi(U) \cap \text{cl}(V) \neq \emptyset$.
- (vii) for each semi-open set V in S , $\psi^-(V) \in \alpha g^*S\text{-int}(\psi^-(\text{cl}(V)))$.

Lemma 4.5: If A and B are disjoint compact subsets of an Urysohn space R then there exist an open set U in R and open set V in S with $A \subset U$, $B \subset V$ and $\text{cl}(U) \cap \text{cl}(V) = \emptyset$.

Theorem 4.6: Let $\psi: R \rightarrow S$ be u.a.c. αg^*S .c M.F. and S is Urysohn space with $\psi(r)$ is compact. Then the space R is $\alpha g^*S\text{-T}_2$.

Proof: Let r and s be any point of R with $\psi(r) \cap \psi(s) = \emptyset$ holds from injectiveness of ψ . Then, there exist open sets V_1 and V_2 with $\psi(r) \subset V_1$ and $\psi(s) \subset V_2$ such that $\text{cl}(V_1) \cap \text{cl}(V_2) = \emptyset$, where $\text{cl}(V_1)$ and $\text{cl}(V_2)$ are regular closed sets. From u.a.c. αg^*S .c of ψ , there exist αg^*S -open sets U_1 and U_2 in R with $\psi(U_1) \subset \text{cl}(V_1)$, $\psi(U_2) \subset \text{cl}(V_2)$. So, $U_1 \cap U_2 = \emptyset$, which holds the conditions of $\alpha g^*S\text{-T}_2$.

Definition 4.7: A M.F. $\psi: R \rightarrow S$ is upper weakly continuous (u.w.C) [12] if for each open set V in S containing $\psi(r)$ holds for each r in R , there exists an open set U in R such that $\psi(U) \subset \text{cl}(V)$.

Theorem 4.8: A space R and an Urysohn space S have the following characteristics:

- (i) $\psi: R \rightarrow S$ is u.w.c. M.F.
- (ii) $\psi: R \rightarrow S$ is an u.a.c. αg^*S .c M.F.
- (iii) $\psi(r)$ and $\Omega(r)$ are compact sets in S , holds for each point r in R .
- (iv) $A = \{r \in R: \psi(r) \cap \Omega(r) \neq \emptyset\}$.

Then the set A is αg^*S -closed. For each point r in R , A is αg^*S -dense in R , if $\psi(r) \cap \Omega(r) \neq \emptyset$.

Proof: Let $r \in R - A$, so $\psi(r) \cap \Omega(r) = \emptyset$. Then by Urysohn space, there exist open sets V_1, V_2 in S with $\psi(r) \subseteq V_1$ and $\Omega(r) \subseteq V_2$ where $\text{cl}(V_1) \cap \text{cl}(V_2) = \emptyset$. As, ψ is u.w.c and $\psi(r) \subseteq V_1$, there exist open sets U_1 in R containing a point r with $\psi(U_1) \subseteq \text{cl}(V_1)$. Since $\text{cl}(V_2)$ is regular closed, $\Omega(r) \subseteq W_1$. By u.a.c. αg^*S .c of ψ , there exist an αg^*S open set U_2 in R with $\Omega(U_2) \subseteq \text{cl}(V_2)$. Consider $U = U_1 \cap U_2$, then U_1 is αg^*S -open set in R with $U_1 \cap A = \emptyset$. So, $r \in R - \alpha g^*S\text{-cl}(A)$.

On the other side, if $\psi(r) \cap \Omega(r) = \Phi$ on a αg^*s -dense set D in R , so $R = \alpha g^*s\text{-cl}(D) \subseteq \alpha g^*s\text{-cl}(A) = A$. Hence, $\psi(r) \cap \Omega(r) = \Phi$ holds for all r in R .

Theorem 4.9. Let $\psi: R \rightarrow S$ be a M.F. with U be an open set in R . If ψ is l.a.c. $\alpha g^*s.c$ (u.a.c. $\alpha g^*s.c$) then $\psi|U: U \rightarrow S$ is l.a.c. $\alpha g^*s.c$ (u.a.c. $\alpha g^*s.c$).

Proof: Let V be regular-closed in S with $r \in U$. Then $r \in (\psi|U)^{-1}(V)$. From the definition of l.a.c. $\alpha g^*s.c$ M.F., there exists an αg^*s -open set G in R with $G \subseteq \psi^{-1}(V)$. So $r \in G \cap U$ which is αg^*s -open set and $G \cap U \subseteq (\psi|U)^{-1}(V)$, which satisfies the condition of l.a.c. $\alpha g^*s.c$ is M.F.

Likewise, we may demonstrate for u.a.c. $\alpha g^*s.c$ M.F.

5.WEAKLY αg^*S -CONTINUOUS MULTIFUNCTION'S

Definition 5.1: A multifunction (briefly m.f) $\psi: R \rightarrow S$ is said to be

- (i) Upper αg^*s -continuous (u. $\alpha g^*s.c$) at each point s in S , if for each open set V in S with $\psi(s) \subseteq V$, there exists an αg^*s -open set U in R such that $\psi(U) \subseteq V$.
- (ii) Lower αg^*s -continuous (l. $\alpha g^*s.c$) at each point r in R , if for each open set V in R with $\psi(r) \cap V \neq \Phi$, there exists an αg^*s -open set U in R such that $\psi(z) \cap V \neq \Phi$ for each $z \in U$.
- (iii) Upper (resp. lower) αg^*s -continuous, if it is upper (resp. lower) αg^*s -continuous at each point of R .

Example 5.2: Let $R = \{r_1, r_2, r_3\}$, $S = \{s_1, s_2, s_3\}$, $\tau = \{R, \phi, \{r_1\}\}$ and $\sigma = \{S, \phi, \{s_1, s_2\}\}$. Let $\psi: R \rightarrow S$ be a M.F and ψ be the identity M.F. from R to S . Consider a point $r_1 \in R$ and $V = \{r_1, r_2\}$ be an open set of S with $\psi(\{r_1\}) = \{r_1\} \subseteq V$. Then, αg^*s -open set $U = \{r_1, r_2\}$ in R containing the point r_1 such that $\psi(U) = \psi(\{r_1, r_2\}) = \{r_1, r_2\} \subseteq V$. So we get, $\psi(U) \subseteq V$ and hence ψ is u. $\alpha g^*s.c$.

Example 5.3: Let $R = S = \{r_1, r_2, r_3\}$. Let $\tau = \{R, \phi, \{r_1\}, \{r_1, r_2\}\}$ and $\sigma = \{S, \phi, \{r_1\}\}$.

Consider a M.F. $\psi: R \rightarrow S$. Let $M = \{r_2\} \in R$. Let us consider $V = \{r_1, r_2, r_3\}$. Then $\psi(\{r_2\}) \cap V = \{r_2\} \cap \{r_1, r_2, r_3\} = \{r_2\} = \phi$.

Again, consider $V = \{r_1, r_2\}$. Then $\psi(\{r_2\}) \cap V = \{r_2\} \cap \{r_1, r_2\} = \{r_2\} = \phi$.

But we have αg^*s -open set containing the set $\{r_2\}$ are: $S, \{r_1, r_2\}$

- (i) $\{r_2\} \cap \{r_1, r_2\} = \{r_2\} = \phi$
- (ii) $\{r_2\} \cap N = \{r_2\} \cap \{r_1, r_2, r_3\} = \{r_2\} = \phi$

Therefore ψ is a l. $\alpha g^*s.c$.

Theorem 5.4: The following holds for a M.F. $\psi: R \rightarrow S$

- (i) ψ is u. $\alpha g^*s.c$.
- (ii) for each point r in R and $V \in O(R)$ with $r \in \psi^+(V)$, there exists $U \in \alpha g^*s\text{-O}(r)$ with $U \subseteq \psi^+(V)$.
- (iii) for each r in R and $K \in C(S)$ with $r \in \psi^+(S - K)$, there exists $H \in \alpha g^*s\text{-C}(R, r)$ such that $r \in R - H$ and $\psi^-(K) \subseteq H$.
- (iv) for each $V \in O(S)$, $\psi^+(V) \in \alpha g^*s\text{-O}(R)$.

(v) for each $K \in C(S)$, $\psi^-(V) \in \alpha g^*s\text{-C}(R)$.

(vi) for each $r \in R$ and for each neighbourhood V of $\psi(S)$, $\psi^+(V)$ is αg^*s -neighbourhood of r .

(vii) for each $r \in R$ and for each neighbourhood V of $\psi(S)$, there exists αg^*s -neighbourhood U of r such that $\psi(U) \subseteq V$.

Theorem 5.5: A M.F. $\psi: R \rightarrow S$ is u. $\alpha g^*s.c$ if and only if for each $B \in O(S)$, $\psi(B) \in O(R)$.

Proof. Let B be an open set in S and $r \in \psi^+(B)$. From the definition, there is $V \in \alpha g^*s\text{-O}(R)$ such that $\psi(V) \subseteq B$, where $\psi^+(B) \in O(R)$. Since $\psi^+(B)$ is open and $r \in \psi^-(B)$, we have $\psi^+(B) = \{r \in B: \psi(r) \subseteq B\}$. Hence ψ is u. $\alpha g^*s.c$.

Theorem 5.6: A M.F. $\psi: R \rightarrow S$ is l. $\alpha g^*s.c$ iff for each $B \in O(S)$, $\psi^-(B) \in O(R)$.

Proof. Let $B \in O(S)$ and $r \in \psi^+(B)$. By l. $\alpha g^*s.c$, there is $V \in \alpha g^*s\text{-O}(R)$ such that $\psi(V) \cap B \neq \phi$. Let $v \in V$, so $\psi^-(B) \in O(R)$. If $\psi^-(B)$ is open with $r \in \psi^-(B)$, then $\psi^-(B) = \{r \in R: \psi(r) \cap B \neq \phi\}$. Thus, ψ is l. $\alpha g^*s.c$.

Theorem 5.7: The following holds good for a M.F. $\psi: R \rightarrow S$:

- (i) ψ is u. $\alpha g^*s.c$.
- (ii) $\psi(\alpha g^*s\text{-cl}(B)) \subseteq \text{cl}(\psi(B))$, holds for all $B \subseteq R$
- (iii) $\alpha g^*s\text{-cl}(\psi^+(A)) \subseteq \psi^+\text{cl}(A)$ holds for all $A \subseteq S$
- (iv) $\psi^-(\text{int}(A)) \subseteq \alpha g^*s\text{-int}(\psi^-(A))$ holds for all $A \subseteq R$
- (v) $\text{int}(\psi(B)) \subseteq \psi(\alpha g^*s\text{-int}(B))$ holds for all $B \subseteq R$

Theorem 5.8: If $\psi: R \rightarrow S$ is u. $\alpha g^*s.c$, injective, S is T_1 -space, then R is $\alpha g^*s\text{-}T_1$.

Proof. Let R be T_1 -space. Then, $r_1, r_2 \in R$ with $r_1 \neq r_2$. So there exist A, B open sets in R with $\psi(r_1) \in A$, $\psi(r_2) \notin A$ and $\psi(r_1) \notin B$, $\psi(r_2) \in B$. As ψ is u. $\alpha g^*s.c$, there exist an αg^*s -open sets U and V in R with $r_1 \in U$, $r_1 \notin V$ and $r_2 \notin U$, $r_2 \in V$, which shows $r_1 \in U$, $r_2 \in V$ with $\psi_1(U) \subseteq A$, $\psi_1(V) \subseteq B$. So, R is $\alpha g^*s\text{-}T_1$.

Theorem 5.9: Let $\psi: R \rightarrow S$ is u. $\alpha g^*s.c$, injective with S is T_2 . Then R is $\alpha g^*s\text{-}T_2$.

Proof. Let $r_1, r_2 \in R$ with $r_1 \neq r_2$. There exist G, H open sets in S with $\psi(r_1) \notin G$, $\psi(r_2) \in H$. As ψ is u. $\alpha g^*s.c$, there exist A, B which are αg^*s -open sets in R with $\psi(G) \subseteq A$, $\psi(H) \subseteq B$ with $A \cap B = \phi$. Thus, $G \cap H = \phi$ and hence R is $\alpha g^*s\text{-}T_2$.

Theorem 5.10: If the graph G_p is u. $\alpha g^*s.c$, then ψ is u. $\alpha g^*s.c$, where $\psi: R \rightarrow S$ be M.F.

Proof. Let $r \in R$ and V be any open set in R . Then $R \times V \in O(R \times S)$, $G_p(r) \subseteq R \times V$. By the definition of G_p , u. $\alpha g^*s.c$, there exists αg^*s -open set U in R containing a point r with $G_p(U) \subseteq R \times V$. Thus, $U \subseteq G_p^+(R \times V)$. From Lemma 1.1, $G_p^+(R \times V) = R \cap \psi^+(V) = \psi^+(V)$, so $U \subseteq \psi^+(V)$. Hence ψ is u. $\alpha g^*s.c$.

Theorem 5.11: Let ψ be a M.F. The graph G_p is l. $\alpha g^*s.c$, then ψ is l. $\alpha g^*s.c$.

Proof. Let r be any point of R and V be an open set in S with $r \in \psi^-(V)$. Then $R \times V \in O(R \times S)$. Again, $G_p(r) \cap (R \times V) = (\{r\} \times \psi(r)) \cap (R \times V) = \{r\} \times \psi(r) \cap V \neq \phi$. As G_p is l. $\alpha g^*s.c$, there exists an αg^*s -open set U in R with $U \in G_p\text{-}(R \times V)$.

From Lemma 1.1, $G_\psi(R \times V) = R \cap \psi^-(V) = \psi^-(V)$ and so $U \subseteq \psi^-(V)$. Hence ψ is $\alpha g^*s.c.$

Theorem 5.12. Let $\psi: R \rightarrow S$ be a compact M.F. Then the graph G_ψ is $\alpha g^*s.c$ if and only if ψ is $\alpha g^*s.c.$

Proof. Let us consider $G_\psi: R \rightarrow S$ be $\alpha g^*s.c.$ holds for each point r in R and an open set V in R . Since, $R \times V \in O(R \times S)$, so there exists αg^*s -open set U in R containing a point r with $G_\psi(U) \subseteq R \times S$. From Lemma 1.1, we have $U \subseteq G_\psi^+(R \times V) = \psi^+(V)$ and $\psi(U) \subseteq V$ and so, ψ is $\alpha g^*s.c.$

On the other hand, assume that ψ is $\alpha g^*s.c.$ Consider $r \in R$ and $A \in O(R \times S)$ containing $G_\psi(r)$. Then, for each p , there exists $U(p) \subseteq R$ and $V(p) \subseteq S$ with $(p, r) \in U(r) \times V(r) \subseteq A$. Then the

family $\{V(p): p \in \psi(p)\}$ is an open cover of $\psi(p)$. Since $\psi(r)$ is compact, there exists finite number of points say $r_1, r_2, \dots, r_k$ in $\psi(r)$ with $\psi(r) \subseteq \{V(p_i): i = 1, 2, k\}$. Consider $U = \bigcap \{U(r_i): i = 1, 2, \dots, k\}$ and $V = \bigcup \{V(p_i): i = 1, 2, \dots, k\}$. Then U is an open set in R , V is an open set in S and $\{r\} \times \psi(r) \subseteq U \times V \subseteq A$. As ψ is $\alpha g^*s.c.$, there exists $H \in \alpha g^*s-O(R, r)$ with $\psi(H) \subseteq V$. From Lemma 1.1, $U \cap H \subseteq U \cap \psi^+(V) = G_\psi^+(U \times V) \subseteq G_\psi^+(A)$ and so $U \cap H \in \alpha g^*s-O(R, r)$ and so $G_\psi(U \cap V) \subseteq A$. Thus G_ψ is $\alpha g^*s.c.$

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