



On Certain Q-Weyl Fractional Integral Operators for Generalized Basic Hypergeometric Functions and Related Q-Polynomials - Part III

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ARTICLE INFO	ABSTRACT
Published Online: 24 July 2026	Fractional calculus and q-calculus have emerged as significant areas of mathematical analysis due to their wide applications in special functions, mathematical physics, and related fields. In this paper, we investigate certain q-Weyl fractional integral operators associated with generalized basic hypergeometric functions. A q-analogue of the Weyl fractional integral operator is employed to derive new integral representations and operational formulas for generalized multibasic hypergeometric series. The obtained results provide extensions of several classical and q-analogue identities within the framework of fractional q-calculus. Furthermore, the action of the proposed q-Weyl fractional integral operator on various important classes of q-polynomials, including q-Laguerre, Wall, Stieltjes–Wigert, q-Konhauser, and other related q-polynomials, is established. Several special cases are derived as consequences of the main results, demonstrating the applicability and generality of the developed formulas. The present work establishes a connection between fractional integral operators, generalized basic hypergeometric functions, and q-polynomial theory, contributing to the advancement of q-analysis and special function theory. These results may be useful for further investigations in fractional q-calculus, orthogonal polynomials, and applications in mathematical physics and related disciplines.
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1. INTRODUCTION

Fractional calculus has become an important branch of mathematical analysis due to its ability to extend the concepts of differentiation and integration to arbitrary non-integer orders. During recent decades, fractional integral and differential operators have attracted considerable attention because of their applications in various fields, including mathematical physics, engineering, signal processing, control theory, and applied sciences. Among the various fractional operators, the Weyl fractional integral operator occupies a significant position, particularly in the study of functions defined on infinite intervals and in the development of integral transformations.

In parallel with the development of fractional calculus, q-calculus (quantum calculus) has emerged as a powerful generalization of classical calculus. Unlike traditional

calculus, q-calculus does not depend on limiting processes and provides an effective framework for investigating q-analogues of special functions, orthogonal polynomials, and difference equations. The combination of fractional calculus and q-calculus has led to the development of q-fractional operators, which provide new possibilities for extending classical results into the q-domain.

The q-Weyl fractional integral operator, being a q-analogue of the classical Weyl fractional integral operator, plays an important role in fractional q-analysis. It has been successfully applied in obtaining transformation formulas, integral representations, and operational identities for several classes of special functions. Such operators establish a meaningful connection between fractional calculus, q-series, and special function theory.

Generalized basic hypergeometric functions form one of the fundamental classes of functions in q-analysis. These functions provide natural extensions of classical hypergeometric functions and appear in various areas such as combinatorics, quantum theory, orthogonal polynomials, and mathematical physics. A number of important q-polynomials, including q-Laguerre, Wall, Stieltjes–Wigert, q-Konhauser, and related polynomial families, can be represented in terms of basic hypergeometric functions and possess rich algebraic properties.

Motivated by the importance of fractional q-operators and generalized basic hypergeometric functions, the present work investigates certain q-Weyl fractional integral operators

acting on generalized basic hypergeometric functions. The main objective is to derive new operational formulas and integral representations in the framework of fractional q-calculus. Furthermore, the applications of these operators to various q-polynomials are established, leading to several interesting special cases.

The results obtained in this paper extend existing theories of q-fractional integral operators and provide a unified approach for studying the interaction between generalized basic hypergeometric functions and q-polynomial systems. These findings may contribute to further developments in fractional q-calculus and may have potential applications in mathematical physics, quantum analysis, and related areas.

For real or complex a, q , the q-shifted factorial is defined by

$$(a; q)_n = 1 \text{ if } n = 0 \quad \text{and} \\ (a; q)_n = (1 - q)(1 - aq)(1 - aq^2) \dots (1 - aq^{n-1}) \text{ if } n \in N \quad (1.1)$$

The q-gamma function is given as

$$\Gamma_q(z) = \frac{(q; q)_\infty}{(q^2; q)_\infty} (1 - q)^{1-z} \quad (1.2)$$

In terms of gamma the equation (1) can be expressed as

$$(a; q)_n = \frac{\Gamma_q(a+n)}{\Gamma_q(a)} (1 - q)^n \quad (1.3)$$

also

$$(x - y)_v = x^v \left(\frac{y}{x}; q \right)_v \quad (1.4)$$

The generalized basic hypergeometric Series [2, (1.2.22), p.4] is defined as

$${}_r\phi_s[a_1, a_2, \dots, a_r; b_1, b_2, \dots, b_s; q, z] \\ = \sum_{n=0}^{\infty} \frac{(a_1, a_2, \dots, a_r)_n}{(b_1, b_2, \dots, b_s)_n} \frac{[(-1)^n q^{n(n-1)/2}]^{1+s-r}}{z^n} \quad (1.5)$$

where r and s are positive integers and $|q| < 1$. The above series converges absolutely for all z if $r \leq s$ and for $|z| < 1$ if $r = s+1$.

The q-analogue of generalized Weyl fractional integral operator (cf. Al Salam [7]) is given by

$$K_q^{\eta, \alpha} \{f(x)\} = \frac{q^{-\eta, \alpha}}{\Gamma_q(\alpha)} \int_x^\infty (x - t)_{\alpha-1} t^{\eta-\alpha} f(tq^{1-\alpha}) d_q t \quad (1.6)$$

Also, the basic integral (cf. Gasper & Rahman [3]) is given by

$$\int_x^\infty f(t) d_q t = x(1 - q) \sum_{k=1}^\infty q^{-k} f(xq^{-k}) \quad (1.7)$$

Also the basic integral (cf. Gasper & Rahman [3]) is given by

$$K_q^{\eta, \alpha} \{f(x)\} = \frac{q^{-\alpha(1-q)} (q; q)_\alpha}{\Gamma_q(\alpha)} \sum_{k=0}^\infty \frac{(q^{\alpha+1}; q)_k q^{k(\eta-1)}}{(q; q)_k (1 - q^{-\alpha-k})} q^{-k} f(xq^{-\alpha-k}) \quad (1.8)$$

Heine's q-analogue of Gauss summation formula is given by

$${}_2\phi_1(a, b; c; q, c/ab) = \frac{(c/a, c/b; q)_\infty}{(c, c/ab; q)_\infty} \quad (1.9)$$

The above formula is valid for $|c/ab| < 1$. For the terminating case when $a = q^{-n}$, (1.9) reduces to

$${}_2\phi_1(q^{-n}, b; c; q, cq^n/b) = \frac{(c/b; q)_\infty}{(c; q)_\infty} \quad (1.10)$$

By changing the order of summation, it follows from (1.10) that

$${}_2\phi_1(q^{-n}, b; c; q, q) = \frac{(c/b; q)_\infty}{(c; q)_\infty} b^n \quad (1.11)$$

The q-binomial theorem due to Cauchy [1843], Heine [1847] is given by

$${}_1\phi_0(a; -; q, z) = \frac{(az; q)_\infty}{(z; q)_\infty} \quad |z| < 1, |q| < 1. \quad (1.12)$$

In the proof of the main result, I will use the following definition of various q-polynomials in the proof of the main results:

The q-Laguerre polynomials

$$L_n^{(\alpha)}(x; q) = \frac{(q^\alpha; q)_n}{(q; q)_n} {}_1\phi_1[q^{-n}; q^{1+n}; q, -xq^n] \quad (1.13)$$

The Wall polynomials (or Little q-Laguerre polynomial)

$$W_n(x; b, q) = (-1)^n (b; q)_n q^{n(n+1)/2} {}_2\phi_1[q^{-n}, 0; b; q, x] \quad (1.14)$$

$$\Gamma_n^{\alpha, \beta}(z; q) = \frac{(q^{\alpha+1}; q)_n}{(q; q)_n} {}_2\phi_1[q^{-n}, q^{\alpha+\beta+n+1}; q^{\alpha+1}; q, -xq] \quad (1.15)$$

The Stieltjes -Wigert polynomials

$$S_n(x; q) = {}_1\phi_0[q^{-n}; -; q, -x] \quad (1.16)$$

The q-Konhauser polynomials of first Kind

$$Z_n^{\{\alpha\}}(z; k; q) = \frac{(q^{\alpha+1}; q)_{nk}}{(q^k; q^k)_n} \sum_{i=0}^n \frac{(q^{-nk}; q^k)_i q^{(1/2)ki(k-1)+ki(n+\alpha+1)}}{(q^k; q^k)_i (q^{\alpha+1}; q)_{ik}} x^{ik} \quad (1.17)$$

The q-Konhauser polynomials of Second Kind

$$Y_n^{\{\alpha\}}(z; k; q) = \frac{1}{(q; q)_n} \sum_{r=0}^n \frac{q^{r(r-1)/2} x^r}{(q; q)_r} \sum_{i=0}^r \frac{(q^{-r}; q)_i (q^{1+\alpha+i}; q^k)_n}{(q; q)_i} q^i \quad (1.18)$$

The bibasic form of the q-analogue of the Bedient polynomials of first kind is given by:

$$R_n(\alpha, \beta; q, x) = \frac{(\alpha; q)_n (2x/\sqrt{\alpha})^n}{(q; q)_n} {}_3\phi_2 \left[q^{-n}, q^{-n+1}, -; \beta, q^{1-n}/\alpha : \beta/\alpha; q^2, q, (q/x)^n \right] \quad (1.19)$$

The bibasic form of the q-analogue of the Bedient polynomials of second kind is given by:

$$G_n(\alpha, \beta; q, x) = \frac{(\alpha; q)_n (2x)^n}{(q; q)_n (\alpha\beta; q)_n} {}_3\phi_2 \left[q^{-n}, q^{-n+1}, -; q^{1-n}/\beta, q^{1-n}/\alpha : q^{1-n}/\alpha\beta; q^2, q, (1/x)^n \right] \quad (1.20)$$

and the Gegenbauer polynomials

$$R_n(\alpha, \beta; q, x) = \frac{(\alpha; q)_n (2x/\sqrt{\alpha})^n}{(q; q)_n} {}_2\phi_1 \left[q^{-n}, q^{-n+1}, -; -; q^{1-n}/\alpha; q^2, q, (q/\alpha x)^n \right] \quad (1.21)$$

2. MAIN RESULTS

$$\begin{aligned} & K_n^{\eta, \alpha} \{ (1 - q^{-\alpha-k}) \times \\ & \phi[a_1, \dots, a_r; c_{11}, \dots, c_{1r_1}; \dots; c_{m1}, \dots, c_{mr_m}; b_1, \dots, b_s; d_{11}, \dots, d_{1s_1}; \dots, d_{m1}, \dots, d_{ms_m}; q, q_1, \dots, q_m; z] \} = \\ & \frac{-q^{-\alpha}(1-q)(q; q)_\infty (q^{\alpha+\eta}; q)_\infty}{\Gamma_q(\alpha)(q^{\eta-1}; q)_\infty} \times \\ & \phi[a_1, \dots, a_r, q^{1-\alpha-\eta}; c_{11}, \dots, c_{1r_1}; \dots; c_{m1}, \dots, c_{mr_m}; b_1, \dots, b_s, q^{2-\eta}; d_{11}, \dots, d_{1s_1}; \dots, d_{m1}, \dots, d_{ms_m}; q, q_1, \dots, q_m; qz] \} \end{aligned} \quad (2.1)$$

$$\begin{aligned} & K_n^{\eta, \alpha} \{ (1 - q^{-\alpha-k}) \times C_n^{\{\alpha\}}(z; q) \} = \frac{-(1-q)(q; q)_\infty (q^{\alpha+\eta}; q)_\infty (\alpha; q)_\infty}{q^\alpha \Gamma_q(\alpha)(q^{\eta-1}; q)_\infty (q; q)_n} \left(\frac{2x}{\sqrt{\alpha}} \right)^n \times \\ & {}_{n+2}\phi_{n+1} \left[-; q^{-n}, q^{-n+1}; q^{\eta-1}, q^\eta, \dots, q^{\eta+n-2}; q^{1-n}/\alpha; -; q^{\alpha+\eta}, q^{\alpha+\eta+1}, \dots, q^{\alpha+\eta+n+1}; q, q^2, q^n; \frac{q^{n(\alpha+1)}}{\alpha x^n} \right] \end{aligned} \quad (2.2)$$

Proof of the Main Results:

To prove the result (2.1), take

$$f(x) = K_n^{\eta, \alpha} \{ (1 - q^{-\alpha-k}) \times \phi[a_1, \dots, a_r; c_{11}, \dots, c_{1r_1}; \dots; c_{m1}, \dots, c_{mr_m}; b_1, \dots, b_s; d_{11}, \dots, d_{1s_1}; \dots, d_{m1}, \dots, d_{ms_m}; q, q_1, \dots, q_m; z] \}$$

in the equation (1.8). After interchanging the order of summation and using the formula (1.12),

we obtain:

$$K_n^{\eta, \alpha} \{ (1 - q^{-\alpha-k}) \times$$

$$\phi[a_1, \dots, a_r; c_{11}, \dots, c_{1r_1}; \dots; c_{m1}, \dots, c_{mr_m}; b_1, \dots, b_s; d_{11}, \dots, d_{1s_1}; \dots, d_{m1}, \dots, d_{ms_m}; q, q_1, \dots, q_m; z] = \frac{-q^{-\alpha}(1-q)(q; q)_{\infty} q^{\alpha+\eta}}{\Gamma_q(\alpha)(q^{\eta-1}; q)_{\infty}} \times \sum_{n=0}^{\infty} \frac{(a_1, \dots, a_r, q^{1-\alpha-\eta}; q)_n}{(q, b_1, \dots, b_s, q^{2-\eta}; q)_n} z^n q^n \prod_{j=1}^m \frac{(c_{j1}, c_{j2}, \dots, c_{jr}; q_j)_n}{(d_{j1}, d_{j2}, \dots, d_{js}; q_j)_n}$$

This in view of definition (1.23) reduces to the result (2.1).

The proof of result (2.2) follows similarly.

4. CONCLUSION

In this work, we have investigated certain q-Weyl fractional integral operators and examined their applications to generalized basic hypergeometric functions and q-polynomials. By employing the framework of fractional q-calculus, we have extended classical Weyl fractional integrals to the q-domain and studied their operational behaviour on important classes of special functions. The results obtained in this study demonstrate the effectiveness of q-Weyl fractional operators as a powerful analytical tool in the theory of special functions. They provide a unified approach for extending classical results to the fractional q-setting and establish connections between fractional calculus, q-series, and orthogonal polynomials. Overall, this work contributes to the growing literature on fractional q-calculus and opens new avenues for further research. Future investigations may focus on the extension of these operators to other classes of q-special functions, multidimensional q-fractional operators, or potential applications in mathematical physics, quantum theory, and applied sciences.

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REFERENCES

1. Agarwal, R.P., Fractional q-derivatives and q-integrals and certain hypergeometric transformations, *Ganita* 27 (1976), 25-32.
2. Gasper, G. and Rahman, M. Basic Hypergeometric Series, Cambridge University Press, Cambridge, 1990. 8
3. H. M. Srivastava and P. W. Karlsson, Multiple Gaussian Hypergeometric Series, John Wiley Sons, Halsted press, New York, 1985. Description
4. Slater, L.J., Generalized Hypergeometric Functions, Cambridge University Press, Cambridge, 1966.
5. Yadav, R.K. and Purohit, S.D., On Applications of q-Fractional Calculus Operators and Transformations Involving Generalized Hypergeometric Basic Functions, *Bull. Pure Appl. Math.*, Vol. 4, No. 1 (2010), 125-132.
6. Srivastava, H.M. and Jain, V.K., q-Series Identities and Reducibility of Basic Double Hypergeometric Functions, *Can. J. Math.*, Vol. xxxviii, No. 1, 1986, pp. 215-231.
7. W.A. Al-Salam, Some fractional q-integrals and q-derivatives, *Proc. Edin. Math. Soc.*, 15(1966), 135-140
8. Yadav, J.P., On Transformation of Basic Hypergeometric Functions Applying Fractional q-Differential Operator, *South East Asian J. of Mathematics and Mathematical Sciences* 19 (3), 75-80, 2023.