

Mathematical Model for Deblurring Images Subjected to Multinoises

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Abstract. In this work, we made two major contributions: proposing a mathematical model for deblurring digital images subject to multiple types of noise, and proposing a new filter, called the CC-Filter (Convex Combination Filter). Our proposed model, equipped with this CC-Filter, is best suited to denoise this type of disturbance in digital images. Firstly, we provide the existence and uniqueness of the solution for the proposed model. Secondly, we elaborate on the associated discretized model for numerical simulation processes. Then, we use image quality metrics such as SSIM and PSNR for evaluating the fidelity of the reconstructed image. Finally, we present numerical results to illustrate the proposed model's effectiveness for denoising mixed noise, typically of Cauchy-Poisson type.

Keywords: Digital Image, Inverse Problem, Poisson Noise, Cauchy Noise, Optimization, CC-Filter.

1 Introduction

Nowadays, advancements in technology have significantly reshaped the framework of digital imaging. Modern digital images extend far beyond their traditional role as visual depictions, serving instead as essential components in numerous scientific and industrial systems. Their utilization spans a wide array of domains, including autonomous systems and automated quality control [1],[2], healthcare diagnostics through medical imaging, and artificial intelligence-driven applications [3],[4],[5],[6], and large-scale image management involving acquisition, compression, storage, and dissemination. Moreover, novel paradigms such as acoustically derived imaging further demonstrate the growing influence of digital imaging in contemporary research and technological development.

The problems of digital image restoration are inverse problems that consist of reconstructing a clear image from a degraded image. For instance, the noise in the initial image is additive, and image processing problems can refer to techniques for recovering an original image $\mathcal{I}(x, y)$ from a noisy one denoted by $\mathcal{I}_0(x, y)$:

$$\mathcal{I}_0(x, y) = \mathcal{I}(x, y) + \xi(x, y), \quad (1)$$

where: ξ is an additive noise.

Generally, digital image deblurring can be considered as an inverse problem that recovers a sharp image from blurred and noise-degraded observations. In fact, the degradation is modeled as a linear operator, typically convolution with a point spread function, mapping the true image to observed data. Solving such an inverse problem is very difficult because it could be ill-posed (in the sense of Hadamard) [7, 8]. In addition, the blurring operator is often ill-conditioned, which implies the attenuation of high frequencies, and noise is amplified during inversion, resulting in unstable and non-unique solutions. To overcome these issues, deblurring is formulated as a regularized inverse problem where prior assumptions, such as smoothness, sparsity, or piecewise regularity, are incorporated through variational, Bayesian, or learning-based methods to ensure stable and physically meaningful reconstructions [9]. As a specialized branch of signal analysis, it focuses on transforming image data into outputs that are more suitable for interpretation, decision-making, or subsequent automated processing. Nevertheless, images captured in practical environments rarely exhibit ideal quality, as they are frequently affected by various forms of corruption introduced during acquisition, transmission, or storage. A critical challenge arises from the fact that real imaging systems are often subject to multiple degradation mechanisms acting simultaneously. These disturbances may include stochastic fluctuations, impulse-like artifacts, photon-limited noise, or multiplicative interference patterns, each originating from distinct physical or technological sources. When such perturbations coexist within a single image, commonly described as mixed-noise degradation, the complexity of restoration and analysis increases substantially. Despite the extensive body of literature devoted to noise reduction techniques, the majority of existing methods are designed to address individual noise characteristics in isolation, for example, Cauchy noise [19], Poisson noise [21], or Gaussian noise [29]. Therefore, their performance typically deteriorates when dealing with images affected by several noise types at once. Research efforts explicitly targeting mixed-noise scenarios remain comparatively scarce, leaving a gap between theoretical developments and the demands of real-world imaging applications. Against this backdrop, investigating robust image processing strategies capable of coping with mixed-noise conditions constitutes a critical research issue. Developing models and some heuristic approaches that can adapt to mixed and multi-source noise is more than necessary for advancing the performance, stability, robustness, and practical applicability of modern digital image processing systems.

The outline of the paper is structured as follows: In Section 2, we recall some literature reviews about mathematical models for digital image denoising. In Section 3, we make a theoretical analysis of our proposed model : (i) Subsection 3.1 presents the existence and uniqueness result, (ii) Subsection 3.2 is the new filtering technique. Section 4 is dedicated to the numerical analysis and simulations. Section 5 concludes our work and provides some perspectives.

2 Literature Reviews of Models for Image Restoration

In the literature, many mathematical models have been proposed for deblurring digital images, such that the pioneering work of Leonid I. Rudin, Stanley Osher and Emad Fatemi suggested in 1992 [28] to minimize a nonlinear total variation defined by: for Ω an open and bounded subset of $\mathbb{R}^N$ ($N \in \{2, 3\}$)

$$\mathcal{I} \longmapsto \text{TV}(\mathcal{I}) = \int_{\Omega} |D\mathcal{I}| d\Omega, \quad (2)$$

amongst functions of bounded variation on Ω , denote by $BV(\Omega)$ under the constraint $\|\mathcal{I} - \mathcal{I}_0\|_2^2 \leq \sigma^2 \text{mes}(\Omega)^2$ (with $\text{mes}(\Omega)$ the Lebesgue measure of the domain Ω which represent the area occupied by the image, and σ is the noise deviation) to solve this considered inverse problem and thus get a clean image $\mathcal{I}$. Furthermore, in [29], Antonin Chambolle and Pierre-Louis Lions proved that one can solve the unconstrained problem equivalently.

$$\min_{\mathcal{I} \in BV(\Omega)} \lambda \int_{\Omega} |D\mathcal{I}| d\Omega + \frac{1}{2} \|\mathcal{I} - \mathcal{I}_0\|_2^2. \quad (3)$$

To improve the quality of restoration of degraded images, non-local models based on the Laplacian have been developed. The first approach was proposed by Stefan Kindermann, Stanley Osher, and Peter W. Jones [30], and is given by:

$$\min_{\mathcal{I}} \left(\frac{1}{2\theta} \int_{\Omega} \int_{\Omega} \mathcal{W}(x, y) |\mathcal{I}(y) - \mathcal{I}(x)|^{\theta} dx dy + \frac{\lambda}{2} \int_{\Omega} (\mathcal{I}(x) - \mathcal{I}_0(x))^2 dx \right), \quad (4)$$

with the boundary of the domain Ω denoted by $\partial\Omega$ enough smooth, $\mathcal{W}$ is a weight function, $1 < \theta < \infty$, and $\lambda > 0$ is a weight parameter. In formula (4), (i) the first integral is a regularization term that allows for suppressing Gaussian noise; (ii) the second one is an observation term (also known as the data fidelity term) that acts like the bridge between mathematical model and reality. In inverse problems like image restoration, its main role is to guarantee that the solution remains faithful to the original image.

To improve the efficiency of this model, F. Karami, K. Sadik, and L. Ziad [31] proposed the following:

$$\min_{\mathcal{I}} \left(\frac{1}{2} \int_{\Omega} \int_{\Omega} \frac{\mathcal{W}(x, y) |\mathcal{I}(y) - \mathcal{I}(x)|^{\theta(x, y)}}{\theta(x, y)} dx dy + \frac{\lambda}{2} \int_{\Omega} (\mathcal{I}(x) - \mathcal{I}_0(x))^2 dx \right). \quad (5)$$

For example, these models allow for restoring images affected by additive Gaussian noise during its acquisition (or manipulations).

Regarding restoring images subject to Poisson noise, Shivarama H. Kayyar and P. Jidesh [21], proposed the model (6) below:

$$E(\mathcal{I}) := \int_{\Omega} (\mathcal{I} - \mathcal{I}_0 \log(\mathcal{I})) + \lambda \int_{\Omega} |\nabla \mathcal{I}|. \quad (6)$$

From his previous work, Fatia Bendaïda [19] proposes the following new non-local model for deblurring images corrupted by Cauchy noise

$$\begin{aligned} \mathcal{L}_{\theta, \lambda, \alpha, \gamma}(\mathcal{I}) = & \frac{1}{2} \int_{\Omega} \int_{\Omega} \frac{\mathcal{W}(x, y) |\mathcal{I}(y) - \mathcal{I}(x)|^{\theta(x, y)}}{\theta(x, y)} dx dy + \frac{\lambda}{2} \int_{\Omega} \log(\gamma^2 + (\mathcal{I}(x) - f(x))^2) dx \\ & + \frac{\alpha}{2} \int_{\Omega} (\mathcal{I}(x) - \tilde{\mathcal{I}}_0(x))^2 dx, \end{aligned} \quad (7)$$

where:

- $\mathcal{I}$ is a non-negative function almost everywhere in Ω .
 - $\tilde{\mathcal{I}}_0$ is obtained by filtering the noisy image $\mathcal{I}_0$.
 - The functions θ , and $\mathcal{W}$ must satisfy the following conditions:
- (C.1) $\theta : \Omega \times \Omega \rightarrow]1, +\infty[$ is a continuous and symmetric such that : $\forall x \in \Omega$

$$\theta_i(x) = \inf_{y \in \bar{\Omega}} \theta(x, y) \in]1, 2[, \quad \text{and} \quad \theta_s(x) = \sup_{y \in \bar{\Omega}} \theta(x, y) \in]1, 2[. \quad (8)$$

- (C.2) $\mathcal{W} : \mathbb{R}^N \times \mathbb{R}^N \rightarrow \mathbb{R}$ is a non-negative symmetric continuous smooth function with compact support contained in $\Omega \times \mathcal{B}(0, d) \subset \mathbb{R}^N \times \mathbb{R}^N$ (for some $d \in \mathbb{R}_+^*$) such that: $\forall x \in \Omega$, we have :

$$0 < \sup_{y \in \mathcal{B}(0, d)} \mathcal{W}(x, y) = \mathcal{R}(x) \in L^\infty(\Omega) \quad \text{and} \quad \int_{\mathbb{R}^N} \mathcal{W}(x, y) dx = 1. \quad (9)$$

In this paper, the Laplacian with variable exponent was proposed to denoise images corrupted by multiple types of noise. In that follows, we will focus on images corrupted by noise composed of a mixture of Cauchy noise and Poisson noise. Since there are the most common factors of image degradation.

3 The proposed model for deblurring corrupted images

The proposed model is a minimization problem defined by:

$$\min_{\mathcal{I}} \mathcal{P}_{\theta, \lambda, \alpha}(\mathcal{I}), \quad (10)$$

with :

$$\begin{aligned} \mathcal{P}_{\theta, \lambda, \alpha}(\mathcal{I}) = & \frac{1}{2} \int_{\Omega} \int_{\Omega} \frac{\mathcal{W}(x, y) |\mathcal{I}(y) - \mathcal{I}(x)|^{\theta(x, y)}}{\theta(x, y)} dx dy \\ & + \frac{\lambda}{2} \int_{\Omega} (\mathcal{I}(x) - \mathcal{I}_0(x) \log(\mathcal{I}(x)))^2 dx + \frac{\alpha}{2} \int_{\Omega} (\mathcal{I}(x) - \tilde{\mathcal{I}}_0(x))^2 dx. \end{aligned} \quad (11)$$

Where:

- $\mathcal{W}(x, y)$ and $\theta(x, y)$ verify conditions (C.1) and (C.2);
- $\tilde{\mathcal{I}}_0$ is obtained by applying our proposed CC-Filter to the noised image (see Subsection 3.2).

3.1 Existence and uniqueness of solution for the new model

Theorem 1. *Let Ω be a bounded open set in $\mathbb{R}^2$ with Lipschitz-continuous boundary. Assume $\mathcal{I}_0 \in L^\infty(\Omega)$ and $\lambda, \alpha > 0$. The functional defined in (11) admits a unique minimizer $\mathcal{I}^*$ over the admissible set $\mathcal{A}_d = \{\mathcal{I} \in L^2(\Omega) \mid \log(\mathcal{I}) \in L^2(\Omega)\}$, i.e.:*

$$\mathcal{P}_{\theta, \lambda, \alpha}(\mathcal{I}^*) = \min_{\mathcal{I} \in \mathcal{A}_d} \mathcal{P}_{\theta, \lambda, \alpha}(\mathcal{I}). \quad (12)$$

Proof. Given the reflexivity of the Banach space $L^2(\Omega)$, it is sufficient to prove that the objective function (11) is coercive, strictly convex, and lower semicontinuous to yield a unique solution to problem (10).

For the coercivity, by expanding the square under the integral, and by using Cauchy-Schwarz inequality, we obtain:

$$\int_{\Omega} (\mathcal{I} - \tilde{\mathcal{I}}_0)^2 \geq \|\mathcal{I}\|_2^2 + \|\tilde{\mathcal{I}}_0\|_2^2 - 2\|\mathcal{I}\|_2 \|\tilde{\mathcal{I}}_0\|_2. \quad (13)$$

Thus, $\mathcal{P}_{\theta, \lambda, \alpha}(\mathcal{I}) \geq \frac{\alpha}{2} \left(\|\mathcal{I}\|_2^2 + \|\tilde{\mathcal{I}}_0\|_2^2 - 2\|\mathcal{I}\|_2 \|\tilde{\mathcal{I}}_0\|_2 \right) \xrightarrow{\|\mathcal{I}\|_2 \rightarrow +\infty} +\infty$.

It follows that : $\mathcal{P}_{\theta, \lambda, \alpha}(\mathcal{I}) \xrightarrow{\|\mathcal{I}\|_2 \rightarrow +\infty} +\infty$. Hence, $\mathcal{P}_{\theta, \lambda, \alpha}$ is coercive.

Let $\text{epi}(\mathcal{P}_{\theta, \lambda, \alpha})$ be the epigraph of the functional $\mathcal{P}_{\theta, \lambda, \alpha}$ defined in the Equation (11), and given by :

$$\text{epi}(\mathcal{P}_{\theta, \lambda, \alpha}) := \{(\mathcal{I}, \eta) \in L^2(\Omega) \times \mathbb{R} \mid \mathcal{P}_{\theta, \lambda, \alpha}(\mathcal{I}) \leq \eta\}. \quad (14)$$

We have to show that $\text{epi}(\mathcal{P}_{\theta, \lambda, \alpha})$ is closed. Let $(v_n, \eta_n) \in \text{epi}(\mathcal{P}_{\theta, \lambda, \alpha})$ a sequence which converge to the couple (v, η) (then, $(v_n) \subset L^2(\Omega)$ and $(\eta_n) \subset \mathbb{R}$ are sequences converging to v and η respectively). Applying the sequential characterization of continuity, it follows that $(v, \eta) \in \text{epi}(\mathcal{P}_{\theta, \lambda, \alpha})$. Then, $\mathcal{P}_{\theta, \lambda, \alpha}$ is lower semicontinuous.

To establish the strict convexity of the functional, we define the function Ψ as follows: $\Psi(s) = \frac{\lambda}{2} (s - \mathcal{I}_0 \log s)^2 + \frac{\alpha}{2} (s - \tilde{\mathcal{I}}_0)^2, \quad \forall s > 0$.

Since this function Ψ is at least of class $\mathcal{C}^2$ on $\mathbb{R}_+^*$. Thus, by computing its second derivative thanks to its strict positivity on $\mathbb{R}_+^*$, we establish the strict convexity of the function by observing that it is composed of a sum of such functions. $\square$

The minimization problem (10) is a Cauchy problem, which means that the solution depends continuously on the initial data. To achieve better restoration quality, we proposed a filter called the CC-Filter and defined below in (3.2).

3.2 The proposed CC-Filter

Filters are a very important component in the denoising methods of digital image processing. The literature proposes a variety of filtering techniques, including the

most popular, such as median, myriad, averaging, and order-based filters [34, 35]. In general, the previously mentioned filters are suited to a single type of noise; for example, order filters and especially values around the median are effective in removing impulsive noise.

The various simulation tests show that they are ineffective against multiple noises. This is why we proposed a new filter called the CC-Filter (CC stands for Convex Combination), which is defined by:

$$\tilde{f} = \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3 + \alpha_4 f_4, \quad \text{with } \alpha_i \in [0; 1] \text{ and } \sum_i \alpha_i = 1, \quad (15)$$

where $\tilde{f}$ is obtained by using median (f_1), Myriad (f_2), order (f_3), and averaging (f_4) filters. Since the new filter $\tilde{f}$ is constructed from a convex combination of existing filters, it offers a valuable advantage in the processing of images corrupted by multiple noises. Indeed, a judicious choice of the different coefficients (α_i) allows for incorporating the advantages of the different filters included in the relation (15). This precise weighting assures a convergence toward the original image, considerably minimizing mean square error and preserving its structural texture.

4 Numerical Analysis

This section provides the numerical scheme of the model (10)-(11) and validates its performance through some numerical results. Moreover, we employ some well-known image quality metrics such as PSNR and SSIM to perform a quantitative analysis of the reconstruction accuracy. Then, numerical applications using some famous pictures illustrate our approaches.

4.1 Numerical scheme

By applying the calculus of variations to our proposed minimization problem (10), we derive the following Euler-Lagrange equation:

$$\int_{\Omega} \mathcal{W}(x, y) |\mathcal{I}(y) - \mathcal{I}(x)|^{\theta(x, y) - 2} (\mathcal{I}(x) - \mathcal{I}(y)) dy - \lambda \frac{\mathcal{I} - \tilde{\mathcal{I}}_0}{\mathcal{I}} (\mathcal{I} - \tilde{\mathcal{I}}_0 \log(\mathcal{I})) + \alpha (\mathcal{I} - \tilde{\mathcal{I}}_0) = 0, \quad (16)$$

with $\tilde{\mathcal{I}}_0$ is the image produced by applying the CC-Filter to the noisy image $\mathcal{I}_0$.

The nonlocal evolution diffusion equation associated with problem (16) can be obtained by applying the backward Euler method to the discretized gradient

flow of the following evolutionary system:

$$\begin{cases} \frac{\partial \mathcal{I}(t, x)}{\partial t} = - \int_{\Omega} \mathcal{W}(x, y) |\mathcal{I}(t, y) - \mathcal{I}(t, x)|^{\theta(x, y) - 2} (\mathcal{I}(t, y) - \mathcal{I}(t, x)) dy \\ \quad + \lambda \left(\frac{\tilde{\mathcal{I}}(t, x) - \mathcal{I}_0(x)}{(\mathcal{I}(t, x))} \right) (\mathcal{I}(t, x) - \mathcal{I}_0(x) \log(\mathcal{I}(t, x))) - \alpha (\mathcal{I}(t, x) - \tilde{\mathcal{I}}_0(x)) \\ \quad \text{in }]0, T[\times \Omega; \\ \mathcal{I}(0, x) = \mathcal{I}_0(x). \end{cases} \quad (17)$$

By setting $\Delta_{NL}\mathcal{I}(x) = \int_{\Omega} \mathcal{W}(x, y) (\mathcal{I}(x) - \mathcal{I}(y)) dy$, we can summarize the main steps of the solution computation procedure in the Algorithm 1.

Algorithm 1 : Numerical Solution Process

input f : original image, Tol, ε , Patch size, search windows.

Output $\mathcal{I}^n$: restaured image.

$\mathcal{I}^0 \leftarrow \mathcal{I}_0$,

$\tilde{\mathcal{I}}_0 \leftarrow$ apply CC-Filter to $\mathcal{I}_0$,

$n \leftarrow 0$,

Tol $\leftarrow 1 \cdot e^{-5}$.

Patch size: 3×3 , a seach Window: 5×5 , Compute $k(q, l)$

While $\frac{\|\mathcal{I}^{n+1} - \mathcal{I}^n\|}{\|\mathcal{I}^n\|} \leq \text{tol}$ **do.** Compute $\Delta_{NL}\mathcal{I}^n$;

$v_q^n \leftarrow |\Delta_{NL}\mathcal{I}_q^n|^{p(i, j) - 2} \Delta_{NL}\mathcal{I}_q^n$,

Compute $\Delta_{NL}v^n$.

$\mathcal{I}_q^{n+1} \leftarrow \mathcal{I}_q^n - \tau \Delta_{NL}v_q^n + \tau \left(\frac{\lambda(\mathcal{I}_q^n - \mathcal{I}_{0q})}{\mathcal{I}_q^n + \varepsilon} (\mathcal{I}_q^n - \mathcal{I}_{0q} \log(\mathcal{I}_q^n + \varepsilon)) - \alpha(\mathcal{I}_q^n - \tilde{\mathcal{I}}_0) \right)$.

$n \leftarrow n + 1$.

The discrete exponent is calculated as follows: $\theta(i, j) = \frac{\pi(i) + \pi(j)}{2}$, where :

$$\pi(i) = 1 + \frac{1}{1 + \varepsilon |\nabla G_{\sigma} * \mathcal{I}_{0i}|^2}, \varepsilon > 0 \text{ and } G_{\sigma} \text{ Gaussian filter kernel.}$$

To compute the discrete weight function, we use the formula: for step $h > 0$,

$$\mathcal{W}(i, j) = \exp \left(-\frac{d_{\sigma}(i, j)}{h^2} \right), \text{ with : } d_{\sigma} = \sum_l G_{\sigma}(l) |\mathcal{I}_0(i + l) - \mathcal{I}_0(j + l)|^2.$$

4.2 Some image quality metrics

Measuring restoration quality is essential for digital image analysis. Indeed, it is essential to ensure that the image being studied is properly denoised. In the present work, we use Image Quality Metrics (IQM) such as: (a) the Structural Similarity Index Measure (SSIM), which measures the perceived similarity between the restored and reference images in terms of luminance, contrast, and structure; (b) the Peak Signal-to-Noise Ratio (PSNR), which quantifies the ratio between the maximum possible signal power and the noise affecting image reconstruction fidelity. The formulae of these two IQM are given below.

1. $SSIM = \frac{1}{m} \sum_{i=1}^m \text{ssim}((a)_i, (b)_i)$, with $\text{ssim}(a, b) := \frac{(2\bar{a}\bar{b} + C_1)(2\sigma_{ab} + C_2)}{(\bar{a}^2 + \bar{b}^2 + C_1)(\sigma_a^2 + \sigma_b^2 + C_2)}$,
where : $\bar{a}, \bar{b}$ are the average; σ_a^2 and σ_b^2 are the variances; σ_{ab} is the covariance of a, b and $C_1 = (k_1 L)^2$ $C_2 = (k_2 L)^2$ are two variables to stabilize the division with weak denominator; L is the dynamic range of the pixel-values. We set $k_1 = 0.01$ and $k_2 = 0.03$.
2. $PSNR = 10 \cdot \log_{10} \left(\frac{255^2 M \times N}{\|\mathcal{I}_1 - \mathcal{I}_2\|^2} \right)$,
with : $M \times N$: image size, $\mathcal{I}_1$ and $\mathcal{I}_2$ are the original image and the reconstructing image respectively.

Table 1 shows the SSIM and the PSNR values of the noised images.

4.3 Numerical simulations

Now, we present the results of our numerical simulations. For these numerical experiments, we have considered the following images : (i) Lena (see Fig. 1); (ii) Wall (see Fig. 2); (iii) Professor Joseph KI-ZERBO (see Fig. 3); (iv) Building of EBOMAF (see Fig. 4). They are corrupted simultaneously by Poisson noise and by Cauchy noise types.

Table 1: SSIM and PSNR of images corrupted by Cauchy and Poisson noises.

Images	SSIM of noisy image	PSNR of noisy image
Lena	0.0629	13.9205
Wall	0.0798	13.8798
KI-ZERBO	0.01107	13.5195
Building	0.0858	13.5857

These very low quantities show a considerable deterioration of the various images considered.

These images, corrupted by at least some types of noise, are therefore used to test the model.

The first evaluation involves comparing the visual quality of the degraded, restored, and original images.

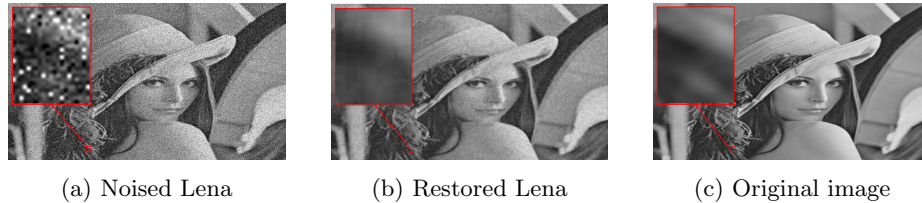


Fig. 1: Lena image restoration processes.

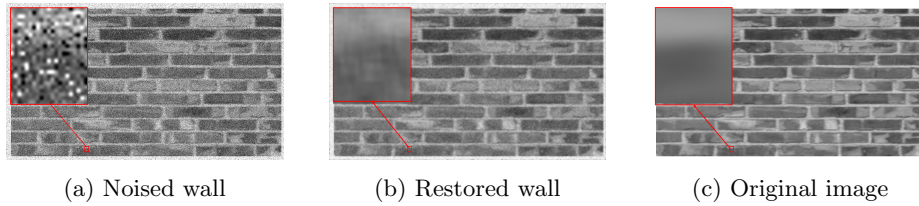


Fig. 2: Wall image restoration processes.

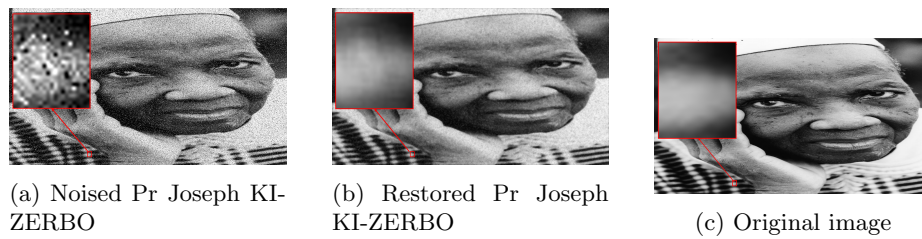


Fig. 3: Pr. Joseph KI-ZERBO image restoration processes.

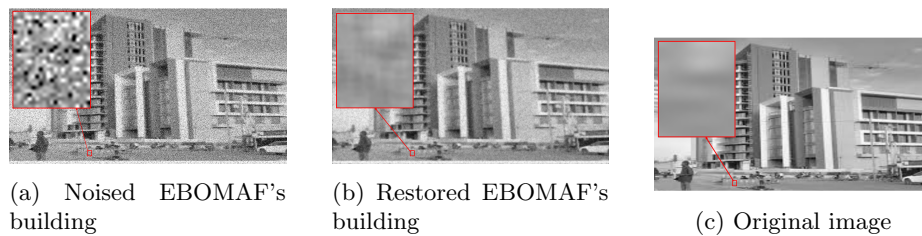


Fig. 4: Building of EBOMAF image restoration processes.

The visual analysis of its different images shows that the model allows an efficient reconstruction of images degraded by various types of noise. We recall here that the images were subject to Cauchy noise degradation at a level of 10% and by Poisson noise, see Fig.(1a), Fig.(2a), Fig.(3a), and Fig.(4a). The second assessment of image quality concerns the use of objective criteria that we have considered in these studies: SSIM and PSNR. The evaluation of the quality of the reconstructed images is summarized in Table 2. The restored images are shown on Fig.(1b) (for Lena), Fig.(2b) (for the Wall), Fig.(3b) (for Pr Joseph KI-ZERBO), and Fig.(4b) (for the EBOMAF's building).

Table 2: Summary of SSIM values and PSNR values of restored images.

Images	SSIM of restored image	PSNR of restored image
Lena	0.7420	28.8376
Wall	0.7655	26.4822
KI-ZERBO	0.6272	25.4285
Building	0.7442	26.1227

The analysis of these results shows an improvement in the quality of the different images processed.

5 Conclusion

In this work, we developed a model based on the variable exponent Laplacian for denoising images corrupted by multiple types of noise, particularly Poisson and Cauchy noise. It is a Cauchy problem whose initial condition is obtained through the application of the proposed CC-Filter. This CC-Filter offers a robust solution to the limitations of traditional filtering methods, which frequently fail when confronted with complex or multi-modal noise. Theoretical and numerical studies demonstrate the effectiveness of the model for denoising multiple types of noise. Future work will focus on proposing new algorithms to improve the results of our numerical simulations.

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