



Intuitionistic Fuzzy Ideal of BS-algebras

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ARTICLE INFO	ABSTRACT
Published Online: 24 April 2026 Corresponding Author: P. Ayesha Parveen	The pupose of this paper is to apply the concept of Intuitionistic Fuzzy Set in BS-algebras. Few algebraic nature of Intuitionistic Fuzzy Ideal is studied in BS-algebras. Homomorphic behaviour is also analysed in IFI of BS-algebras.
KEYWORDS: BS-algebras, Fuzzy ideal, IFI, Homomorphism.	

1. INTRODUCTION

After the introduction of fuzzy subsets by L.A.Zadeh[7], several researches explored on the generalization of the notion of fuzzy subset. In 1966, Imai and Iseki introduced two classes of abstract algebras viz. BCK-algebras and BCI-algebras[5]. J.Negggers and H.S. Kim introduced the notion of B-algebras[6] which is a generalisation of BCK-algebras. I introduce the notion of BS-algebras which is a generalisation of B-algebras and applied the concept of Fuzzy ideal in it[3]. We established the notion of Doubt fuzzy bi-ideal of BS-algebras[1]. We extended the concept of BS-Algebras to Neutrosophic Fuzzy Bi-ideal of BS-algebras[2] and Neutrosophic Doubt Fuzzy Bi-ideal of BS-algebras[4]

In this paper, I study the notion of Intuitionistic Fuzzy Ideal in BS-algebras and investigate some of its algebraic properties. Finally, the homomorphic behaviour of IFI in BS-algebras is also analysed.

2. PRELIMINARIES

In this Section, We recall some basic definitions which are needed for our study.

Definition 2.1 [3]. A BS-Algebra $\mathfrak{B}$ is a non empty set with a constant 1 and a binary operation $*$ satisfying the following axioms

- (i) $a*a=1$
- (ii) $a*1=a$
- (iii) $(a*b)*c = a*(c*(1*b)) \forall a, b, c \in \mathfrak{B}$

Definition 2.2 [5]. An Intuitionistic Fuzzy Set (IFS) in a non empty set $\mathfrak{B}$ is an object having the form IFS $A = \{x, \mu_A(x), \lambda_A(x)/x \in \mathfrak{B}\}$ where the function $\mu_A : \mathfrak{B} \rightarrow [0, 1]$ and

$\lambda_A : \mathfrak{B} \rightarrow [0, 1]$ denote the degree of membership and the degree of non-membership of each element $x \in \mathfrak{B}$ to the set A respectively and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1 \forall x \in \mathfrak{B}$.

In simple, $A = (\mu_A, \lambda_A)$ for the IFS

$A = \{x, \mu_A(x), \lambda_A(x)/x \in \mathfrak{B}\}$

Definition 2.3 [3]. A fuzzy set A in $\mathfrak{B}$ is called a fuzzy ideal of $\mathfrak{B}$ if it satisfies

- i) $\mu(1) \geq \mu(y)$
- ii) $\mu(y) \geq \min\{\mu(y * x), \mu(x)\}$

Example 2.4 [3]. Let $\mathfrak{B} = \{1, a, b, c\}$ be a set with the following Cayley table

*	1	a	b	c
1	1	a	b	c
a	a	1	c	b
b	b	c	1	a
c	c	b	a	1

Then $(\mathfrak{B}, *, 1)$ is a BS-algebra. Define a fuzzy set $\mu : \mathfrak{B} \rightarrow [0, 1]$ by $\mu(1) = \mu(b) = 0.7$ and $\mu(a) = \mu(c) = 0.3$ is the fuzzy ideal of $\mathfrak{B}$.

3. INTUITIONISTIC FUZZY IDEAL OF BS-ALGEBRAS

In this section, we study about Intuitionic Fuzzy Ideal of BS-Algebras and studied some of their algebraic properties.

Definition 3.1. An IFS $A = (\mu_A, \lambda_A)$ in $\mathfrak{B}$ is called an IFI of $\mathfrak{B}$ if it satisfies the following inequalities

- i) $\mu_A(1) \geq \mu_A(x)$
- ii) $\mu_A(y) \geq \min\{\mu_A(x), \mu_A(y * x)\}$
- iii) $\lambda_A(1) \leq \lambda_A(x)$
- iv) $\lambda_A(y) \leq \max\{\lambda_A(x), \lambda_A(y * x)\} \forall x, y \in \mathfrak{B}$.

Example 3.2 . Let $\mathfrak{B} = \{1, a, b, c\}$ be a set with the following Cayley table

*	1	a	b	c
1	1	a	b	c
a	a	1	c	b
b	b	c	1	a
c	c	b	a	1

Then $(\mathfrak{B}, *, 1)$ is a BS-algebra. Define a fuzzy set μ_A in $\mathfrak{B}$ by $\mu_A(1) = \mu_A(b) = 0.8$ and $\mu_A(a) = \mu_A(c) = 0.4$ and the fuzzy set λ_A in $\mathfrak{B}$ by $\lambda_A(1) = \lambda_A(a) = 0.3$ and $\lambda_A(b) = \lambda_A(c) = 0.7$. Then $A = (\mu_A, \lambda_A)$ is IFI of $\mathfrak{B}$.

Theorem 3.3. Let $A = (\mu_A, \lambda_A)$ be an IFI of $\mathfrak{B}$. If $y \leq x$ in $\mathfrak{B}$, then $\mu_A(y) \geq \mu_A(x)$, $\lambda_A(y) \leq \lambda_A(x)$ that is μ_A is order reversing and λ_A is order preserving.

Proof. Let $x, y \in \mathfrak{B}$ be such that $y \leq x$ then $y * x = 1$ and

$$\begin{aligned} \text{Now, } \mu_A(y) &\geq \min\{\mu_A(x), \mu_A(y * x)\} \\ &= \min\{\mu_A(x), \mu_A(1)\} \\ &= \mu_A(x) \end{aligned}$$

Therefore, $\mu_A(y) \geq \mu_A(x)$

$$\begin{aligned} \text{Now, } \lambda_A(y) &\leq \max\{\lambda_A(x), \lambda_A(y * x)\} \\ &= \max\{\lambda_A(x), \lambda_A(1)\} \\ &= \lambda_A(x) \end{aligned}$$

Therefore, $\lambda_A(y) \leq \lambda_A(x)$.

Hence, μ_A is order reversing and λ_A is order preserving.

Theorem 3.4. An IFS $A = (\mu_A, \lambda_A)$ in $\mathfrak{B}$ is an IFI of $\mathfrak{B}$ if and only if the fuzzy sets μ_A and λ_A^c are fuzzy ideals of $\mathfrak{B}$.

Proof. Let $A = (\mu_A, \lambda_A)$ be an IFI of $\mathfrak{B}$. Clearly, μ_A is a fuzzy ideal of $\mathfrak{B}$. For every $x, y \in \mathfrak{B}$, We have

$$\lambda_A^c(1) = 1 - \lambda_A(1) \geq 1 - \lambda_A(x) = \lambda_A^c(x)$$

Therefore, $\lambda_A^c(1) \geq \lambda_A^c(x)$.

$$\begin{aligned} \text{Now, } \lambda_A^c(y) &= 1 - \lambda_A(y) \\ &\geq 1 - \max\{\lambda_A(x), \lambda_A(y * x)\} \\ &= \min\{1 - \lambda_A(x), 1 - \lambda_A(y * x)\} \\ &= \min\{\lambda_A^c(x), \lambda_A^c(y * x)\} \end{aligned}$$

Therefore, $\lambda_A^c(y) \geq \min\{\lambda_A^c(x), \lambda_A^c(y * x)\}$.

Hence λ_A^c is a fuzzy ideal of $\mathfrak{B}$.

Conversely, assume that μ_A and λ_A^c are fuzzy ideals of $\mathfrak{B}$. For every $x, y, z \in \mathfrak{B}$,

we get $\mu_A(1) \geq \mu_A(x)$ and

$$1 - \lambda_A(1) = \lambda_A^c(1) \geq \lambda_A^c(x) = 1 - \lambda_A(x).$$

(i.e.,) $\lambda_A(1) \leq \lambda_A(x)$

Also, $\mu_A(y) \geq \min\{\mu_A(x), \mu_A(y * x)\}$ and

$$\begin{aligned} 1 - \lambda_A(y) &= \lambda_A^c(y) \\ &\geq \min\{\lambda_A^c(x), \lambda_A^c(y * x)\} \\ &= \min\{1 - \lambda_A(x), 1 - \lambda_A(y * x)\} \end{aligned}$$

$$= 1 - \max\{\lambda_A(x), \lambda_A(y * x)\}$$

$$(i.e) \lambda_A(y) \leq \max\{\lambda_A(x), \lambda_A(y * x)\}$$

Hence $A = (\mu_A, \lambda_A)$ is an IFI of $\mathfrak{B}$.

Theorem 3.5. Let $A = (\mu_A, \lambda_A)$ be an IFS in $\mathfrak{B}$. Then $A = (\mu_A, \lambda_A)$ is an IFI of $\mathfrak{B}$ iff $A = (\mu_A, \mu_A^c)$ and $A = (\lambda_A^c, \lambda_A)$ are IFI of $\mathfrak{B}$.

Proof. If $A = (\mu_A, \lambda_A)$ is an IFI of $\mathfrak{B}$, then $\mu_A = \mu_A^c$ and λ_A are fuzzy ideals of $\mathfrak{B}$ from previous theorem. Hence $A = (\mu_A, \mu_A^c)$ and $A = (\lambda_A^c, \lambda_A)$ are IFI of $\mathfrak{B}$. Conversely, if $A = (\mu_A, \mu_A^c)$ and $A = (\lambda_A^c, \lambda_A)$ are IFI of $\mathfrak{B}$, then the fuzzy set μ_A and λ_A^c are fuzzy ideals of $\mathfrak{B}$. Hence $A = (\mu_A, \lambda_A)$ is an IFI of $\mathfrak{B}$.

Theorem 3.6. An IFS $A = (\mu_A, \lambda_A)$ is an IFI of $\mathfrak{B}$ iff for all $s, t \in [0, 1]$, the sets $U(\mu_A : t)$ and $L(\lambda_A : s)$ are either empty or ideals of $\mathfrak{B}$.

Proof. Let $A = (\mu_A, \lambda_A)$ be an IFI of $\mathfrak{B}$ and $U(\mu_A : t) \neq \emptyset$

$L(\lambda_A : s) \neq \emptyset$ for any $s, t \in [0, 1]$. It is clear that

$$1 \in U(\mu_A : t) \cap L(\lambda_A : s).$$

Since $\mu_A(1) \geq t$ and $\lambda_A(1) \leq s$.

Let $x, y \in \mathfrak{B}$ be such that $(y * x) \in U(\mu_A : t)$ and

$x \in U(\mu_A : t)$. Then $\mu_A(x) \geq t$ and $\mu_A(y * x) \geq t$. It follows

that $\mu_A(y) \geq \min\{\mu_A(x), \mu_A((y * x))\} \geq t$ so that

$y \in U(\mu_A : t)$. Hence $U(\mu_A : t)$ is an ideal of $\mathfrak{B}$.

Now let $x, y \in \mathfrak{B}$ be such that $x \in L(\lambda_A : s)$ and

$(y * x) \in L(\lambda_A : s)$. Then $\lambda_A(x) \leq s$ and $\lambda_A(y * x) \leq s$ which imply

that $\lambda_A(y) \leq \max\{\lambda_A(x), \lambda_A(y * x)\} \leq s$.

Thus $y \in L(\lambda_A : s)$ and therefore $L(\lambda_A : s)$ is an ideal of $\mathfrak{B}$.

Conversely, assume that for each $t, s \in [0, 1]$, the sets

$U(\mu_A : t)$ and $L(\lambda_A : s)$ are either empty or ideal of $\mathfrak{B}$.

For any $y \in \mathfrak{B}$, let $\mu_A(y) = t$ and $\lambda_A(y) = s$. Then

$y \in U(\mu_A : t) \cap L(\lambda_A : s)$ and so $U(\mu_A : t) \neq \emptyset \neq L(\lambda_A : s)$.

Since $U(\mu_A : t)$ and $L(\lambda_A : s)$ are ideals of $\mathfrak{B}$, therefore

$$1 \in U(\mu_A : t) \cap L(\lambda_A : s). \text{ Hence } \mu_A(1) \geq t = \mu_A(x) \text{ and}$$

$$\lambda_A(1) \leq s = \lambda_A(x) \forall x \in \mathfrak{B}.$$

If there exists $x', y' \in \mathfrak{B}$ such that

$\mu_A(y') < \min\{\mu_A(x'), \mu_A(y' * x')\}$, then by taking

$t_0 = (1/2)(\mu_A(y') + \min\{\mu_A(x'), \mu_A(y' * x')\})$, we have

$$\mu_A(y') < t_0 < \min\{\mu_A(x'), \mu_A(y' * x')\}.$$

Hence $y' \in U(\mu_A : t_0)$, $x' \in U(\mu_A : t_0)$ and

$(y' * x') \in U(\mu_A : t_0)$ that is $U(\mu_A : t_0)$ is not an ideal of $\mathfrak{B}$,

which is a contradiction.

Finally assume that there exists $a, b \in \mathfrak{B}$ such that

$$\lambda_A(b) > \max\{\lambda_A(a), \lambda_A(b * a)\}$$

Taking $s_0 = (1/2)(\lambda_A(b) + \max\{\lambda_A(a), \lambda_A(b * a)\})$, then

$$\max\{\lambda_A(a), \lambda_A(b * a)\} < s_0 < \lambda_A(b).$$

Therefore $(b * a) \in L(\lambda_A : s_0)$ and $a \in L(\lambda_A : s_0)$

but $b \notin L(\lambda_A : s_0)$ which is a contradiction,

this completes the proof.

4. HOMOMORPHISM ON INTUITIONISTIC FUZZY IDEAL OF BS-ALGEBRAS

In this section, we discuss about homomorphism

Definition 4.1. A mapping $f : X \rightarrow Y$ of BS-algebra is called a homomorphism if $f(x * y) = f(x) * f(y) \forall x, y \in \mathfrak{B}$.

Let $f: X \rightarrow Y$ be a homomorphism of $\mathfrak{B}$ for any IFS

$A = (\mu_A, \lambda_A)$ in Y . We define new IFS $A^f = (\mu_A^f, \lambda_A^f)$ in $\mathfrak{B}$ by $\mu_A^f = \mu_A(f(x))$, $\lambda_A^f = \lambda_A(f(x))$

Theorem 4.2. Let $f: X \rightarrow Y$ be a homomorphism of BS-algebra $\mathfrak{B}$. If an IFS

$A = (\mu_A, \lambda_A)$ is an IFI of Y , then an IFS

$A^f = (\mu_A^f, \lambda_A^f)$ in $\mathfrak{B}$ is an IFI of $\mathfrak{B}$.

Proof. We first have that

$$\begin{aligned}\mu_A^f(x) &= \mu_A(f(x)) \\ &\leq \mu_A(1) \\ &= \mu_A(f(1)) \\ &= \mu_A^f(1) \quad \forall x \in X\end{aligned}$$

Therefore, $\mu_A^f(x) \leq \mu_A^f(1)$.

$$\begin{aligned}\text{Similarly, } \lambda_A^f(x) &= \lambda_A(f(x)) \\ &\geq \lambda_A(1) \\ &= \lambda_A(f(1)) \\ &= \lambda_A^f(1).\end{aligned}$$

Therefore, $\lambda_A^f(x) \leq \lambda_A^f(1) \quad \forall x \in X$

Let $x, y \in X$.

$$\begin{aligned}\text{Then } \mu_A^f(y) &= \mu_A(f(y)) \\ &\leq \min\{\mu_A(f(x)), \mu_A(f(y * x))\} \\ &= \min\{\mu_A^f(x), \mu_A^f(y * x)\}\end{aligned}$$

Therefore, $\mu_A^f(y) \leq \min\{\mu_A^f(x), \mu_A^f(y * x)\}$.

similarly, $\lambda_A^f(y) \geq \max\{\lambda_A^f(x), \lambda_A^f(y * x)\}$.

Hence, $A^f = (\mu_A^f, \lambda_A^f)$ in $\mathfrak{B}$ is an IFI of $\mathfrak{B}$.

Theorem 4.3. Let $f: X \rightarrow Y$ be an epimorphism of BS-algebra

$\mathfrak{B}$. Let $A = (\mu_A, \lambda_A)$ be an IFS of Y .

If $A^f = (\mu_A^f, \lambda_A^f)$ is an IFI of $\mathfrak{B}$, then $A = (\mu_A, \lambda_A)$ is an IFI of Y .

Proof. For any $x, y \in X$, there exists $a, b \in Y$ such that

$$f(x) = a, f(y) = b.$$

$$\begin{aligned}\text{Now, } \mu_A(a) &= \mu_A(f(x)) \\ &= \mu_A^f(x) \\ &\leq \mu_A^f(1) \\ &= \mu_A(f(1)) \\ &= \mu_A(1)\end{aligned}$$

$$\mu_A(a) \leq \mu_A(1)$$

Similarly, $\lambda_A(a) \geq \lambda_A(1)$.

$$\begin{aligned}\text{Now, } \mu_A(b) &= \mu_A(f(y)) \\ &= \mu_A f(y) \\ &= \mu_A^f(y) \\ &\leq \min\{\mu_A^f(x), \mu_A^f(y * x)\} \\ &= \min\{\mu_A(f(x)), \mu_A(f(y * x))\} \\ &= \min\{\mu_A(f(x)), \mu_A(f(y) * f(x))\} \\ &= \min\{\mu_A(a), \mu_A(b * a)\}\end{aligned}$$

Therefore, $\mu_A(b) \leq \min\{\mu_A(a), \mu_A(b * a)\}$.

Similarly, $\lambda_A(b) \geq \max\{\lambda_A(a), \lambda_A(b * a)\}$.

Hence, $A = (\mu_A, \lambda_A)$ is an IFI of Y .

CONCLUSION

In this paper, the notion of Intuitionistic fuzzy ideal of BS-algebras are introduced and studied their algebraic properties. We obtained the homomorphic behaviour of Intuitionistic fuzzy ideal for BS-algebras.

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