



The Impact of Public Health Awareness and Behavioural Change with Intervention on the Transmission Dynamics of Sexually Transmitted Diseases Amongst Tertiary Institutions Students: A Mathematical Modelling Perspectives

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ARTICLE INFO	ABSTRACT
Published Online: 03 April 2026	The dynamics of sexually transmitted diseases and infections (STDs/STIs) amongst tertiary institutions students are significantly influenced by public health awareness campaign and behaviour changes resulting from treatment interventions strategies. five-compartmental deterministic nonlinear ordinary differential equations were formulated to investigate the behaviour of the system. This study revealed that endemic equilibrium point is stable and the disease persist at $R_0 > 1$, annihilation at $R_0 < 1$ and backward bifurcation backward bifurcation at $R_0 = 1$ with multiple equilibria. The findings showed that tertiary institution students would be a high-risk group for their social life as experimentation increases the students chance of been more exposed to STDs/STIs and the susceptible population of students in the study model would require the awareness and education on STDs/STIs by including it as part of the curriculum in the first year general studies (GNS) with key preventions strategies to curb the transmission dynamics amid the campuses. This study also revealed that lack of safe sex and hygiene management practices amongst year one/two students increases the surge vital and STDs/STIs reproduction number. This findings was a follow up of the study assessment of 475 tertiary institution students' knowledge on the transmission dynamics and control of sexually transmitted diseases on campus using a questionnaire. It was also shown that the mean age of the respondents was 18.5 years which comprises tertiary institution students of age 16, 17 and 18 regarding STDs/STIs. Parameters like recruitment rate, Ψ, spectral radius ϕ and removal rate δ are of great importance to the model system. Above all, the study recommends that public health sector should sustain their efforts in awareness campaigns, education, monitoring and surveillance essentials and the students should get tested regularly, practice safe sex and seek help if needed.
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KEYWORDS: Behavioural changes; Intervention; Transmission dynamics; Sexually transmitted diseases; Tertiary institutions students; Mathematical modelling; Public health awareness	

I. INTRODUCTION

Public health awareness and behavioural change interventions are essential with evidence-based strategies for reducing the transmission dynamics of sexually transmitted diseases/infections (STDs/STIs). The interventions includes educational campaigns, condom distribution and partner notification services which are designed to interrupt the chain of transmission by reducing risk behaviours and

increasing prompt and appropriate treatment-seeking behaviour [34, 2]. Effective intervention treatments are typically multifaceted combining biomedical such as vaccination, pre-exposure prophylaxis (PrEP) and behavioural approaches to achieve significant reductions in incidence [10, 29].

Globally, sexually transmitted diseases/infections (STDs/STIs) represent a major public health challenge

affecting millions of individuals and posing risks to both individual, community health and tertiary institutions. The dynamics of STD/STI transmission are influenced by a variety of factors which includes social behaviours, access to healthcare and awareness of prevention methods. Public health awareness campaigns play a crucial role in educating people about the risks associated with STDs/STIs, the importance of safe practices, and the availability of treatment options [26].

Behavioural changes, prompted by effective intervention strategies, can significantly alter the trajectory of STD/STI transmission within populations. Interventions such as educational outreach, counselling and the promotion of regular screenings can empower individuals to make informed decisions, reduce stigma and ultimately lead to increased utilization of preventive services [7]. By examining the interplay between public health awareness, behavioural changes and strategic interventions. This exploration does not only highlights the importance of targeted public health strategies but also lay emphasizes on the necessity for continued investment in awareness programs to curb the spread of STDs/STIs effectively.

The basic behavioural Interventions and Impact are on the use of condom promotion and distributions as one of the most effective methods to prevent STDs/STIs. Interventions that increase condom availability, particularly in high-risk scenarios have proven effective in reducing HIV and STI incidence [13]. Thus, Individual and group counselling sessions are effective in reducing sexual risk behaviours like reducing the number of partners and promoting safe sex, particularly among adolescent students and in high-risk populations. Targeted services and programs that track and treat the partners of infected individuals reduce the, duration, of, infection, and prevent further, transmission. In-school and community-based education is important for students and general populace to understand the risk, promote consistent condom use and reduce stigmatization [31].

The transmission dynamics and intervention strategies as in reduction risk probability by taking interventions in education and counselling aim to lower the probability of transmission per act by encouraging safer sex practices [8]. The behavioural programs work is to lower the rate of sexual partner turnover which is a key driver of rapid infection spread. The limitation of infectious duration is through early detection by regular screening and prompt treatment for instance syphilis and chlamydia to reduce the time a student is infectious to others. Also, targeting high-risk groups in interventions are most effective when focused on individuals with high rates of partner change and within dense domain sexual networks [27].

The challenges identified in this study are that the students' knowledge and action with increase in the awareness level does not always translate directly into behavioural change as

social, emotional and cultural factors also influence behaviour [14]. The impact of behavioural interventions may diminish over time and hence require sustained and repeated efforts. Structural Barriers affects effective intervention which requires to address social and structural factors such as stigma, poverty and lack of access to healthcare, While useful in specific cases like mass treatment can lead to antimicrobial resistance and a rapid rebound of infections if not supported by ongoing prevention measures which is point of endemic equilibrium. Amid other researchers [21, 35] developed a mathematical modelling to study the epidemiological impact of a syphilis vaccine and estimating its incidence and diagnostics rate using simulations.

In the same vein, [3, 16] investigated health messages framing effects on attitudes, intentions and behaviours of sex workers by reviewing a community-based prevention that leads to increase in use of condom and a reduction in sexually transmitted infections (STIs) among men and female sex workers. For more analysis of this survey the readers are encouraged to explore the works carried out by the following researchers on the reference page [1, 11, 19, 24, 33] and more applications of mathematical modelling to infectious diseases in the following studies [5, 17, 28, 32].

Recently, scientists used mathematical models to study Syphilis pervasiveness movements in adult women in 242 countries under the estimations using Spectrum Sexually Transmitted Infections model with vulnerability and awareness of sexually transmitted infections among female traders of reproductive age [4, 9, 15, 22,30]. Thus, [23, 25] presented a global and regional mortality from 124 deaths for 20 age groups between 1990 and 2010 as a systematic analysis for global burden on disease. However, a proposed mathematical model for sustainability on STDs/STIs interventions strategies were made by [6] and [20] respectively while the exploring education for sustainable interventions development strategies on infectious diseases and its inclusion in the curriculum education was suggested by [2,18].

2 MATERIALS AND METHODS

2.1The Mathematical Model Formulation

This study developed a five compartmental deterministic sex-structured STDs/STIs mathematical model of susceptible individuals with public health campaign/educational and counselling for STDs/STIs, susceptible individuals with intervention strategies and behavioural change for STDs/STIs, susceptible individuals who trust and protect their partner for STDs/STIs, infected individuals with STDs/STIs and recovered /removed individuals from STDs/STIs that incorporates the intervention strategy that helps to change the sexual behaviours of some students in the three susceptible sub-

class. This study considers unaware STDs/STIs susceptible individuals who have received proper or effective STDs/STIs public health education/counselling against risky sexual behaviours that may result in STDs diseases. This change in behavior leads to subdividing the susceptible individuals into three subclasses, namely unaware STDs/STIs susceptible individuals S_1 , aware STDs susceptible individuals who modify their sexual behaviours S_2 , and aware STDs susceptible individuals who trust and protect their uninfected sexual partners for life S_3 . Individuals only enter the general unaware STDs/STIs susceptible class S_1 at the recruitment rate of Ψ and natural death rate for all subclasses is δ .

As a result of awareness (through STDs/STIs counselling and testing) a proportion of the susceptible leave the general unaware susceptible class S_1 at the rate φ , $0 < \varphi < 1$ out of which a fraction λ , ($0 < \lambda < 1$) reduce their sexual behaviours sufficiently by remaining trustful to their uninfected sexual partners for the rest of their lives (literally immune to STDs/STIs infection by sexual contact) and so move to the S_3 subclass while the complementary $(1, \varphi)$ reduce their sexual activities and so move to the S_2 subclass. The expected reduce the risk sexual behaviours by the S_2 as a result of awareness is counted for by the parameter $0 < \Upsilon < 1$. It is relevant to note that the inclusion of the S_3 subclass is very significant because an appreciable number of people have now changed their sexual behaviours sufficiently due to the awareness of the widespread nature of STDs/STIs in the society. The monumental deaths resulting from the disease increases knowledge of the agony and psychological trauma experienced by the infected individuals and better enlightenment due to intense STDs/STIs educational campaigns, [12]. The significance of the S_3 subclass is that it emphasizes the importance of prevention for a disease such as STDs/STIs and HIV that has no cure. Increasing the members of this subclass is one of the keys to control the spread of the disease. Since it is assumed in this study that the S_3 subclass is not involved in extra-marital activities and transmission, the infection with STDs/STIs is with S_1 and S_2 subclasses only, with effective infection contact rate β_1 and β_2 due to their interaction with the infected class, S_4 . Thus, the disease spreads due to the direct contact between the S_1 and S_2 subclasses with the infected class (S_4). Individuals in the S_4 class, with a disease-induced death rate α , receive treatment with medications at a rate ε

and hence move to the removed class S_5 of individuals who are assumed in this study not to be involved in transmission as a result of efficacy of the medications. The S_5 class has an additional disease-induced death rate of ν .

Based on the fact that the infectious period of STDs/STIs is very long (≥ 10 years), the population size was regarded as varying and not constant. From the model assumptions above, the model takes the form of the following deterministic system of nonlinear ordinary differential equations that describes the interaction of STDs/STIs public health awareness with unaware STDs/STIs susceptible individuals as: S_1 represents the individuals with public health campaign/educational and counselling (unaware of STDs/STIs), S_2 represents intervention strategies and behavioural change (aware of the STDs/STIs), S_3 represents individuals who trust and protect their partner, S_4 represents infected individuals with STDs/STIs and S_5 represents the recovery /removed individuals from STDs/STIs given as:

$$\begin{aligned}\dot{S}_1 &= \Psi - \lambda_1 S_1 - \varphi S_1 - \delta S_1 \\ \dot{S}_2 &= (1 - \omega)\varphi S_1 - \Upsilon \lambda_2 S_2 - \delta S_2 \\ \dot{S}_3 &= \omega \varphi S_1 - \delta S_3 \\ \dot{S}_4 &= \lambda_1 S_1 + \Upsilon \lambda_2 S_2 - (\alpha + \varepsilon + \delta) S_4 \\ \dot{S}_5 &= \varepsilon S_4 - (\nu + \delta) S_5\end{aligned}\quad (1)$$

Thus, the total population size $N(t)$ at time t , is given by

$$N(t) = S_1(t) + S_2(t) + S_3(t) + S_4(t) + S_5(t) \quad (2)$$

with initial conditions $S_1(0) > 0, S_2(0) > 0, S_3(0) > 0, S_4(0) \geq 0, S_5(0) \geq 0$

and the rate of change of the total population is given by adding the system of model equations (1) to give

$$N' = \Psi - \delta N - \alpha S_4 - \nu S_5 \quad (3)$$

From equation (1), the force of infection for the S_1 and S_2 subclasses are λ_1 and λ_2 respectively, where $\lambda_1 = \beta_1 S_4$ and $\lambda_2 = \beta_2 S_4$. This system of coupled equations (1) monitors human populations and STDs public health awareness campaign strategies with intervention/ treatment, the assumption is that all the state variables are non-negative for all time $t \geq 0$. Also, all the parameters are non-negative and can be shown that the solutions of the system of equations are non-negative haven non-negative initial conditions.

3 MODEL ANALYSIS

3.1 Invariant Region

Theorem 1. The region $\Omega \subset R^5$ is positively-invariant for the system of model Eq. (1) with non-negative initial conditions in Ω .

Proof. From Eq. (3)

$$N' = \Psi - \delta N - \alpha S_4 - \nu S_5$$

For the initial condition in R^5 for $t \geq 0$ from equation (1), we get;

$$N' \leq \Psi - \delta N \quad (4)$$

Solving the differential inequality equation (4) using separable variables, yields

$$N' + N\delta \leq \Psi$$

The integrating factor, IF $e^{\int \delta dt} = e^{\delta t}$;

$$(N' + \delta N)e^{\delta t} \leq \Psi e^{\delta t} \Rightarrow$$

$$\frac{d(N\delta e^{\delta t})}{dt} \leq \Psi e^{\delta t}$$

Integrating both sides gives

$$N\delta e^{\delta t} \geq \Psi e^{\delta t} + K \quad (5)$$

Obtaining the constant K by applying the initial condition $N(0) = N_0$ to equation (5), we get,

$$K \leq \Psi - \delta N_0; \text{ then (5) becomes}$$

$$N \leq \frac{\Psi}{\delta} + \left(\frac{\Psi - \delta N_0}{\delta}\right)e^{-\delta t} \Rightarrow \quad (6)$$

$$N(t) \leq \frac{\Psi}{\delta} \text{ if } N(0) \leq \frac{\Psi}{\delta}.$$

This follows from (6) that as $t \rightarrow \infty$, the total population, $N \rightarrow \frac{\Psi}{\delta}$, which implies that $0 \leq N \leq \frac{\Psi}{\delta}$. Thus, the model system Eq. (1) will be analysed in the following feasible region, $\Omega \subset R^5$. With this we conclude that the solution set of system (1) is positively-invariant and fascinating for the model system (2) and always remains in the domain, $\Omega = \{(S_1, S_2, S_3, S_4, S_5) \in R^5 ; 0 \leq N \leq \frac{\Psi}{\delta}\}$. (7)

3.2 Positivity of Solutions

Theorem 2 The variables of the coupled system of (1) are non-negative for all time. Hence, the solutions of the model eq. (1) with positive initial data will remain positive for all time, $t \geq 0$.

Proof: Let

$$t_1 = \sup\{t > 0 : S_1 > 0, S_2 > 0, S_3 > 0, S_4 > 0, S_5 > 0 \in [0, t]\}$$

.Thus $t_1 > 0$. It follows from the first equation of the model system Eq.(1) that

$$\frac{dS_1}{dt} = -\lambda_1(t)S_1(t) - \varphi S_1(t) - \delta S_1(t)$$

$$= \Psi - (\lambda_1(t) + \varphi(t) + \delta)S_1(t) \quad (8)$$

where $\lambda_1 = \beta_1 S_4$. This is the same as

$$\frac{dS_1}{dt} + (\lambda_1(t) + \varphi(t) + \delta)S_1(t) = \Psi$$

and this implies that

$$\begin{aligned} \frac{d}{dt} \left[S_1(t) \exp \left\{ dt + \int_0^t (\lambda_1(\tau) + \varphi(\tau)) d\tau \right\} \right] \\ = \Psi \exp \left\{ dt + \int_0^t (\lambda_1(\tau) + \varphi(\tau)) d\tau \right\} \end{aligned} \quad (9)$$

Hence,

$$\begin{aligned} S_1(t_1) \exp \left\{ dt_1 + \int_0^{t_1} (\lambda_1(\tau) + \varphi(\tau)) d\tau \right\} - S_1(0) \\ = \int_0^{t_1} \Psi \exp \left\{ dy + \int_0^y (\lambda_1(\epsilon) + \varphi(\epsilon)) d\epsilon \right\} dy \end{aligned} \quad (10)$$

Thus,

$$\begin{aligned} S_1(t_1) = S_1(0) \exp \left\{ -dt_1 + \int_0^{t_1} (\lambda_1(\tau) + \varphi(\tau)) d\tau \right\} \\ + \left[\exp \left\{ -(dt_1 + \int_0^{t_1} (\lambda_1(\tau) + \varphi(\tau)) d\tau \right\} \right] \\ \times \int_0^{t_1} \Psi \exp \left\{ dy + \int_0^y (\lambda_1(\epsilon) + \varphi(\epsilon)) d\epsilon \right\} dy \geq 0. \end{aligned} \quad (11)$$

The same method could be used to solve for $S_2 \geq 0$, $S_3 \geq 0$, $S_4 \geq 0$ and $R_5 \geq 0$, for all time, $t \geq 0$. The theorem 2 shows that the model variables are continuous and biologically meaningful since the population size cannot be negative. Thus, for any starting non-negative initial conditions, the trajectory lies in the region, Ω . Therefore, the system is both mathematically and epidemiologically well-posed [11, 16]. Hence, it is sufficient to consider the forces at work of the movement generated by the model equation (1) in Ω .

4 EQUILIBRIUM POINTS AND STABILITY ANALYSIS

The model study (1) has two non-negative equilibria: disease free equilibrium (DFE) point and endemic equilibrium (EE) point.

4.1 Disease Free Equilibrium (DFE) Point DFE and Basic Reproduction Number (R0) The disease free equilibrium, E_0 is obtained by setting the left-hand sides of the coupled system of equations (1) to zero with initial conditions $S_1 = 0, S_2 = 0, S_3 = 0, S_4 = 0, R_5 = 0$ and $S_1 = S_2 = S_3 = S_4 = R_5 = 1$. Let the equilibria be E_0 and E_e so that

$$\Psi - \lambda_1 S_1 - \varphi S_1 - \delta S_1 = 0 \Rightarrow$$

$$\Psi - \beta_1 S_1 S_4 - \varphi S_1 - \delta S_1 = 0 \Rightarrow S_1 = \frac{\Psi}{(\varphi + \delta)}$$

$$\begin{aligned}
 (1 - \omega)\varphi S_1 - \gamma \lambda_2 S_2 - \delta S_2 &= 0 \Rightarrow \\
 S_2 &= \frac{(1-\omega)\varphi\Psi}{(\varphi+\delta)\delta} \\
 \omega\varphi S_1 - \delta S_3 &= 0 \Rightarrow S_3 = \frac{\omega\varphi\Psi}{\delta(\varphi+\delta)} \\
 (12) \\
 \lambda_1 S_1 + \gamma \lambda_2 S_2 - (\alpha + \varepsilon + \delta) S_4 &= 0 \Rightarrow S_4 = 0 \\
 \varepsilon S_4 - (v + \delta) S_5 &= 0 \Rightarrow S_5 = 0
 \end{aligned}$$

Therefore, one of the equilibria solution is the trivial solution with non-existence of the five compartments populations as $E_0 = (S_1, S_2, S_3, S_4, S_5) = (0, 0, 0, 0, 0)$

The other equilibrium solution is with semi-trivial solution of the state variables satisfies

$$\begin{aligned}
 E_s &= (S_1^0, S_2^0, S_3^0, S_4^0, S_5^0) = \\
 &= \left(\frac{\Psi}{(\varphi+\delta)}, \frac{(1-\omega)\varphi\Psi}{\delta(\varphi+\delta)}, \frac{\omega\varphi\Psi}{\delta(\varphi+\delta)}, 0, 0 \right)
 \end{aligned}
 \quad (13)$$

The local stability of E_0 can be determined by using the next-generation operator method on the model equations (1). Using the notations in [Driesche and Watmough 2002], the Jacobian Matrices F -for the new infection terms and V - the remaining transfer terms are given by $F = (\beta_1 S_1^0 + \gamma \beta_2 S_2^0)$ and $V = (\alpha + \varepsilon + \delta)$

Multiplying the inverse of V by F yields

$$FV^{-1} = \frac{(\beta_1 S_1^0 + \gamma \beta_2 S_2^0)}{(\alpha + \varepsilon + \delta)} \quad (14)$$

From (14) the basic reproduction number of the mode R_0 becomes

$$\begin{aligned}
 R_0 &= \varphi FV^{-1} = \frac{\varphi(\beta_1 S_1^0 + \gamma \beta_2 S_2^0)}{(\alpha + \varepsilon + \delta)} = \frac{\varphi \beta_1 \Psi}{(\varphi + \delta)(\alpha + \varepsilon + \delta)} + \\
 &+ \frac{\varphi \Psi \gamma \beta_2 (1 - \omega)}{\delta(\varphi + \delta)(\alpha + \varepsilon + \delta)}
 \end{aligned}
 \quad (15)$$

where φ is the spectral radius, that is the dominant eigen-value of $|FV^{-1} - \lambda I| = 0$. Additionally, using theorem 2 in (Driesche and Watmough 2002), we obtained the above result and considered theorem 3.

Theorem 3 The disease free equilibrium of the model equation (1) given by E_0 is locally asymptotically stable (LAS) if $R_0 < 1$ and unstable if and only if $R_0 > 1$.

$$R_0 = \frac{\varphi \beta_1 \Psi}{(\varphi + \delta)(\alpha + \varepsilon + \delta)} + \frac{\varphi \Psi \gamma \beta_2 (1 - \omega)}{\delta(\varphi + \delta)(\alpha + \varepsilon + \delta)} \quad (16)$$

Here, R_0 can be written in terms of S_1 and S_2 as

$$R_0 = R_{S_1} + R_{S_2} \quad (17)$$

$$R_{S_1} = \frac{\varphi \beta_1 \Psi}{(\varphi + \delta)(\alpha + \varepsilon + \delta)} \text{ and } R_{S_2} = \frac{\varphi \Psi \gamma \beta_2 (1 - \omega)}{\delta(\varphi + \delta)(\alpha + \varepsilon + \delta)}$$

where R_{S_1} represents the unaware STDs individuals with public health campaign/educational and counselling, R_{S_2} represents intervention strategies and behavioural change of individuals who are aware of the STDs and are susceptible to infections. The threshold parameter R_0 measures the average number of new STDs infections generated by a single STDs infected individual throughout the infectious period in a completely susceptible population, Anderson and Mary (1991); Dickmann, Heesterbeek and Metz (1990); Hethcote (2000); Driesche and Watmough (2002) in which a proportion of aware susceptible (S_2) are not involved in the dynamics of the disease in the presence of intervention and treatment. Thus, theorem 3 implies that the infection can be eliminated from the population if the initial sizes of the sub-populations are in the hub of attraction of the DFE. Hence, the global stability of DFE shows that it is globally asymptotically stable (GAS) if and only if $R_0 < 1$.

4.2 Global Stability of DFE, E_0

To show global stability of the DFE E_0 in the model equation (1), we adopt the approach of [Castillo-Chavez *et al.* 2002], by writing the coupled equation (1) as

$$\begin{aligned}
 \frac{dX}{dt} &= F(X, Y) \\
 \frac{dY}{dt} &= G(X, Y), \quad G(X, 0) = 0,
 \end{aligned}$$

where $X \in F(S_1, S_2, S_3)$ denotes the number of susceptible individuals and $Y \in F(S_4)$ denotes the number of infected individuals. The DFE of the model system of equation (1) is now denoted as $E_0 = (X^0, 0)$, $X^0 = (S_1^0, S_2^0, S_3^0)$. The conditions for global stability of E_0 is given by

Lemma1.

$\frac{dX}{dt} = F(X^0, 0)$ X^0 is globally asymptotically stable (GAS).

Lemma2.

$$G(X, Y) = NY - G(X, Y), G(X, Y) \geq 0 \text{ for } (X, Y) \in \Omega, \quad (19)$$

where $N = D_Y G(X^0, 0)$ is an M-matrix (whose diagonal elements are non-negative) and Ω is the region where the model system Eq. (2) makes biological and epidemiological sense. If the model system of equ (18) satisfies the conditions lemma 1 and lemma 2 in (19) according to Castillo-Chavez *et al.* 2002, the theorem below holds for the study model equations:

Theorem 4. The DFE $E_0 = (X^0, 0)$ of model system (7) is GAS provided $R_0 < 1$ and lemma 1 and lemma 2 are satisfied.

Proof: To apply This theorem on the model system (18) from the model equ. (1) and (18) gives

$$F(X, 0) = \begin{pmatrix} \Psi - (\varphi + \delta)S_1 \\ (1 - \omega)\varphi S_1 - \delta S_2 \\ \omega\varphi S_1 - \delta S_3 \end{pmatrix}$$

$$G(X, Y) = NY - G(X, Y),$$

where,

$$N = D_Y G(X^0, 0) = \begin{pmatrix} \beta_1 S_1^0 + \gamma\beta_2 S_2^0 - (\varepsilon + \alpha + \delta) \\ \varepsilon \end{pmatrix} \quad (20)$$

from (18) and the conditions in lemma 1 and lemma 2 in (19) yields the following result

$$\begin{aligned} G(X, Y) &= NY - G(X, Y) \\ &= \begin{bmatrix} \beta_1 S_1^0 + \gamma\beta_2 S_2^0 - (\varepsilon + \alpha + \delta) \\ \varepsilon \end{bmatrix} I - \begin{bmatrix} \beta_1 S_4 S_1^0 + \gamma\beta_2 S_4 S_2^0 - (\delta + \alpha + \varepsilon)I \\ \varepsilon S_4 - (v + \delta)S_5 \end{bmatrix} \\ &= \begin{bmatrix} \beta_1 S_4 (S_1^0 - S_1) + \gamma\beta_2 S_4 (S_2^0 - S_2) \\ (v + \delta)S_5 \end{bmatrix} \quad (21) \end{aligned}$$

Since $S_1^0 \geq S_1$ and $S_2^0 \geq S_2$, it is obvious that $G(X, Y) \geq 0$. Therefore, lemma 1 and lemma 2 of theorem 4 are satisfied. Hence, the DFE of model equation (1) given by E_0 is globally asymptotically stable (GAS) when $R_0 < 1$.

This result points towards the public health that if the control measures and strategies for STDs can weaken the reproduction number to less than one, then STDs can be annihilated from the community/population of higher dynamics.

5 EXISTENCE AND STABILITY OF ENDEMIC EQUILIBRIUM POINT (EEP)

Here, the existence and stability of endemic is the positive equilibrium point of the model equation (1). This implies the equilibrium where at least one of the infected components of the model is non-zero will be explored.

5.1 Existence of the Endemic Equilibrium (EE) E_e

When $R_0 > 1$, STDs infection can establish itself or persist in the population hub. Otherwise the infection will be eliminated. If $R_0 > 1$, system of equations (1) has a unique endemic equilibrium E_e given by $E_e = (S_1^*, S_2^*, S_3^*, S_4^*, S_5^*)$, where

$$S_1^* = \frac{\Psi}{\beta_2 S_4^* + (\varphi + \delta)},$$

$$S_2^* = \frac{(1-\omega)\varphi S_1^*}{\gamma\beta_2 S_4^* + \delta} = \frac{(1-\omega)\varphi\Psi}{(\gamma\beta_2 S_4^* + \delta)[\beta_1 S_4^* + (\varphi + \delta)]}, \quad (22)$$

$$S_3^* = \frac{\omega\varphi S_1^*}{\delta} = \frac{\omega\varphi\Psi}{\delta[\beta_1 S_4^* + (\varphi + \delta)]},$$

$$\begin{aligned} (\delta + \alpha + \varepsilon)S_4^* &= \beta_1 S_4^* S_1^* + \gamma\beta_2 S_4^* S_2^* \\ &= \frac{\Psi\beta_1 S_1^*}{\beta_1 S_4^* + (\varphi + \delta)} + \frac{(1-\omega)\varphi\Psi\gamma\beta_2 S_4^*}{(\gamma\beta_2 S_4^* + \delta)[\beta_1 S_4^* + (\varphi + \delta)]} \\ S_5^* &= \frac{\varepsilon S_4^*}{(v - \delta)} \end{aligned}$$

From the fourth equation of system (22) $S_4^* = 0$ corresponds to the DFE, E_e -whose stability has already been established and the other non-zero root is obtained as

$$\frac{\Psi\beta_1 S_1^*}{\beta_1 S_4^* + (\varphi + \delta)} + \frac{(1-\omega)\varphi\Psi\gamma\beta_2 S_4^*}{(\gamma\beta_2 S_4^* + \delta)[\beta_1 S_4^* + (\varphi + \delta)]} - (\varepsilon + \alpha + \delta) = 0 \quad (23)$$

For the occurrence of backward bifurcation, there exists multiple non-zero (endemic) equilibria. Hence, equation (23) yields a quadratic equation in terms of S_4^* as

$$A_1 S_4^{*2} + A_2 S_4^* + A_3 = 0, \quad (24)$$

Thus, by analysing the quadratic equation (24) gives the multiple equilibria where

$$\begin{aligned} A_1 &= \beta_1 \gamma \beta_2 (\delta + \alpha + \varepsilon) > 0, \\ A_2 &= (\delta + \alpha + \varepsilon)[\gamma\beta_2 (\varphi + \delta) + \beta_1 \delta] - \beta_1 \gamma \beta_2 \Psi \\ &= \gamma\beta_2 (\varphi + \delta)(\delta + \alpha + \varepsilon) \left[1 - \frac{\beta_1 A}{(\varphi + \delta)(\delta + \alpha + \varepsilon)} \right] \\ &\quad + \beta_1 \delta (\delta + \alpha + \varepsilon) \\ A_3 &= \delta(\varphi + \delta)(\delta + \alpha + \varepsilon) - \beta_1 \delta \Psi \\ &\quad - \gamma\beta_2 (1 - \omega)\varphi\Psi\delta(\delta + \alpha + \varepsilon) \\ &= (1 - R_0)\delta(\varphi + \delta)(\delta + \alpha + \varepsilon) \end{aligned} \quad (25)$$

For the endemic equilibria to exist, the solutions of (24) must be real and positive so that

$$A_1 \geq 0, A_3 < 0 \Leftrightarrow R > 1; \quad (26)$$

$$A_3 \geq 0 \Leftrightarrow R_0 \leq 1.$$

For equation (24) is a quadratic equation with respect to S_4^* since $A_1 > 0$. Let the discriminant be $\Delta = A_2^2 - 4A_1A_3$. If $A_2 \geq 0$ then (24) has no real solution. Similarly, if $\Delta < 0$, equation (24) has no positive solution. Also if $A_2 < 0$ and $\Delta \geq 0$, then (24) has two positive solutions. For $\Delta = 0$ in terms of R_0 , gives $R_0 = R_0^C$, where

$$R_0^C = 1 - \frac{A_2^2}{4\beta_1 \gamma \beta_2 \delta (\varphi + \delta) (\delta + \alpha + \varepsilon)^2} \quad (27)$$

Thus, the equivalent relations are:

$$\begin{aligned} \Delta < 0 &\Leftrightarrow R_0 < R_0^C, \Delta = 0 \Leftrightarrow R_0 = R_0^C, \\ \Delta > 0 &\Leftrightarrow R_0 > R_0^C. \end{aligned} \quad (28)$$

The proofs and results clearly shows that there is existence of the endemic equilibrium on STDs amongst tertiary institution students.

Theorem 5. *Considering the STDs model system of coupled equations (1) there exists:*

- (i) a unique endemic equilibrium whenever $R_0 > 1$;
- (ii) a unique endemic equilibrium whenever $R_0 = 1$ and $A_2 < 0$;
- (iii) a unique endemic equilibrium of multiplicity 2 when $R_0 = R_0^c$ and $A_2 < 0$;
- (iv) two endemic equilibria $E_1(S_{11}, S_{21}, S_{31}, S_{41}, S_{51})$ and $E_2(S_{12}, S_{22}, S_{32}, S_{42}, S_{52})$ when $R_0^c < R_0 < 1$ and $A_2 < 0$, when

$$S_{41} = \frac{-A_2 + \sqrt{\Delta}}{2\gamma\beta_1\beta_2(\delta + \alpha + \varepsilon)} \quad \text{and} \quad S_{42} = \frac{-A_2 - \sqrt{\Delta}}{2\gamma\beta_1\beta_2(\delta + \alpha + \varepsilon)}$$

$$\text{Setting } S_{11} = \frac{\Psi}{\beta_1 S_{41} + (\varphi + \delta)} \text{ and } S_{12} = \frac{\Psi}{\beta_1 S_{42} + (\varphi + \delta)};$$

$$S_{31} = \frac{(1 - \omega)\varphi\Psi}{(\gamma\beta_2 S_{41} + \delta)[\beta_1 S_{41} + (\varphi + \delta)]};$$

$$S_{42} = \frac{(1 - \omega)\varphi\Psi}{(\gamma\beta_2 S_{42} + \delta)[\beta_1 S_{42} + (\varphi + \delta)]} \quad (29)$$

$$S_{21} = \frac{\omega\varphi A}{\delta[\beta_1 S_{41} + (\varphi + \delta)]} \text{ and } S_{22} = \frac{\omega\varphi A}{\delta[\beta_1 S_{42} + (\varphi + \delta)]};$$

$$S_{51} = \frac{\varepsilon S_{41}}{(v + \delta)} \text{ and } S_{52} = \frac{\varepsilon S_{42}}{(v + \delta)};$$

then $E_i = (S_{1i}, S_{2i}, S_{3i}, S_{4i}, S_{5i})$ $i=1,2$ are endemic equilibria of the STDs model study equation (1)

- (v) has no endemic equilibria whenever $R_0 < R_0^c$ and $A_2 > 0$ or whenever $R_0 \leq 1$ and $A_2 > 0$.

From Theorem 5, there exists a unique EE when $R_0 > 1$ which approaches zero as $R_0 \rightarrow 1$ and there is no EE if $R_0 < 1$. At this point a backward bifurcation is possible at $R_0 = 1$.

However, if $A_2 < 0$, in (1) has a unique EE when $R_0 > 1$ and has two different endemic equilibria when $R_0^c < R_0 < 1$ and system equations (1) has no EE when $0 < R_0 < R_0^c$.

Hence the system of equations (1) has a backward bifurcation at $R_0 = 1$ from the DFE to two endemic equilibria. It also follows from theorem 6 below

Theorem 6 *The STDs model system of equations (1) has a backward bifurcation at $R_0 = 1$ if and only if $A_2 < 0$.*

Public health implication of backward bifurcation: The public health implication of backward bifurcation is that the classical requirement of having the basic reproduction number less than unity, although necessary, is not sufficient for disease control.

The backward bifurcation in this study of STDs models revealed that Multiple equilibria exist hence, the disease persists in our tertiary institutions even if $R_0 < 1$. The Control measures may not lead to annihilation of the diseases. The initial conditions of this disease dynamics depends on the starting point and nature of area. There is need for the control strategies to be refined by targeting the main factors, hence, prevention is the optimum key as to avoid reaching the tipping point.

5.2 Local Stability of the EE E_e

Using the model (1) to establish the local asymptotic stability of the EE, the study employs Theorem 4.1 in [Casitillo-Chev2002] and the following change of variables as to apply the Centre Manifold Theory:

$$S_1 = x_1, S_2 = x_2, S_3 = x_3, S_4 = x_4, S_5 = x_5 \quad (30)$$

Using the vector $X = (x_1, x_2, x_3, x_4, x_5)^T$. Then model system (1) can be written in the form $\frac{dX}{dt} = F$, where

$F = (f_1, f_2, f_3, f_4, f_5)^T$ such that

$$\frac{dx_1}{dt} = f_1 = A - \beta_1 x_4 x_1 - \varphi x_1 \delta x_1$$

$$\frac{dx_2}{dt} = f_2 = (1 - \omega)\varphi x_1 - \gamma\beta_2 x_4 x_2 - \delta x_2$$

$$\frac{dx_3}{dt} = f_3 = \omega\varphi x_1 - \delta x_3 \quad (31)$$

$$\frac{dx_4}{dt} = f_4 = \beta_1 x_4 x_1 + \gamma\beta_2 x_4 x_2 - (\delta + \alpha + \varepsilon)x_4$$

The Jacobian matrix of the model system of equations (1) at E_0 is given by

$$J(E_0) = \begin{bmatrix} -(\varphi + \delta) & 0 & 0 & \frac{\beta_1 \Psi}{(\varphi + \delta)} & 0 \\ (1 - \omega)\varphi & -\delta & 0 & \frac{(1 - \omega)\varphi \Psi}{\delta(\varphi + \delta)} & 0 \\ \omega\varphi & 0 & -\delta & 0 & 0 \\ 0 & 0 & 0 & \frac{\beta_1 \Psi}{(\varphi + \delta)} + \frac{\gamma\beta_2(1 - \omega)\varphi \Psi}{\delta(\varphi + \delta)} - (\delta + \alpha + \varepsilon) & -(v + \delta) \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (32)$$

From (32), the reproduction number is

$$R_0 = \left(\frac{\beta_1 \Psi}{(\varphi + \delta)} + \frac{\gamma\beta_2(1 - \omega)\varphi \Psi}{\delta(\varphi + \delta)} \right) \left(\frac{1}{(\delta + \alpha + \varepsilon)} \right)$$

If β_1 is a bifurcation point and if $R_0 = 1$, Then, solving for β_1 gives

$$\beta_1 = \beta^* = \frac{\delta(\varphi + \delta)(\delta + \alpha + \varepsilon) - \gamma\beta_2(1 - \omega)\varphi \Psi}{\delta \Psi} \quad (33)$$

It is observed that the Jacobian matrix $J_{(E_0)}$ of the linearized system of the transformed equation (32) with $\beta_1 = \beta^*$ has a simple zero eigenvalue and the rest of the eigenvalues have negative real parts. Therefore, the Centre Manifold Theory in [Castillo-Chevs4] is appropriate to analyse the dynamics of the STD near $\beta_1 = \beta^*$. For the case where $R_0 = 1$, it can be shown that the Jacobian matrix $J_{(E_0)}$ at $\beta_1 = \beta^*$ has a right eigenvalue which corresponds to the zero eigenvalue expressed as $U = (u_1, u_2, u_3, u_4, u_5)^T$

where

$$u_1 = \frac{r_1}{(\varphi + \delta)} u_4, u_2 = \frac{(1 - \omega)\varphi u_1 + r_2 u_4}{\delta}, u_3 = \frac{\omega\varphi}{\delta} u_1 = \frac{r_1 \omega \varphi}{\delta(\varphi + \delta)} u_4$$

$$u_4 = \frac{\varepsilon}{(v + \delta)} u_4, u_4 = u_4 > 0 \quad (34)$$

The left-eigenvector of $J_{(E_0)}$ associated with the zero eigenvalue at $\beta_1 = \beta^*$ is given by

$$V' = (v_1, v_2, v_3, v_4, v_5)^T, \text{ where}$$

$$v_1, v_3, v_5 = 0, -\frac{r_3}{r_1}, v_4, v_5 = v_4 > 0. \quad (35)$$

Here, theorem 4.1 in Castillo-chaen is used to establish the condition for the existence of backward bifurcations.

Theorem 7 As contained in the Theorem 4.1 in [Castillo-chaen 4], Consider the following general system of ordinary differential equations with a parameter ψ :

$$\frac{dx}{dt} = f(x, \psi), f: R^n \times R \rightarrow S^n \text{ and } f \in C^2(R^n \times R) \quad (36)$$

However, it is assumed that 0 is an equilibrium for system (36) for all values of the parameter ψ ; that is, $f(0, \psi) = 0$ for all ψ . Assume

Lemma 3. $A = D_x f(0, 0) = \left(\frac{\partial f_i}{\partial x_j}(0, 0) \right)$ is the linearization of system (36) around the equilibrium 0 with ψ evaluated at 0. Zero is a simple eigenvalue of A and other eigenvalues of A have negative real parts.

Lemma 4: Matrix A has a right eigenvector w and a left eigenvector v corresponding to the zero eigenvalue. Let f_k be the k^{th} component of f and

$$a = \sum_{k,i,j=1}^n v_k w_i w_j \frac{\partial^2 f_k}{\partial x_i \partial x_j}(0, 0),$$

$$b = \sum_{k,i=1}^n v_k w_i \frac{\partial^2 f_k}{\partial x_i \partial \theta}(0, 0). \quad (37)$$

The local dynamics of (36) around 0 are governed by a and b in the following manner.

- i. $a > 0, b > 0$, when $\theta < 0$ with $|\theta| \ll 1, 0$ is locally asymptotically stable, and there exists a positive unstable equilibrium; when $0 < \theta \ll 1, 0$ is unstable and there exists a negative and locally asymptotically stable equilibrium;
- ii. $a < 0, b < 0$. When $\theta < 0$ with given by $|\theta| \ll 1, 0$ is unstable; when $0 < \theta \ll 1$ is locally asymptotically stable, and there exists a positive unstable equilibrium;
- iii. $a > 0, b < 0$. When $\theta < 0$ with $|\theta| \ll 1, 0$ is unstable, and there exists a locally asymptotically stable negative equilibrium; when $0 < \theta \ll 1, 0$ is stable, and a positive unstable equilibrium is shown
- iv. $a < 0, b > 0$. When θ changes from negative to positive, θ changes its stability from stable to unstable. Correspondingly, a negative equilibrium becomes positive and locally asymptotically stable.

$$a = \sum_{h,i,j=1}^5 v_h u_i u_j \frac{\partial^2 f_i}{\partial x_i \partial x_j}(E_0, \beta^*)$$

$$b = \sum_{k,t=1}^5 v_h u_i \frac{\partial^2 f_h}{\partial x_i \partial \beta^*}(E_0, \beta^*).$$

The system of equations (31), the non-zero partial derivatives of f associated with a at the

DFE is given as

$$\frac{\partial^2 f_1}{\partial x_1 \partial x_4} = \frac{\partial^2 f_1}{\partial x_4 \partial x_1} = -\beta^*,$$

$$\frac{\partial^2 f_2}{\partial x_2 \partial x_4} = \frac{\partial^2 f_2}{\partial x_4 \partial x_2} = -\gamma \beta_2, \quad (38)$$

$$\frac{\partial^2 f_4}{\partial x_1 \partial x_4} = \frac{\partial^2 f_4}{\partial x_4 \partial x_1} = \beta^*$$

$$\frac{\partial^2 f_4}{\partial x_2 \partial x_4} = \frac{\partial^2 f_4}{\partial x_4 \partial x_2} = \gamma \beta_2$$

Taking into account (34) and (35), it follows from the expressions in (38) that

$$a = \sum_{h,i,j=1}^5 v_h u_i u_j \frac{\partial^2 f_i}{\partial x_i \partial x_j}(E_0, \beta^*),$$

$$a = v_1 u_1 u_4 \frac{\partial^2 f_1}{\partial x_1 \partial x_4} + v_1 u_4 u_1 \frac{\partial^2 f_1}{\partial x_4 \partial x_1}$$

$$+ v_2 u_2 u_4 \frac{\partial^2 f_2}{\partial x_2 \partial x_4} + v_2 u_4 u_2 \frac{\partial^2 f_2}{\partial x_4 \partial x_2}$$

$$\begin{aligned}
 & +v_4u_1u_4\frac{\partial^2 f_4}{\partial x_4\partial x_1} + v_4u_4u_1\frac{\partial^2 f_4}{\partial x_2\partial x_4} \\
 & +v_2u_2u_4\frac{\partial^2 f_4}{\partial x_4\partial x_2} + v_4u_4u_2\frac{\partial^2 f_4}{\partial x_4\partial x_2} \\
 & v_1u_1u_4(-\beta^*) + v_1u_4u_1(-\beta^*) + v_2u_2u_4(-\gamma\beta_2) \\
 & +v_2u_4u_2(-\gamma\beta_2) + v_4u_1u_4(\beta^*) + v_4u_4u_1(\beta^*) \\
 & +v_2u_2u_4(\gamma\beta_2) + v_4u_4u_2(\gamma\beta_2) \\
 & = -2v_1u_1u_4\beta^* - 2v_2u_2u_4\gamma\beta_2 \\
 & + 2v_4u_4u_1\beta^* + 2v_4u_2u_4\gamma\beta_2 \\
 & = -2v_1u_1u_4\beta^* + 2v_4u_4u_1\beta^* + 2v_4u_2u_4\gamma\beta_2 \\
 & = -2\frac{[\beta^*\Psi^2\mu^2 + Q + P - \beta^*\Psi(\varphi + \delta)\delta^2]}{\delta(\varphi - \delta)^3} \quad (39)
 \end{aligned}$$

Where,

$$Q = \gamma^2\beta_2^2(1 - \omega)\varphi\Psi(\varphi + \delta)^2$$

$$P = \beta(\delta + \alpha + \varepsilon)(\varphi + \delta)^2\delta^2$$

For the sign of b , it is associated with the following non-vanishing partial derivatives off:

$$\frac{\partial^2 f_1}{\partial x_4\partial\beta} = -\frac{\Psi}{\varphi + \delta}, \quad \frac{\partial^2 f_4}{\partial x_4\partial\beta} = \frac{\Psi}{\varphi + \delta} \quad (40)$$

And taking account of. (34) and (35), it follows from the expression of (40) that

$$\begin{aligned}
 b & = v_1u_4\frac{\partial^2 f_1}{\partial x_4\partial\beta^*} + v_4u_4\frac{\partial^2 f_4}{\partial x_4\partial\beta^*} \\
 & = v_1u_4\left(-\frac{\Psi}{\varphi + \delta}\right) + v_4u_4\left(\frac{\Psi}{\varphi + \delta}\right) \quad (41) \\
 & = \frac{(\delta + \alpha + \varepsilon)(\varphi + \delta)\delta - \gamma\beta_2(1 - \omega)\varphi\Psi}{\delta(\varphi + \delta)\beta^*}
 \end{aligned}$$

From equations (39) and (41), and since $a < 0$ and $b > 0$, according to Theorem 7 lemma 4 condition (iv) the result is established.

The results revealed that the endemic equilibrium E_e of the STDs model system in (1) is locally asymptotically stable for $R_e > 1$, but close to 1.

6 GLOBAL STABILITY OF THE EEP E

Theorem 8 If $R_0 > 1$, then there exist an endemic equilibrium E_e and it is globally asymptotically stable (GAS).

Proof: Given that $R_0 > 1$, then the existence of the endemic equilibrium is guaranteed. Considering the Lyapunov function $V(S_1, S_2, S_3, I_4, R_5)$ given as

$$V = \frac{1}{2}(S_1 - S_1^*)^2 + \frac{1}{2}(S_2 - S_2^*)^2 + \frac{1}{2}(S_3 - S_3^*)^2$$

$$+ \epsilon \left(S_4 - S_4^* - r^* \ln \left(\frac{I}{S_4^*} \right) \right) + \frac{1}{2}(S_5 - S_5^*)^2 \quad (42)$$

where ϵ is a positive constant to be chosen suitably

Upon differentiating with respect to time along the solutions path of the model system equations (1), gives

$$\begin{aligned}
 \dot{S} & = (S_1 - S_1^*)S_1' + (S_2 - S_2^*)S_2' + (S_3 - S_3^*)S_3' \\
 & + \frac{\epsilon}{I}(S_4 - S_4^*)S_4' + (R_5 - S_5^*)S_5' \quad (43)
 \end{aligned}$$

Substituting the model system (1) into the relation equation (43) gives

$$\begin{aligned}
 \dot{S} & = (S_1 - S_1^*)(\Psi - \beta_1S_4S_1 - \varphi S_1 - \delta S_1) \\
 & + (S_2 - S_2^*)((1 - \omega)\varphi S_2 - \gamma\beta_2S_2 - \delta S_2) \\
 & + (S_3 - S_3^*)(\omega\varphi S_1 - \delta S_3) \\
 & + \frac{\epsilon}{I}(S_4 - S_4^*)(\beta_1S_4S_1 + \gamma\beta_2S_4S_1 \\
 & - (\varepsilon + \delta)S_4) + (S_5 - S_5^*)(\varepsilon S_4 - (v + \delta)S_5) \quad (44)
 \end{aligned}$$

Recall that at the endemic equilibrium point E_e , gives

$$\Psi = \beta_1S_4^*S_1^* + (\varphi + \delta)S_1^*,$$

$$(1 - \omega)\varphi = \omega\beta_2S_4^*\frac{S_2^*}{S_1^*} + \delta\frac{S_2^*}{S_1^*},$$

$$\omega\varphi = \delta\frac{S_3^*}{S_1^*},$$

$$(\delta + \alpha + \varepsilon) = \beta_1S_1^* + \gamma\beta_2S_2^* \quad (45)$$

$$\varepsilon = (v + \delta)\frac{S_5^*}{S_4^*}$$

Using the equilibrium conditions equation (45), equation (44) becomes:

$$\begin{aligned}
 \dot{V} & = (S_1 - S_1^*)(\beta_1S_4^*S_1^* + (\varphi + \delta)S_1^* - \beta_1S_4S_1 - (\varphi + \delta)S_1) \\
 & + (S_2 - S_2^*)R \\
 & + (S_3 - S_3^*)\left(\delta S_3^*\frac{S_1}{S_1^*} - \delta S_3\right) \\
 & + \frac{\epsilon}{I}(S_4 - S_4^*)(\beta_1S_4S_1 + \gamma\beta_2S_4S_2) \\
 & - (\beta_1S_1^* + \gamma\beta_2S_4S_1^*) \\
 & + (S_5 - S_5^*)\left[(v + \delta)S_5^*\frac{S_4}{S_4^*} - (v + \delta)S_5\right] \\
 & \dot{V} = (S_1 - S_1^*)\left(\beta_1S_4^*S_1^* + (\varphi + \delta)S_1^*\right. \\
 & \left. - \beta_1S_4S_1 - (\varphi + \delta)S_1\right) \\
 & + (S_3 - S_3^*)[\delta(S_3^* - S_3)] \\
 & + \epsilon(S_4 - S_4^*)[\beta_1(S_1 - S_1^*) + \gamma\beta_2(S_2 - S_2^*)] \\
 & - (v + \delta)(S_5 - S_5^*)^2 \\
 & = (S_1 - S_1^*)W + (S_1S_1^*)R + (S_3 - S_3^*)[\delta(S_3^* - S_3)]
 \end{aligned}$$

$$+\epsilon(S_4 - S_4^*)K - (v + \delta)(S_5 - S_5^*)^2$$

where,

$$R = Y\beta_2(S_2^* - S_2S_4) + \delta(S_2^* - S_2)$$

$$K = [\beta_1(S_1 - S_1^*) + Y\beta_2(S_2 - S_2^*)]$$

$$W = \beta_1(S_1^*S_4^* - S_1S_4) + (\varphi + \delta)(S_1^* - S_1)$$

$$(since S_1 \leq S_4 \leq S_4^*)$$

$$\begin{aligned} &= -(\varphi + \delta)(S_1 - S_1^*)^2 - \delta(S_2 - S_2^*)^2 - \delta(S_3 - S_3^*)^2 \\ &\quad - (v + \delta)(S_5 - S_5^*)^2 + \beta_1(S_1 - S_1^*)(S_4^*S_1^* - S_1S_4SI) \\ &\quad + Y\beta_2(S_2 - S_2^*)(S_2^*S_4^* - S_2^*) + \epsilon\beta_1(S_4 - S_4^*)(S_1 - S_1^*) \\ &\quad + \epsilon Y\beta_2(S_4 - S_4^*)(S_2 - S_2^*) \end{aligned}$$

$$\begin{aligned} &= -(\varphi + \delta)(S_1 - S_1^*)^2 - \delta(S_2 - S_2^*)^2 - \\ &\quad \delta(S_3 - S_3^*)^2 - (v + \delta)(S_5 - S_5^*)^2 \\ &\quad + \beta_1(S_1 - S_1^*)[S_1^*(S_4^* - S_4) - S_4(S_1 - S_1^*)] \\ &\quad + Y\beta_2(S_2 - S_2^*)[S_2^*(S_4^* - S_4) - S_4(S_2 - S_2^*)] \\ &\quad + \epsilon\beta_1(S_4 - S_4^*)(S_1 - S_1^*) \\ &\quad + \epsilon Y\beta_2(S_4 - S_4^*)(S_2 - S_2^*) \end{aligned}$$

$$\begin{aligned} &= -(\varphi + \delta)(S_1 - S_1^*)^2 - \delta(S_2 - S_2^*)^2 \\ &\quad - \delta(S_3 - S_3^*)^2 - (v + \delta)(S_5 - S_5^*)^2 \\ &\quad - \beta_1S_4(S_1 - S_1^*)^2 - \beta_1S_1^*(S_1 - S_1^*)(S_4 - S_4) \\ &\quad - Y\beta_2S_4(S_2 - S_2^*)^2 - Y\beta_2S_2(S_2 - S_2^*)(S_4 - S_4^*) \\ &\quad + \epsilon\beta_1(S_4 - S_4^*)(S_1 - S_1^*) \\ &\quad + \epsilon Y\beta_2(S_4 - S_4^*)(S_2 - S_2^*) \end{aligned}$$

$$\begin{aligned} &= -(\varphi + \delta + \beta_1S_4)(S_1 - S_1^*)^2 \\ &\quad - (\delta + Y\beta_2S_4)(S_2 - S_2^*)^2 - \delta(S_3 - S_3^*)^2 \\ &\quad - (v + \delta)(S_5 - S_5^*)^2 + \beta_1(\epsilon - S_1^*)(S_1 - S_1^*)(S_4 - S_4^*) \\ &\quad + Y\beta_2(\epsilon - S_2^*)(S_2 - S_2^*)(S_4 - S_4^*) \end{aligned}$$

$$\leq 0. \quad (46)$$

where, $\epsilon = S_1^*, \epsilon = S_2^*$

It is important to note that $V^* = 0$, holds only at E_e . By LaSalle's invariant principle [17], every solution to the system (1) with positive initial conditions in Ω , approaches E_e as $t \rightarrow \infty$ if $R_0 > 1$. Hence, the endemic equilibrium E_e , is globally asymptotically stable (GAS) in Ω if $R_0 > 1$.

The public health implications of endemicity of STDs are the increased disease burden even with control measures (disease can persist). For $R_0 > 1$, more resources would be needed for sustained control. Reducing the control measures can lead to outbreaks of STD, the target interventions is important as it focus on high risk areas and tracking the disease trends adjust strategies.

CONCLUSIONS

The findings of this mathematical model on STD/STIs transmission dynamics revealed that endemic equilibrium point is stable and the disease can persist because $R_0 > 1$, also, STD in tertiary institutions is possible to be eradicated for the reproduction number is less than one. The backward bifurcation is observed with multiple equilibria having $R_0 = 1$ thereby leading to surge or hysteresis due to the presence of the three susceptible subpopulation in the model study has made the virus and bacteria in the infected phenomenon has made the transmission dynamics to lag behind changes in the effect of the public health awareness campaign and intervention strategies. The presence of Public health awareness strategies and behavioural changes reduces the transmission dynamics and control the disease (STDs). The public health targeted interventions is effective since it is co-opted in the model study as its Focus is on high-risk groups and areas in tertiary institutions. This study showed that stability of DFE is stable if and only if $R_0 < 1$, EE is stable if $R_0 > 1$. The disease persistence is at EE if and only if $R_0 > 1$ and a change in stability and behavior at the bifurcation with $R_0 = 1$. At endemic equilibrium (EE), the STDs/STIs infectious population settles at a constant level and balance between transmission and removal or recovery or death.

This study also showcased that the STDs/STIs is locally asymptotically stable for $R_0 > 1$ showing no immunity and the students cycle between two groups of susceptible and infectious populations and those exposed are at the latent infection stage.

The findings showed that tertiary students is would be a high risk group for social life as experimentation increases the students chance of been more exposed to STDs/STIs, the susceptible population of students in the study model would require. The study revealed that the awareness and education on STDs/STIs by including it as part of the curriculum in the first year general studies (GNS) with key preventions strategies to curb the transmission dynamics. Also, it has shown that the persistence of STDs/STIs with $R_0 > 1$ in tertiary institutions is as a result of no testing and screening centre for early detection and treatment. This study also revealed that lack of safe and hygiene management practices amongst year one/two increases the surge vital and reproduction number. This finding was a follow up of the study assessment of 475 tertiary institution students' knowledge on the transmission dynamics and control of sexually transmitted diseases on campus. The mean age of the respondents is 18.5 years. This study tried to establish the knowledge base of tertiary institution students of age 16, 17 and 18 regarding STDs/STIs. The findings presented that the threshold R_0 determines the Endemic Equilibrium surge when $R_0 > 1$ and that the equilibrium prevalence depends on

model structure of five-compartmental subpopulations and parameters such as the recruitment rate Ψ , spectral radius φ and removal rate δ are of great importance to the system.

The public health implications of the study to students is that prevalence of STDs/STIs might be high due to unprotected sex and multiple partners. Stigmatization and shame hinder help-seeking students from normalize testing and support needed. There should be interventions needed for campus programs and peer education.

The recommendations from findings are behavioural interventions in combination with biomedical approaches can lead to a 38% reduction in STD/STI acquisition. Female condoms can reduce STDs/STIs incidence by 24% in high-risk populations. Comprehensive sexuality education for adolescents is strongly associated with reduced risk of HIV and improved sexual health outcomes and treatment with 100% condom policy can cause dramatic declines in STDs/STIs incidence to eradication. Also, this study recommends that public health sector should sustain their efforts in awareness campaigns, education, monitoring and surveillance essential. The students should get tested regularly, practice safe sex and seek help if needed.

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