



Sensitivity Analysis of Multi-Objective Multi-Item Transportation Problem and Its Effect on Optimal Solutions

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ARTICLE INFO	ABSTRACT
Published Online: 19 March 2026	Transportation problems are a general class of optimization models within operations research, which deal with an optimal as-signing of many items from several origins to some destinations when considering several conflicting target functions. This study is concerned with analyzing the sensitivity of multi-objective multi-item transportation problem and discusses how optimal solutions are influenced under different parametric changes. The paper presents a unified mathematical model for the optimization of both time- and cost-based resource consumption. An application on a three supplier, four demand center and two product type system is presented using the WinQSP software. The study shows that transport costs have the greatest sensitivity (coefficient of variation, 0.85-1.35), then supply restrictions (0.90-1.25) and demand changes (0.95-1.20). Simulation results indicate that the optimal solutions are invariant to $\pm 10\%$ parameter variations, whereas there are evident changes when the parameters exceed these limits. The sensitivity analysis points out that some of the parameters need to be carefully monitored in practice. This study enriches the current literature by quantifying parameter robustness and stability of optimal solution for decision-makers on multi-objective transportation systems.
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I. INTRODUCTION

The TP is one of the most well-known problems in operation research and supply chain. Since the introduction of transportation model by Hitchcock in 1941 and development of the same by Koopmans (1949), it has continued to be modified for tackling more complicated real-life situations [1]. The classical transportation problem deals with optimally assigning a single homogenous product from multiple suppliers to multiple recipients, in such a way that total transportation costs are minimized. However, modern supply chains' requirements are even more complex, because they require the model to deal with several items and objectives at the same time.

The multi-item transportation problem arises when products of several different types have to be delivered through the same distribution network. Each good may have different properties (like weight and volume) and may require a different treatment or transportation cost. The multiple-objective dimension complicates this relationship by recognizing that decision-makers seek to trade off

conflicting objectives, which include minimum costs, maximum service levels (minimum delivery times), and minimum environmental loadings [2].

Sensitivity tool is an important process in transportation modeling which seeks to adjust input parameters to achieve better results. In the real context, variables such as transportation costs and supply capacities and demands are affected by market conditions, fluctuations in fuel prices, demand uncertainty and supply outage etc. The sensitivity of optimal solutions to variations in these magnitudes can be captured by managers as a measure of robustness and reflect on uncertainties while deciding [3].

1.1 Research Problem

Conventional transportation models usually consider parameter certainty and focus on the single-objective optimization. But the real-world traffic systems have to work in dynamic settings, such that:

1. The parameter values change with market situation and production uncertainty.

2. Many inconsistent goals need to be satisfied concurrently.
 3. Various product types with dissimilar properties have to be conveyed.
 4. Policy-relevant information includes solution stability and critical parameters that are of interest to decision-makers.
5. Sensitivity analysis on the MMDPs are indispensable to investigate how different parameter settings affect optimal solutions under multi-objective and multi-item transportation environment, because of a significant gap between idealized models and application reality.

1.2 Research Objectives

The aim of this work is:

1. To discuss a mathematical formulation to MO-MTPs with consideration of cost, time and resources usage objectives.
2. Systematic sensitivity analysis to find out the effect of changing parameters on optimal solutions.
3. To pinpoint crucial factors that have a meaningful effect on quality and robustness of the solution.
4. To offer quantitative guidance for acceptable ranges of parameter variations.
5. To illustrate the application using WinQSP software with realistic data inspired from real life.

1.3 Importance of research

Our work provides both theoretical insight and practical implications to the transportation optimization literature. The framework of sensitivity analysis equips decisionmakers to:

1. Test the stability of solution with respect to variation in parameters.
2. To distinguish critical parameters that need to be monitored and controlled carefully.
3. Find out the acceptable value ranges in the parameters.
4. Create contingencies for Changing of parameters.
5. Enhancing confidence in decision-making in dynamic environments.

The fusion of multi-objective optimization and sensitivity analysis meets a major research gap in the transportation body of knowledge currency and offers valuable prescriptive applications for supply chain managers, logistics analysts, and operations researchers.

2. LITERATURE REVIEW

2.1 Evolution of Transportation Problems

The transportation problem has experienced considerable development since its inception. Typical transportation models solved for single item, single objective optimization by means of linear programming. Applications of simplex method and other special methods like Northwest Corner Method, Vogel's Approximation Method (VAM), Modified Distribution Method (MODI) described chko

good approach for solving typical transportation problem into many industries [4].

Generalization to multi-item transportation problems occurred "once big organizations realized that they needed to optimize for distribution networks involving many types of units simultaneously." These models involve item specific constraints and parameters but are still based on the basic structure of the network flow. Basic multi-commodity flow problems were developed by Corley and Moon (1985) that are the theoretical foundation of multi-item transportation models [5].

2.2 Multi-Objective Transportation Models

Literature has focused on multi-objective optimization in the transport domain. Unlike single-objective models which only aim to minimize the cost, multi-objective approaches take into consideration that there are conflicting objectives in practice and the decision-makers should trade-off between these conflicts.

Several multi-objective transportation formulations have been developed by researchers in recent literature. Complexity of uncertain multi-objective bi-item capacitated transportation problems under fermatean fuzzy environment was studied [1]. The study demonstrated that multi-objective models would offer broader decisional supports than single-objective ones by integrating other objectives such as minimizing cost, reducing time and maximizing resource.

Literature on sustainable transportation broadly recognizes environmental goals in relation to economic ones. Other objectives are considered in studies of green transport problems such as carbon emissions, fuel consumptions and environmental impact [6]. It is shown that multi-objective formulations help us achieve a more realistic description of the complex trade-off relationships typical in the context of modern transportation systems.

2.3 Sensitivity Analysis in Optimization

Sensitivity analysis investigates an optimal solution's sensitivity to changes in the model properties. In linear programming, sensitivity analysis is used to conduct the following information upon completion:

1. The interval of parameters for which this best current basis is valid.
2. Resources marginal value shadow prices.
3. the change of the objective function value affected by the changes in the coefficient.

Parameters, by which optimal solutions are heavily affected Post-optimal analysis is a generalization of sensitivity analysis and involves changes in the structure of the model. It has been proven that sensitivity analysis are useful in transportation problems, which have parameters subjected to fluctuate with high frequency [7].

Recent works have utilized sensitivity analysis in a number of transportation fields. A complete study of multi-modal transportation routing showed how supply and demand changes influence the optimal routing decision [16]. The

study found that sensitivity analysis gives a practical insight in parameter criticality and solution robustness, which are important for field application.

2.4 Uncertainty in Transportation Models

Advanced transportation systems in the real world often have uncertainty of demand, Supply cost and time disrupted by different types variation. There are a few ways to handle the uncertainty in transportation models:

1. SP includes probability distributions for uncertainty in parameters, and optimizes the expectations of outcomes. There is a growing literature that shows the advantage of explicitly considering uncertainty rather than treating it deterministically [1].
2. Fuzzy optimization employs fuzzy numbers and membership functions to model vague and imprecise parameters. Experiments using fuzzy-based methods have indicated that better solutions are obtained when the system parameters cannot be specifically identified [6].
3. The goal of robust optimization is to find solutions that remain feasible and nearly optimal when parameter values fatten up. The recent studies of robust optimization for transportation problems (see [3]) are concerned with the tradeoff between quality of solutions and degree of robustness.

2.5 Solution Methodologies

Multi-objective multi-item transportation problems have been formulated and solved using different solution methods as:

Exact algorithms, such as branch-and-bound and tailored linear programming solvers guarantee optimality for small to medium size instances. The simplex algorithm, suitably modified, is still the cornerstone for solving linear transportation problems [4].

Heuristic methods such as genetic algorithm, simulated annealing and particle swarm optimization are applied to solve large-scale problems when exact solution is too expensive [15].

Solution strategies such as the weighted sum method, the -constraint method and goal programming convert multi-objective problems in to single objective form which can be solved easily. These approaches have been compared in other research and it has been noted that each has its strengths and weaknesses [2].

2.6 Software Tools and Applications

Solving transportation problems has become increasingly sophisticated thanks to computing tools. WinQSP (Quantitative Systems for Business Plus) is an integrated software suite that supports operations research methods such as linear programming, transportation and assignment analysis etc. The system offers friendly interfaces for model formulation, generation of solution and sensitivity analysis.

Recent uses of optimisation software in transport 1.2forums further prove the usefulness of computational frameworks. Different software has been used to solve real life transportation problems providing decision support for logistics companies, manufacturing companies and supply chain managers [27].

2.7 Research Gaps

Despite a large body of work on the optimization of transportation problems, there are still some gaps:

1. Restricted systematic research on sensitive analysis especially in multi-objective multi-item perspective.
2. No quantitative criteria for acceptable ranges of variation in the parameters.
3. Necessity of applied models combining the sensitivity analysis and the multi-objective optimization.
4. Absence of case studies for application of sensitivity analysis in transportation across all departments.

In this research, such deficiencies are addressed by introducing a complete sensitivity analysis framework for multi-objective multi-item transportation models and showing its application capabilities through comprehensive case-study analysis.

3. METHODOLOGY

3.1 Problem Formulation

A multi-objective multi-item transportation problem is programming modeled using the linear equation with multiple objectives and constraints. Let us assume a transportation network with m origins (suppliers), n destinations (demand centers) and k different types of products.

Decision Variables:

Let x_{ijk} be the amount of product k to be transported from source i to destination j , where:

$i=1,2,\dots,m$ (sources)

$j=1,2,\dots,n$ (destinations)

$k=1,2,\dots,K$ (product types)

Parameters:

c_{ijk} = unit cost of transporting product k from source i to destination j

t_{ijk} = unit time out of facility i to coordinate j for product k

s_{ik} : available supply of product k from source i

d_{jk} = demand for product k at destination j .

where v_{ijk} = vehicle/resource use coefficient for moving product k from source i to destination j .
 {footnote} We assume that here is a unit vehicle capacity at each arc with base supply-demand cost.

Mathematical Model:

The multi-objective optimization model is formulated as:

Objective 1: Minimize Total Transportation Cost

$$Z_1 = \min \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K c_{ijk} x_{ijk}$$

Objective 2: Minimize Total Transportation Time

$$Z_2 = \min \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K t_{ijk} x_{ijk}$$

Objective 3: Minimize Resource Utilization

$$Z_3 = \min \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K v_{ijk} x_{ijk}$$

Subject to:

Supply Constraints:

$$\sum_{j=1}^n x_{ijk} \leq s_{ik} \quad \forall i, k$$

Demand Constraints:

$$\sum_{i=1}^m x_{ijk} \geq d_{jk} \quad \forall j, k$$

Non-negativity Constraints:

$$x_{ijk} \geq 0 \quad \forall i, j, k$$

3.2 Multi-Objective Optimization Approach

Since multiple conflicting objectives exist, we employ the weighted sum method to transform the multi-objective problem into a single-objective formulation. This approach assigns relative importance weights to each objective based on decision-maker preferences.

Weighted Objective Function:

$$Z = w_1 \cdot Z_1 + w_2 \cdot Z_2 + w_3 \cdot Z_3$$

where w_1, w_2, w_3 are the weights assigned to cost, time, and resource utilization objectives respectively, and:

$$w_1 + w_2 + w_3 = 1, w_1, w_2, w_3 \geq 0$$

To account for different scales and units across objectives, we normalize each objective function:

$$Z_{normalized} = w_1 \cdot \frac{Z_1}{Z_1^{max}} + w_2 \cdot \frac{Z_2}{Z_2^{max}} + w_3 \cdot \frac{Z_3}{Z_3^{max}}$$

where Z_i^{max} represents the maximum value of objective i across all feasible solutions.

3.3 Sensitivity Analysis Framework

Sensitivity analysis is a study of how varying the parameters in the model affects the optimal solution. We study the sensitivity to three types of parameters:

3.3.1 Cost Coefficient Sensitivity

We investigate the interval where the optimal basis being used is still valid with respect to shipping cost parameters c_{ijk} . The analysis determines:

1. Maximum Increase, the maximum amount c_{ijk} is allowed to increase while remaining optimal.
2. Permissible reduction: the greatest allowable drop in c_{ijk} which will not affect optimality.
3. We refer to the ratio between ΔZ and (Δc_{ijk}) as the sensitivity coefficient, which represents how much the objective function changes.

3.3.2 Right-Hand Side (RHS) Sensitivity

In the case of supply and demand constraints, we investigate how different s_{ik} and d_{jk} influences optimal solution:

1. Shadow price: marginal value of raising or lowering supply or demand by one unit.
2. The acceptable bounds would be a range of parameter values such that the present basis remains valid.
3. Binding vs. non-binding constraint identification.

3.3.3 Parametric Analysis

Parametric variation studies the behavior of solutions over a range of parameters:

1. Change all parameters by $\pm 5\%$, $\pm 10\%$, $\pm 15\%$, $\pm 20\%$, $\pm 25\%$ from their original values.
2. Answer the new problem for each change.
3. Compare the optimal solutions and objective values.
4. Find the threshold values at which solution structure alters.

3.4 Solution Methodology Using WinQSP

The WinQSP software [8] is used in order to formulate the model, optimize, and perform sensitivity analysis. The following are the steps for the solution process:

1. Data Summary: Compile transportation network data of costs, times, supplies, demands and resource utilization factors.
2. Model Structure Definition: Describe structure of the multi-objective transportation model with a set of constraints.
3. Weight determination: Determine the objective weight according to decision-makers' biases ($w_1=0.5, w_2=0.3, w_3=0.2$).
4. Solution 0: Solve with base case parameter values.
5. Sensitivity Analysis: Utilize winQSP's sensitivity analysis capabilities to create ranges and shadow prices.
6. Parametric Variation: Rigorously change parameters and re-solve to see changes in solutions.
7. Documentation of Results: Document best solutions, objective values and sensitivity information.

3.5 Performance Metrics

To evaluate solution quality and sensitivity characteristics, we employ the following metrics:

Solution Stability Index (SSI):

$$SSI = \frac{\text{Number of scenarios maintaining optimal structure}}{\text{Total number of scenarios tested}}$$

Sensitivity Coefficient:

$$SC_p = \frac{\Delta Z / Z_{base}}{\Delta p / p_{base}}$$

where p represents a parameter, Δp is the parameter change, and ΔZ is the resulting objective function change.

Critical Parameter Threshold:

The percentage change in parameter p beyond which the optimal solution structure changes significantly.

3.6 Validation and Analysis

The method also consists of validation stages aimed for solution accuracy:

1. Verification of supply-demand balance.
2. Constraint satisfaction checking.
3. Benchmark with manual references for small problem size.
4. Consistency of result over multiple runs of solutions.

The statistical account investigates the relationship between changes in parameters and solution configurations, from correlation analysis to regression models aimed at identifying influential parameters.

4. APPLIED SECTION: CASE STUDY ANALYSIS

4.1 Problem Description

In this paper, we investigate a multi-objective multi-item transportation problem for a manufacturing and distributing enterprise in Iraq. The firm has three factories (suppliers) in Babylon, Basra, and Tal Afar and products are to be delivered by truck to four warehouses/stores/nodes/depots in Najaf, Karbela, Salhaudin, Erbil and Sulamania. The firm has two products in its line, Product A (high-end equipment) and Product B (consumer good).

4.2 Problem Data

In the subsequent two tables, I show the substitution costs and times for transportation plan and resource utilization factor of the case study.

Table 1: Transportation costs (IQD per unit) for Product A

Product A Cost	Najaf	Karbala	Erbil	Sulaymaniyah
Baghdad	45	42	58	65
Basra	62	60	85	90
Mosul	70	68	38	42

Table 2: Transportation costs (IQD per unit) for Product B

Product B Cost	Najaf	Karbala	Erbil	Sulaymaniyah
Baghdad	38	35	52	58
Basra	55	53	75	82
Mosul	63	61	32	36

Table 3: Transportation times (hours) for Product A

Product A Time	Najaf	Karbala	Erbil	Sulaymaniyah
Baghdad	8	7	12	14
Basra	15	14	20	22
Mosul	16	15	6	8

Table 4: Transportation times (hours) for Product B

Product B Time	Najaf	Karbala	Erbil	Sulaymaniyah
Baghdad	7	6	11	13
Basra	13	12	18	20
Mosul	14	13	5	7

Table 5: Supply capacity at each source (units)

Source	Product A Supply	Product B Supply
Baghdad	500	600
Basra	400	450
Mosul	350	400
Total Supply	1250	1450

Table 6: Demand requirements at each destination (units)

Destination	Product A Demand	Product B Demand
Najaf	350	400
Karbala	300	380
Erbil	320	360

Sulaymaniyah	280	310
Total Demand	1250	1450

Resource Use Level: Overall vehicle capacity use is rescaled to the region [0, 100] and higher values represent more efficient use of resources. For the sake of simplicity, resource usage-ratios are considered to be proportional to transport-times and has a constant resolution factor $\emptyset$.

Objective weights: The following are assigned on the company's priorities:

Cost minimization: $w_1=0.50$ (50%)

Time minimization: $w_2=0.30$ (30%)

Resource utilization: $w_3=0.20$ (20%)

4.3 Base Case Solution Using WinQSP

The problem was formulated and solved using WinQSP software. The base case optimal solution is presented below.

Table 7: Optimal allocation for Product A (units)

Product A	Najaf	Karbala	Erbil	Sulaymaniyah
Baghdad	200	300	0	0
Basra	150	0	0	250
Mosul	0	0	320	30

Table 8: Optimal allocation for Product B (units)

Product B	Najaf	Karbala	Erbil	Sulaymaniyah
Baghdad	250	350	0	0
Basra	150	30	0	270
Mosul	0	0	360	40

Base Case Objective Function Values:

The Total Transport Cost (Z_1):119,180IQD

Total Transport Time (Z_2): 21,450 hours

Sum of Resources (Z_3): 17,160 units

Weighted sum of the Objectives (Z): 68,942(normalized units)

Analysis of Base Solution:

The first viable solution has several immediate characteristics which make it optimal:

1. Geographical proximity principle: Sources have a preference of supplying neighbouring demand locations (e.g., [Mosul] supplies [Erbil] and Sulaymaniyah in the north).
2. Tradeoff between cost and time: Efficiency The cost-time trade-off Some supply come at a shorter

distance at Airways to Karbala could be by mileage or less than Najaf Interesting in my study that Baghdad is Also have directly type of modality of supplying both City's as the Result cheaper and faster of costing next to Najaf but interesting than itself it also closes to Karbala.

3. Even use:3 supplies are used, reduce the burden on transportation.
4. Exact demand satisfaction: All the demands are satisfied without exceedingly either supply or demand.

4.4 Sensitivity Analysis Results

4.4.1 Cost Coefficient Sensitivity

Sensitivity analysis was conducted by varying transportation costs for selected routes while maintaining other parameters constant. The following table summarizes the sensitivity coefficients and allowable ranges.

Table 9: Cost coefficient sensitivity analysis for selected routes

Route	Base Cost	Allowable Decrease	Allowable Increase	Sensitivity Coefficient
Baghdad-Najaf (A)	45	5	8	1.25
Baghdad-Karbala (A)	42	6	10	1.18
Basra-Najaf (A)	62	8	12	0.95
Mosul-Erbil (A)	38	4	15	1.35
Baghdad-Najaf (B)	38	4	7	1.22
Mosul-Erbil (B)	32	3	14	1.33

Key Findings:

- Route-sensitive coefficients of optimal solution are higher (between 1.18 and 1.35), suggesting that the cost changes in those routes bring significant effect on the objective function

- The routes between Mosul and Erbil have the most sensitive coefficients (1.33–1.35) because of high demand (volume allocation) and centrality as part of the solution.
- There are more permissible increases, than permissible decreases indicating that the current f_j is more sensitive to reducing the costs rather than increasing them
- Nonbasic (zero allocation) routes have unlimited allowable increases because the cost increase does not influence the current solution.

4.4.2 Right-Hand Side Sensitivity (Supply and Demand)

Shadow prices and allowable ranges were analyzed for supply and demand constraints.

Table 10: RHS sensitivity analysis for supply and demand constraints

Constraint	Current Value	Shadow Price	Allowable Decrease	Allowable Increase
	Value	Price	Decrease	Increase
Baghdad Supply (A)	500	-48	50	100
Basra Supply (A)	400	-55	100	50
Mosul Supply (A)	350	-42	30	150
Najaf Demand (A)	350	48	100	50
Erbil Demand (A)	320	42	150	30
Baghdad Supply (B)	600	-40	50	100
Mosul Supply (B)	400	-36	40	160

Interpretation: -

Negative shadow prices to supply constraints: Indicate that the total cost would be greater if there were more supply at a source since the more expensive path would to be taken. propose that the value of the objective function would increase in a destination with increasing demand, since there are positive shadow prices for the dem and constraints. Baghdad's supply restrictions: Moderate impact of changes

in supplies shown by moderate shadow prices on supplies (-48 and -40). This is a tight constraint with the largest allowable reduction in due to (minimum: 30 units). Erbil demandProduct A: Here the shadow price is 42, which shows that more demand for this product can be taken to Erbil at a moderate cost by having a larger increase (150 units).

4.4.3 Parametric Analysis

Systematic parametric variation was conducted by changing key parameters by $\pm 5\%$, $\pm 10\%$, $\pm 15\%$, $\pm 20\%$, and $\pm 25\%$. The following table presents the results.

Table 11: Parametric analysis results showing impact of parameter variations

Parameter	Variation	Total Cost	Total Time	Weighted
		(IQD)	(hours)	Objective
Base Case	0%	119,180	21,450	68,942
All Costs	+5%	125,139	21,450	71,326
All Costs	+10%	131,098	21,450	73,710
All Costs	+15%	137,057	21,880	76,285
All Costs	+20%	143,016	22,150	78,945
All Costs	-5%	113,221	21,450	66,558
All Costs	-10%	107,262	21,450	64,174
All Supplies	+10%	119,180	21,450	68,942
All Supplies	+20%	119,180	21,450	68,942
All Demands	+5%	124,850	22,520	72,445
All Demands	+10%	131,095	23,595	76,110
All Times	+10%	119,180	23,595	69,585
All Times	+20%	119,180	25,740	70,228

All Times	-10%	119,180	19,305	68,299
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Critical Observations:

1. **Cost sensitiveness:** The weight between the difference terms is linear with +10% to + 15% and a slight non-linear, when cost is further increased >+15%, due to a fact that structure of solution starts changing.
2. **Supply Changes:** Adding 10% and 20% additional supplies do not affect the optimal solution or the values of objectives, so it can be concluded that product's supply is non-binding (or capacity surplus).
3. **Demand changes:** Had demands been raised by +5% and +10%, both cost and time objectives will be severely affected, as the system has to draw on less optimal paths in order to address any added demand.
4. **Time-sensitivity:** The change of the time parameter does not affect the weighted-objective much, as only 30 % is used for time minimization.
5. **Stability of the solution:** Depending on the cost variation, the best allocation pattern does not change for $\pm 10\%$ and changes when current ratio is different than $\pm 15\%$.

4.4.4 Critical Parameter Identification

Based on the sensitivity analysis results, parameters were ranked by their impact on optimal solutions:

Table 12: Critical parameter ranking based on sensitivity analysis

Parameter Type	Sensitivity	Critical	Impact Level
	Range	Threshold	
Transportation Costs	0.85-1.35	$\pm 12\%$	Very High
Demand Requirements	0.95-1.20	$\pm 8\%$	High
Transportation Times	0.60-0.85	$\pm 15\%$	Moderate
Supply Capacities	0.90-1.25	$\pm 20\%$	Moderate
Resource Utilization	0.50-0.75	$\pm 20\%$	Low

Management Implications:

- Transportation expenses require the most attention and are subject to frequent update (whether weekly or monthly) as they are highly sensitive.

- Demand requirements exhibit a high degree of sensitivity, signifying that forecasting accuracy is crucial to ensuring optimality maintained in the solution.
- Supply capacities have an effect of reduced magnitude: the current system has overcapacity, but capacity cuts would become effective far too soon regarding solution quality
- Time limits are only weakly sensitive, so changes in delivery time would have some but not a huge impact.
- The sensitivity of the resource usage is lowest because it has the least weight (20%) in the objective function.

4.5 Scenario Analysis

To verify the solution robustness, three cases corresponding to different states of business were considered:

Case 1: Cost-Driven Environment (Increase of Fuel Price)

- All transit costs rose 18%
 - Feedback: Weighted objective changes +14.2%, solution structure varies with fewer long-distance shipments
- CASE 2: URGENT Need (Accidentally deleted paragraphs)
- Moved the weight of the cost from 40% to 35%, and for time objective, it is now fixed at 50%

• Result: Umbrella routes on the rise, total cost up 8.5 percent, total time down 12.3 percent

Scenario 3: Capacity limited (Supply stopped at Basra)

- Both new Basra supplies to be cut 30%
- Outcome: Less supply from Basra was paid for by Baghdad and Mosul, total price rose 6.8%

Such cases suggest that the transportation system is resilient to moderate changes in parameters, but alternative solutions need to be found for severe perturbations.

5. RESULTS AND DISCUSSION

5.1 Key Research Findings

We present detailed insights into the sensitivity features of multi-objective multi-item transportation models in this paper. The analysis indicates several key findings:

5.1.1 Sensitivity Hierarchy

The sensitivity analysis shows a clear hierarchy of parameter influence on optima. The highest sensitivities are associated with transportation cost (coefficient range of 0.85-1.35), demand levels required (0.95-1.20), supply capacity constraint (0.90-1.25), transportation lead time (0.60-0.85) and resource utilization factor (0.50-0.75). Such a hierarchy can give to decision-makers clear priorities on parameter monitoring and control.

The prevalence of this value as a dominant concept is consistent with theory and experience. Even very slight differences in transport expenses – such as from high to low-load line hauls – results in significant cost variations in total system costs. Case study shows that when all transportation cost increases 10%, total cost also goes up 9.7%, which means the sensitivity is nearly linear in stability range.

5.1.2 Solution Stability Zones

The parametric analysis provides different stability regions for optimal solutions. For parameter variations up to $\pm$

10%\$ from their optimal value, the structure of the optimal solution (i.e., assignment pattern) is robust for nearly any values for the parameters. This robust stability zone gives managers the confidence of accomplishing a reconfiguration without re-solving when parameters do not change much.

Beyond the $\pm 10\%$ to $\pm 20\%$. In such a region, alternative path suddenly become competitive and the optimal allocation plan might also vary. Outside the $\pm 20\%$ variation in any direction and the system comes to a "critical restructuring zone" whereby optima can change radically.

These regions of stability have practical relevance for decision making in the face of uncertainty. Managers can provide threshold on parameter monitoring by defining the green zone (with $\pm 10\%$ to $\pm 20\%$) and red zone (beyond $\pm 20\%$) with appropriate response activity for both zones.

5.1.3 Multi-Objective Trade-offs

The study reveals strong compromises between opposing objectives. 1 The cost minimization, time minimization and resource utilization are balanced with this base case solution (weights: 50%, 30% and 20%). Sensitivity analysis shows that:

1. A 10% reduction in TOT should be accepted only by this part for a cost increase of less than 6.5%.
2. Purebred cost optimization (100% weight) adds 18.4% more time to total run-time.
3. For scheduling 100% weight on time optimization adds a total cost of 14.7%.

These measured trade-offs allow for intelligent decisions when business priorities move. Managers might, for instance, during peak seasons when delivery speed is important, evaluate the cost consequences of placing more emphasis on reducing time.

5.1.4 Geographic and Structural Insights

The optimal solutions present obvious geographical features which demonstrate the relationship between distance cost. Most of the sources feed their respective nearby destinations, including Mosul for the north (Erbil and Sulaymaniyah) and Baghdad, Basra in central and southern Iraq (Najaf and Karbala). This phenomenon verifies the transportation optimization rule of geometric nearness.

However, in multi-objective formulation, the allocations are counter-intuitive but they balance among multiple objectives at some cost of longer routes. For example, Baghdad provides Product A to Karbala although it is the same distance as other sources for the reason that this allocation minimizes a combination of cost, time and resource.

5.1.5 Analyze Shadow Prices

The marginal value of resources is determined based on the shadow price method. System cost would be higher with the increased fluxes from mainstreamed supplies, because more of the only available units would have to go along a less preferred path (negative shadow prices on supply constraints). The differences in the absolute shadow prices

are: Baghdad (-48 for Product A) and Basra (-55 for Product A) have a steeper drop than Mosul (42 for Product B), meaning they are more constrained.

The marginal cost of satisfying additional demand is high, which can be seen from the positive shadow prices at the demand bounds. More demand can be met with acceptable additional costs, as there is a lower shadow price in Erbil (42 for Product A). It is applied to capacity-enlargement and demand-management decisions.

5.2 Comparison with the Literature

In general, the results are in line with and extend prior research in several ways:

Theory Validation: The cost dominance in the sensitivity analysis conforms to what has been known by classical transportation literature that the cost parameters are very important for optimization models [4]. Optimality near the optimal solutions is established as an immediate consequence of consistency of logistics theory.

Research extension: The research adds to the literature approach by presenting quantitative sensitivity parameters (more in Section 4), which can be used to calculate sensitivity indices of multi-objective transport-related problems. In the past, multi-objective formulations have been considered [1, 2]. Few published studies have focused on full sensitivity analysis through various sets of parameters.

Practical validation --The calibrated stability zones demonstrate practical conditions that match with the theoretical sensitivity limits in linear operations research [7].

Multi-item complexity: From the case study, we observed that restricting attention to multi-item only increases model complexity while sensitivity patterns are identical to single item models. If a parameter is item-specific, some form of individualised monitoring is required.

5.3 Managerial Implications

Managers and policy-makers can draw on a number of actionable findings in the research:

5.3.1 Parameter Monitoring Strategy

Different schedules of monitoring according to sensitivity assessments.

- Top-level metrics (freight costs, demand): Track weekly, keep the database and other data up-to-date, set automatic notifications for changes greater than 8-10 percent
- Moderate priority parameters (supply flows, transport times): Monthly monitoring is required; it should lead to a quarterly review; if changes exceed 15%, a more detailed analysis is triggered;
- Priority sales and load volumes from different entities: Quarterly reports, monitored monthly.
- Secondary indicators (resource utilization factors): Patrol, make an annual assessment

5.3.2 Contingency Planning

Identification of thresholds allows the anticipation of critical events. Managers need to plan for alternative transport where sensitivities become sensitive himself. For example:

- Develop contingency plans where costs escalate beyond 12% of estimate
- Create plans to diversify your supply base if you face disruptions of >20%
- Contract with substitute carriers for demand spikes over 8_reason_table_split

5.3.3 Contract Negotiation

Price/margin information helps drive contract negotiations with carriers and vendors. The fact that supply constraints have a shadow price of zero or negative indicates to a manager that they should not pay premiums for additional capacity at existing sources as the utilization rates imply underutilized capacity. Instead, investment should emphasize cost reductions on high-sensitivity routes or be directed at developing alternative routes with attractive cost-time characteristics.

5.3.4 Objective Weight Calibration

The study shows that objective weights have a major impact on optimal solutions. Managers should regularly reevaluate the relative weights based on evolving business priorities. The sensitivity analysis yields a numerical value of what it costs to change priorities, arming decision makers with this information.

5.4 Methodological Contributions

Methodological contributions of this study are as follows:

1. Global approach: Multiple objectives optimization and global sensitivity analysis are combined to realize an integrated decision support rather than independent optimization or sensitivity analysis.
2. Practical consequences: The application of WinQSP software showed that using standard commercial software, complex multi-objective multi-item models can be solved with minimum difficulty which contributes to practical use.
3. Quantitative guidelines: The definition of sensitivity ratios and stability ranges makes qualitative notions of sensitivities become quantitative decision principles.

5.5 Limitations and Considerations

Several limitations should be acknowledged:

1. **Linear approximation:** Multi-objective optimisation with sum weighted method implies linear trade-offs between objectives, which may not be able to account for all non-linear related interactions in complicated systems.
2. **Deterministic model:** The analysis is deterministic in that parameter variations were treated deterministically using parametric study, NAHST and MI processes but not used on probabilistic distributions by which extra information could have been added under uncertainty.

3. **Scope of computations:** the problem addressed in the case study is a small sized one (3 sources, 4 destinations, 2 products). Problems with larger scales can now have the different sensitivities due to more complicated solution space.
4. **Resource utilization distilled down:** There is a simplified resource utilization goal, with simplified factors that are proportional to travel times. In the real world, resources may be more complicated: as types of vehicle/shift or using capacity limits.

Nevertheless, the study has added some useful inputs for sensitivity analysis of multi-objective multi-item transportation models and gives a good direction for real life implications.

6. CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

In this paper a comprehensive investigation and analysis on the sensitivity of multi-objective multi-item transportation problems about optimal solutions is presented. The following conclusions are derived:

1. Sensitivity analysisThe proposed model is mostly sensitive to shipping cost (0.85–1.35) respectively by demand amount, supply capacity and advancement periods of transportation (0.95–1.20and 0.90–1.25and 0.60–0.85), sequentially. This design allows for concise communication about what's most important to monitor and manage parameter-wise.
2. The stable zones of solution As shown in Fig. 2(b), the optimal case (the highest IQEE) does not vary while considering 10% variation in parameters. It's what is known as a “robust zone.” 50) but not further: beyond a certain point "solutions" begin to "flow" into the region of transition (10–20%), where some changes indeed should take place — and then into the critical area (beyond 20%) where large scale structural peculiarities are anticipated.
3. Conflicting priorities: the paper examines conflicting goals. Up to a somewhat 10% smaller total travel time, the cost is about $\Delta c \approx 6.5\%$. Furthermore, the total travel time has been extended by 18.4%, even if fares had been lowered. It helps gauge these connections so that educated decisions can be made in the event of a turn-around.
4. Discovery of significant parameters the type of travel cost function in the high-volume flows (e.g., Baghdad-Karbala and Mosul-Erbil according to the highest sensitivity values) requires further study, for there are higher sensitivity values. A shadow price analysis indicates that supply restrictions tend to be weak and demand constraints that are

only weakly binding have limited development potential.

5. **Two Geographical optimization** There is a geographical proximity to the best solutions, which are also consisting of multiple aims. The source/s normally remain in the shops next door, although multi-objective optimization means that sometimes distance is traded against other parameters.

6. **Practical application:** The research demonstrates that the WinQSP program is useful for solving practical transportation problems that have more than one objective and more than one unit. i.e., professional can work with it and to extract the best solutions, but determine if those solutions are sensitive enough too.

6.2 Recommendations

Implications Based on the findings of this study, recommendations for practitioners, researchers and decision-makers are offered:

6.2.1 For Practitioners and Managers

1. **Tiered monitoring:** A three-tier parameter monitoring system should be developed with high-priority parameters (costs, demand) monitored weekly, moderate-priority parameters (supply and time), monthly and lowpriority parameters every three months.

2. **Set threshold points:** Program management information systems with alarms which sounded whenever accepted measures plunged more than 8-10% from the norm so immediate indications were available, paving way for response in advance of when optimal solutions would need re-configuration.

3. **Develop alternative plans:** Develop alternative transportation scenarios of when major variables are converging to a critical point (12% costs, 8% demand, 15% times) in the reference scenario. These plans should be reviewed and revised every three months.

4. **Regular objective calibration:** Review objective weights every quarter to ensure they are currently relevant. Interpret sensitivity analysis on the costs of shifts in priorities preimplementation.

5. **Spend in the reduction:** Deploy purposeful CI on high-sensitivity routes that offer outsized reduction. The results of this research show that any cost reduction on the Baghdad-Karbala and Mosul-Erbil would significantly contribute to enhancing the objective function.

6. **Utilize spare capacity:** There seems to be some slack available according to the current shadow price analysis. Think about lowering inventory holding costs closer to the source or shifting capacity elsewhere rather than building additional.

7. **Flexible contracts:** Embed flexibility clauses in transport contracts to account for modification of routes when parameters are getting closer to threshold values. Constructed penalty terms which mirror shadow fees for incentive compatibility.

6.2.2 For Researchers

1. **Expand to stochastic models:** Future studies may consider using stochastic programming methods to introduce probability distributions for uncertain parameters served as the counterpart of deterministic sensitivity analysis.

2. **Non-linear multi-objective approaches:** Explore non-linear multi-objective optimization approaches, such as epsilon-constraint method, goal programming or evolutionary operations that would better reflect non-linear trade-offs instead of the weighted sum formulations used in this study.

3. **Large-scale applications:** Perform sensitivity analysis on large scale transportation networks (10+ origins, 20+ destinations, 5+ commodities) to check whether the sensitivity patterns scale in an expected manner, have emergent complexity.

4. **Temporal sensitivity analysis:** Establish techniques for sensitivity analysis in dynamic transportation models whose parameters are time-varying instead of fixed snapshots.

5. **Machine Learning Integration:** Investigate hybrid method optimization-based sensitivity analysis and ML prediction of parameter changes to establish proactive decision support systems.

6. **Multi-stage transport:** Generalize the system to multi-tiered supply chains including intermediate DCs and multiple levels of transportation.

7. **Uncertainty quantification** – Tools to quantify the total system uncertainty resulting from combining of individual parameter sensitivities (with respect to particular outputs) taking into account correlations between parameters.

6.2.3 For Policy Makers

1. **Infrastructure investment prioritization:** Use sensitivity analysis outcomes to rank transportation corridors according to their relative economic effectiveness. Maximum network efficiency savings are realized on high-sensitivity routes by enhancing infrastructure.

2. **Local development:** Take into account the sensitivity of transportation costs when deciding where to locate ZEDES. The preferred locations of manufacturing facilities are regions which have good connections to many demand centers.

3. **Regulatory:** Formulate regulations that incentivizes transportation companies to have updated parameter databases and perform sensitivity analysis at regular intervals, which will improve the resilience of the system.

6.3 Future Research Directions

This study has a few limitations and many promising future research directions can be derived.

- **Integration of robust optimization:** Join Sensitivity analysis with robust optimization such that solutions are explicitly formulated to include parameter uncertainty in an applied mathematical model.

- **Dynamic optimization in real-time applications:** Generalize the model to include changes of parameters over

time, so that solutions should be adapted dynamically and synchronization behavior for models evolving with different timescales.

- Integrate sustainability: Introduce environmental sustainability goals (carbon emission, fuel consumption) directly as objectives in the multi-objective framework along with associated sensitivity analysis.
- Sensitivity indicators according to risk Based on risk weight, calculate sensitivity index based on probability on parameters of change rather than equal parametric variations.
- Machine learning for parameter prediction: Construct predictive models of transportation parameters based on historical data and incorporate the predictions into optimization models to make proactive decisions.
- Multi-modal transportation: Generalize the analysis to multi-modal transportation systems with modes of truck, rail, air and sea, each being defined by its own set of parameters and constrains.
- Behavioral issues: Include behavioral elements of decision-making under uncertainty and study how managers really use sensitivity information in decisions.

6.4 Final Remarks

1. This study reveals the fact that sensitivity analysis is no longer a theoretical activity but a real necessity when applying transportation optimization models to flexible service designs. By determining which species are more or less sensitive, the stability zones, and the critical breaking points this work translates general optimization theory into concrete management application.
2. The multi-objective multi-item model captures a realistic situation in the modern supply chain which is concerned with transporting several items under multiple performance measures. The sensitivity analysis shows which parameters need to be carefully controlled and which can be relaxed thus determining an efficient allocation of scarce monitoring resources.
3. As the supply chains continue to grow in size and dynamics, combinational optimization modeling combined with comprehensive sensitivity analysis will be a competitive necessity. "The ones that are monitoring the critical parameters, have knowledge of how sensitive the solutions are and plan for parameter change will be more adaptable and resilient.
4. A decision-making basis is also proposed for transportation and logistics management based on the methodological framework of researches. This research, in quantifying the change of a solution resulting from changes of parameters, enables decision makers to handle uncertainty confidently and efficiently optimize multi-objective multi-items transportation systems.

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