



A Study on Relaxed Skolem Mean Labeling of 4 - Star Graph with Partition (3,1)

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ARTICLE INFO	ABSTRACT
Published Online: 18 March 2026	To prove that the 4 - star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta}$ where
Corresponding Author: Dr. G. Padma Karthiyayini	$\alpha_1 \leq \alpha_2 \leq \alpha_3$ is a relaxed skolem mean graph if $\beta - \alpha_1 - \alpha_2 - \alpha_3 = -6$ is the core objective of this article.
KEYWORDS: Relaxed skolem mean graphs, relaxed skolem mean labeling, 4-star graph.	

Note: It has already been proved that a 4-star graph is a relaxed Skolem mean graph if $\beta - \alpha_1 - \alpha_2 - \alpha_3 = 6$

1. INTRODUCTION

Relaxed skolem mean labeling of a graph was first introduced by V.Balaji et al.in [5].In that paper , they clearly defined this type of labeling for the first time. They also explained the basic properties and conditions that a graph must satisfy to have a relaxed skolem mean labeling.

2. PRELIMINARIES

Definition 2.1: A graph $G = (V, E)$ with p vertices and q edges is said to be a **skolem mean graph** if there exists a function $f : V \rightarrow \{1, 2, 3, \dots, p = |V|\}$ such that the induced map $f^* : E \rightarrow \{2, 3, \dots, p = |V|\}$ given by

$$f^*(e = uv) = \begin{cases} \frac{f(u) + f(v)}{2} & \text{if } (f(u) + f(v)) \text{ is even} \\ \frac{f(u) + f(v) + 1}{2} & \text{if } (f(u) + f(v) + 1) \text{ is even} \end{cases}$$

then, the resulting distinct edge labels are from the set $\{2, 3, \dots, p = |V|\}$.

Definition 2.2 : A graph $G = (V, E)$ with p vertices and q edges is said to be a relaxed skolem mean graph if there exists a function $f : V \rightarrow \{1, 2, 3, \dots, p+1 = |V| + 1\}$ such that induced edge map $f^* : E \rightarrow \{2, 3, \dots, p+1 = |V| + 1\}$ given by

$$f^*(e = uv) = \begin{cases} \frac{f(u) + f(v)}{2} & \text{if } (f(u) + f(v)) \text{ is even} \\ \frac{f(u) + f(v) + 1}{2} & \text{if } (f(u) + f(v) + 1) \text{ is even} \end{cases}$$

The resulting distinct edge labels are from the set $\{2, 3, \dots, p+1 = |V| + 1\}$.

Note 2.3: There are p vertices and available vertex labels are $p + 1$ and hence one number from the set $\{1, 2, 3, \dots, p+1 = |V| + 1\}$ is not used and we call that number as the relaxed label. When the relaxed label is $p + 1$, the relaxed mean labeling becomes a skolem mean labeling.

Result 2.4: In the relaxed skolem mean labeling $p \geq q$.

Result 2.5: The three-star graph $K_{1,a} \cup K_{1,b} \cup K_{1,c}$ satisfies relaxed skolem mean labeling if $a + b \leq c \leq a + b + c$.

3. MAIN RESULT

Theorem 3.1: The 4-star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta}$ where $\alpha_1 \leq \alpha_2 \leq \alpha_3$ is a relaxed skolem mean graph if $\beta - \alpha_1 - \alpha_2 - \alpha_3 = -6$

Proof: Let $\sigma_1 = \alpha_1; \sigma_2 = \alpha_1 + \alpha_2; \sigma_3 = \alpha_1 + \alpha_2 + \alpha_3$

Consider the 4 - star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta}$

The condition $\beta - \alpha_1 - \alpha_2 - \alpha_3 = -6$ gives rise to the case $\beta = \sigma_3 - 6$. In this case we will establish that the graph G is relaxed skolem mean. Let $V = V_1 \cup V_2 \cup V_3 \cup V_4$ be the vertex set of G where $V_k = \{v_{k,i} : 0 \leq i \leq \alpha_k\}$; for $1 \leq k \leq 3$ and

$V_4 = \{v_{4,i} : 0 \leq i \leq \beta\}$. Let $E = \bigcup_{k=1}^3 \{v_{k,0}v_{k,i} : 1 \leq i \leq \alpha_k\} \cup \{v_{4,0}v_{4,i} : 1 \leq i \leq \beta\}$ be the edge set of G .

Let $\beta = \sigma_3 - 6$

G has $\sigma_3 + \beta + 4 = 2\sigma_3 - 2$ vertices and $\sigma_3 + \beta = 2\sigma_3 - 6$ edges.

The rsv function $f : V \rightarrow \{1, 2, \dots, p+1 = \sigma_3 + \beta + 4 + 1 = 2\sigma_3 - 1\}$ as follows:

$$f(v_{1,0}) = 2; \quad f(v_{2,0}) = 4; \quad f(v_{3,0}) = 6;$$

$$f(v_{4,0}) = \sigma_3 + \beta + 4 + 1 = 2\sigma_3$$

$$f(v_{1,j}) = 2j - 1 \quad 1 \leq j \leq \alpha_1$$

$$f(v_{2,j}) = 2\sigma_1 + 2j - 1 \quad 1 \leq j \leq \alpha_2$$

$$f(v_{3,j}) = 2\sigma_2 + 2j - 1 \quad 1 \leq j \leq \alpha_3$$

$$f(v_{4,j}) = 2j + 8 \quad 1 \leq i \leq \beta = \sigma_3 - 6$$

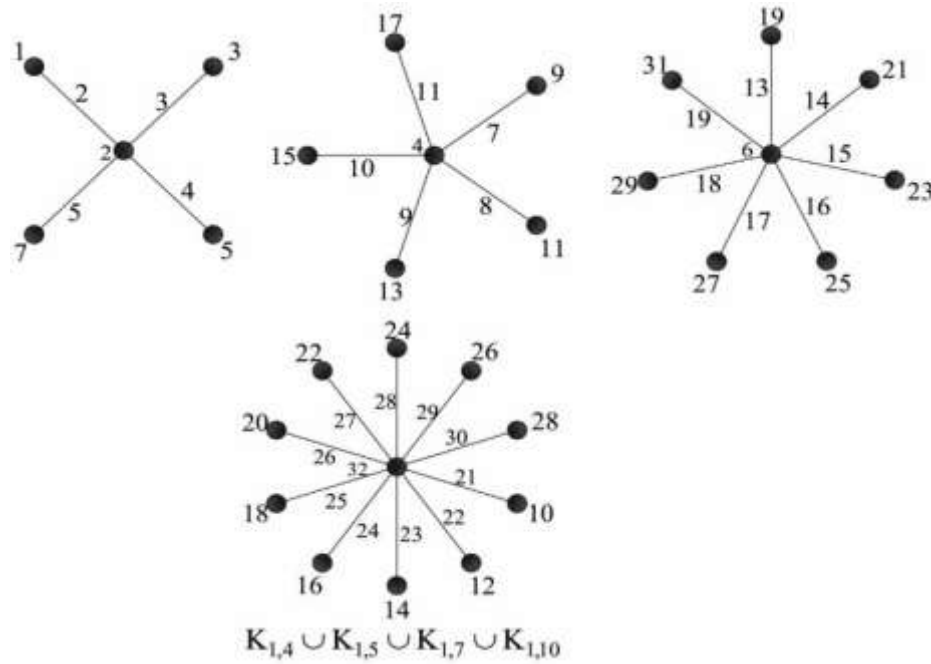
Here $2\sigma_3 - 2$ is the relaxed label.

The edge labels of $v_{1,0}v_{1,j}$ is $j + 1$ for $1 \leq i \leq \alpha_1$ (edge labels are $2, 3, \dots, \alpha_1 + 1 = \sigma_1 + 1$), $v_{2,0}v_{2,j}$ is $\sigma_1 + j + 2$ for $1 \leq i \leq \alpha_2$ ($\sigma_1 + 3, \sigma_1 + 4, \dots, \sigma_1 + \alpha_2 + 2 = \sigma_2 + 2$), $v_{3,0}v_{3,j}$ is $\sigma_2 + j + 3$ for $1 \leq i \leq \alpha_3$ ($\sigma_2 + 4, \sigma_2 + 5, \dots, \sigma_2 + \alpha_3 + 3 = \sigma_3 + 3$), $v_{4,0}v_{4,j}$ is $\sigma_3 + j + 4$ for $1 \leq i \leq \beta = \sigma_3 - 6$ ($\sigma_3 + 6, \dots, \sigma_3 + 5, \sigma_3 + (\sigma_3 - 6) + 3 = 2\sigma_3 - 3$)

The edge labels are therefore $2, 3, \dots, \sigma_1 + 1, \sigma_1 + 3, \sigma_1 + 4, \dots, \sigma_2 + 2, \sigma_2 + 4, \sigma_2 + 5, \dots, \sigma_3 + 3, \sigma_3 + 5, \sigma_3 + 6, \dots, 2\sigma_3 - 3$.

These induced edge labels of the graph G are therefore distinct. Hence G is a relaxed skolem mean graph.

Example



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