



Triangular Path Decomposition of Pentagonal Cyclic Snake Graphs

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Abstract

Let $G = (V, E)$ be a simple connected graph of order p and size q . If $\{G_1, G_2, \dots, G_n\}$ are edge disjoint subgraphs of G such that $E(G) = E(G_1) \cup E(G_2) \cup E(G_3) \cup \dots \cup E(G_n)$ then $\{G_1, G_2, \dots, G_n\}$ is said to be a Decomposition of a graph G . A graph of size $q = \binom{n+2}{3}$ is said to have a Triangular decomposition (TD) if G can be decomposed into n - subgraphs $\{G_1, G_2, \dots, G_n\}$ such that each subgraphs G_i is connected and $|E(G_i)| = \binom{i+1}{2}$ for $1 \leq i \leq n$. A Triangular decomposition (TD) in which each G_i is a path is said to be a Triangular path decomposition (TPD). In this paper we discuss about the necessary and sufficient conditions for the existence of Triangular path decomposition of Pentagonal cyclic snake graphs.

Keywords: Triangular decomposition; Triangular path decomposition; Pentagonal cyclic snake graphs.

1 Introduction

A graph G , referred to here is an undirected connected graph without loops or multiple edges. A path is a type of open walk where neither edges nor vertices are allowed to repeat. A path of length n is denoted by P_n Harary [1972]. The concept of Even Decomposition was introduced by Suthiesh Goldy and Ebin Raja Merly [2018]. The concept of Continuous Monotonic Decomposition of graph was introduced by Gnanadhas and Paulraj Joseph [2000]. Different types of decomposition of a graph G have been studied in the literature by imposing suitable conditions on the subgraphs G_i . Motivated by the above we have introduced the concept of Triangular Decomposition of a graph. A TD in which each G_i is a path of length $\binom{i+1}{2}$ in G , then we say that G admits Triangular path decomposition (TPD). In this paper we discuss about the necessary and sufficient conditions for the existence of Triangular path decomposition in Pentagonal snake graphs.

2 Basic Definitions

Definition 2.1. A graph G of size $q = \binom{n+2}{3}$ is said to have a Triangular decomposition (TD) if G can be decomposed into n - subgraphs $\{G_1, G_2, G_3, \dots, G_n\}$ such that each G_i is connected and $|E(G_i)| = \binom{i+1}{2}$ for $1 \leq i \leq n$.

Definition 2.2. A Triangular decomposition (TD) in which each G_i is a path is said to be Triangular path decomposition (TPD).

Definition 2.3. A Pentagonal snake graph kC_5 is obtained from a path $v_0v_1v_2 \dots v_m$ by joining v_i and v_{i+1} for $1 \leq i \leq m-1$, to new vertices $u_i^{(1)}, u_i^{(3)}$ and then joining $u_i^{(1)}, u_i^{(2)}$ and $u_i^{(2)}, u_i^{(3)}$. That is the path P_m by replacing each edge of the path by a cycle C_5 .

3 Triangular Decomposition of Pentagonal cyclic snake graphs

Theorem 3.1. Any Pentagonal snake graph $\frac{(5m-2)(5m-1)(m)}{6}C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_n)$ if and only if $n = 5m-2, m \in N$.

Proof. Assume that $\frac{(5m-2)(5m-1)(m)}{6}C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_n)$. Now $q(\frac{(5m-2)(5m-1)(m)}{6})C_5 = 5(\frac{(5m-2)(5m-1)(m)}{6})$. Then $5(\frac{(5m-2)(5m-1)(m)}{6}) = \frac{n(n+1)(n+2)}{6}$. This implies $n = 5m-2, m \in N$.

Conversely, suppose $n = 5m-2, m \in N$. Let $V = \{v_0, v_1, v_2, \dots, v_{\frac{(25m^3-15m^2+2m)}{6}}\}$ be the set of vertices of the base path of $\frac{(5m-2)(5m-1)(m)}{6}C_5$ of length $\frac{(5m-2)(5m-1)(m)}{6}$.

Let v_0 and $v_{\frac{(25m^3-15m^2+2m)}{6}}$ are the beginning and end vertices of the base path respectively. Let the base path be $P_{\frac{(25m^3-15m^2+2m)}{6}}$. Also we consider the path $v_0, u_1^{(1)}, u_1^{(2)}, u_1^{(3)}, v_1, u_2^{(1)}, u_2^{(2)}, u_2^{(3)}, v_2, \dots, u_{\frac{(25m^3-15m^2+2m)}{6}}^{(1)}, u_{\frac{(25m^3-15m^2+2m)}{6}}^{(2)}, u_{\frac{(25m^3-15m^2+2m)}{6}}^{(3)}, v_{\frac{(25m^3-15m^2+2m)}{6}}$ of length $4(\frac{(25m^3-15m^2+2m)}{6})$. To prove $\frac{(5m-2)(5m-1)(m)}{6} C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_n)$ where $n = 5m - 2, m \in N$. We prove this theorem by using induction on ' m '. When $m = 1$, $q(\frac{(5m-2)(5m-1)(m)}{6} C_5) = \frac{(5-2)(5-1)(5)}{6}$. Clearly $\frac{(5-2)(5-1)(5)}{6}$ admits TPD (P_1, P_2, P_3) . Suppose the result is true when $m = k$. That is $\frac{(5k-2)(5k-1)k}{6} C_5$ admits TPD $(P_1, P_2, \dots, P_n)$ where $n = 5k - 2, m \in N$. To prove the result is true for $m = k+1$. That is to prove $\frac{(5(k+1)-2)(5(k+1)-1)(k+1)}{6} C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_{5(k+1)-2})$. Now $q(\frac{(5(k+1)-2)(5(k+1)-1)(k+1)}{6} C_5) = (\frac{(25k^3+75k^2-15k^2+75k-30k+25-15+2+2k)}{6})5 = (\frac{(25k^3-15k^2+2k)}{6})5 + (\frac{(75k^2+75k-30k+12)}{6})5$ and the sum of edges of $(P_{5k-1}, P_{5k}, \dots, P_{5(k+1)-2})$ equal to $(\frac{(75k^2+75k-30k+12)}{6})5$ and hence $(\frac{(5(k+1)-2)(5(k+1)-1)(k+1)}{6}) C_5$ can be decompose into $(P_1, P_2, P_3, \dots, P_{5(k+1)-2})$. We conclude by the induction principle that any pentagonal snake graphs $\frac{(5m-2)(5m-1)m}{6} C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_n)$ if and only if $n = 5m - 2, m \in N$. $\square$

Table 1: $(\frac{(5m-2)(5m-1)m}{6}) C_5$

m	q	TPD $(P_1, P_2, P_3, \dots, P_n)$
1	10	(P_1, P_2, P_3)
2	120	$(P_1, P_2, P_3, \dots, P_8)$
3	455	$(P_1, P_2, P_3, \dots, P_{13})$
4	1140	$(P_1, P_2, P_3, \dots, P_{18})$
5	2300	$(P_1, P_2, P_3, \dots, P_{23})$
6	4060	$(P_1, P_2, P_3, \dots, P_{28})$
7	6545	$(P_1, P_2, P_3, \dots, P_{33})$
8	9880	$(P_1, P_2, P_3, \dots, P_{38})$
9	14190	$(P_1, P_2, P_3, \dots, P_{43})$
10	19600	$(P_1, P_2, P_3, \dots, P_{48})$

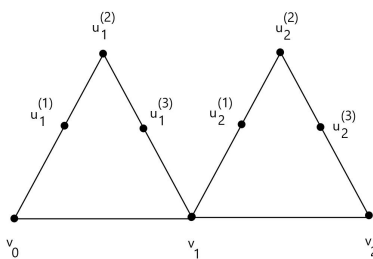


Figure 1: Pentagonal Snake Graph $2C_5$

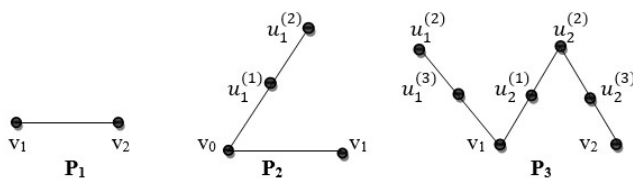


Figure 2: TPD (P_1, P_2, P_3) of $2C_5$

Theorem 3.2. Any Pentagonal snake graph $\frac{(5m-1)m(5m+1)}{6}C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_n)$ if and only if $n = 5m - 1, m \in N$.

Proof. Assume $\frac{(5m-1)m(5m+1)}{6}C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_n)$. Then $q(\frac{(5m-1)m(5m+1)}{6}C_5) = 5(\frac{(5m-1)m(5m+1)}{6})$. That is $\frac{(5m-1)m(5m+1)}{6} = \frac{n(n+1)(n+2)}{6}$. Implies $n = 5m - 1, m \in N$.

Conversely, suppose $n = 5m - 1, m \in N$. Let $V = \{v_0, v_1, v_2, \dots, v_{\frac{(25m^3-m)}{6}}\}$ be the set of vertices of the base path of $(\frac{(5m-1)m(5m+1)}{6})C_5$ of length $\frac{(5m-1)m(5m+1)}{6}$. Let v_0 and $v_{\frac{(25m^3-m)}{6}}$ are the beginning and end vertices of the base path respectively.

Let the base path be $P_{\frac{(25m^3-m)}{6}}$. Also consider the path $v_0, u_1^{(1)}, u_1^{(2)}, u_1^{(3)}, v_1, u_2^{(1)}, u_2^{(2)}, u_2^{(3)}, v_2, \dots, u_{\frac{(25m^3-m)}{6}}^{(1)}, u_{\frac{(25m^3-m)}{6}}^{(2)}, u_{\frac{(25m^3-m)}{6}}^{(3)}, v_{\frac{(25m^3-m)}{6}}$ of length $4(\frac{(5m-1)m(5m+1)}{6})$.

To prove $(\frac{(5m-1)m(5m+1)}{6})C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_n)$ where $n = 5m - 1, m \in N$. We prove this theorem by using induction on 'm'. When $m = 1, q(\frac{(5m-1)m(5m+1)}{6}C_5) = \frac{(5-1)5(5+1)}{6}$. Clearly $\frac{(5-1)5(5+1)}{6}$ admits TPD (P_1, P_2, P_3, P_4) . Suppose the result is true when $m = k$. That is $(\frac{(5k-1)k(5k+1)}{6})C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_n)$ where $n = 5k - 1, m \in N$. To prove the result is true for $m = k + 1$. That is to prove $(\frac{(5(k+1)-1)(k+1)(5(k+1)+1)}{6})C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_{5(k+1)-1})$.

Now $q(\frac{(5(k+1)-1)(k+1)(5(k+1)+1)}{6}C_5) = \frac{(25k^3+75k^2+75k-k+25-1)}{6}5 = (\frac{(25k^3-k)}{6})5 + (\frac{(75k^2+75k+24)}{6})5$ and the sum of edges of $(P_{5k}, P_{5k+1}, P_{5k+2}, \dots, P_{5(k+1)-1})$ equal to $\frac{(75k^2+75k+24)}{6}5$ and hence $\frac{(5(k+1)-1)(k+1)(5(k+1)+1)}{6}C_5$ can be decompose into $(P_1, P_2, P_3, \dots, P_{5(k+1)-1})$. We

conclude by the induction principle that any pentagonal snake graphs $\frac{(5m-1)m(5m+1)}{6}C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_n)$ if and only if $n = 5m - 1$, $m \in N$. $\square$

Table 2: $\left(\frac{(5m-1)m(5m+1)}{6}\right) C_5$

m	q	TPD $(P_1, P_2, P_3, \dots, P_n)$
1	20	(P_1, P_2, P_3, P_4)
2	165	$(P_1, P_2, P_3, \dots, P_9)$
3	560	$(P_1, P_2, P_3, \dots, P_{14})$
4	1330	$(P_1, P_2, P_3, \dots, P_{19})$
5	2600	$(P_1, P_2, P_3, \dots, P_{24})$
6	4495	$(P_1, P_2, P_3, \dots, P_{29})$
7	7140	$(P_1, P_2, P_3, \dots, P_{34})$
8	10660	$(P_1, P_2, P_3, \dots, P_{39})$
9	15180	$(P_1, P_2, P_3, \dots, P_{44})$
10	20825	$(P_1, P_2, P_3, \dots, P_{49})$

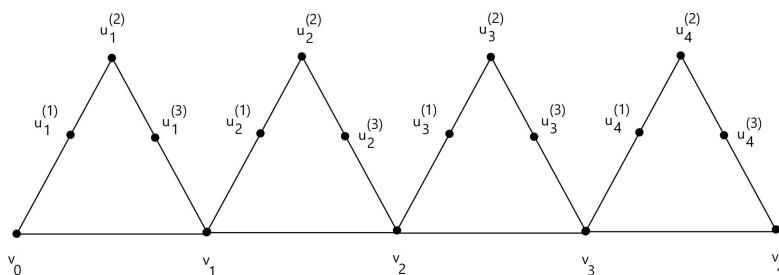


Figure 3: Pentagonal Snake Graph $4C_5$

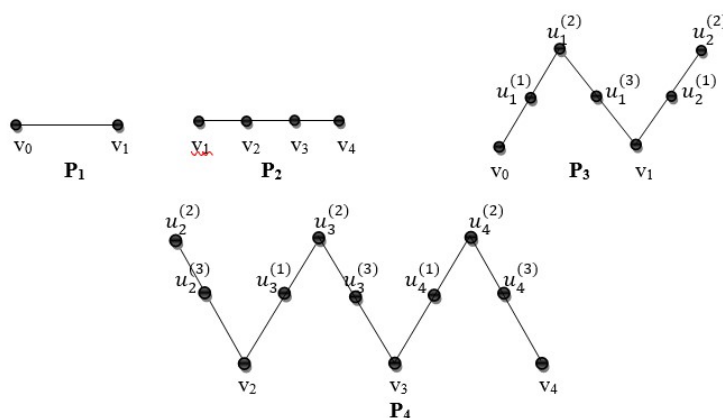


Figure 4: TPD (P_1, P_2, P_3, P_4) of $4C_5$

Theorem 3.3. Any Pentagonal snake graph $\frac{m(5m+1)(5m+2)}{6}C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_n)$ if and only if $n = 5m$, $m \in N$.

Proof. Assume $\frac{m(5m+1)(5m+2)}{6}C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_n)$. Now $q(\frac{m(5m+1)(5m+2)}{6}C_5) = 5(\frac{m(5m+1)(5m+2)}{6})$. That is $\frac{5m(5m+1)(5m+2)}{6} = \frac{n(n+1)(n+2)}{6}$. Implies $n = 5m$, $m \in N$.

Conversely, suppose $n = 5m$, $m \in N$. Let $V = \{v_0, v_1, v_2, \dots, v_{\frac{25m^3+15m^2+2m}{6}}\}$ be the set of vertices of the base path of $\frac{m(5m+1)(5m+2)}{6}C_5$ of length $\frac{m(5m+1)(5m+2)}{6}$. Let v_0 and $v_{\frac{25m^3+15m^2+2m}{6}}$ are the beginning and end vertices of the base path respectively. Let the base path be $P_{\frac{25m^3+15m^2+2m}{6}}$. Also we consider the path $v_0, u_1^{(1)}, u_1^{(2)}, u_1^{(3)}, v_1, u_2^{(1)}, u_2^{(2)}, u_2^{(3)}, v_2, \dots, u_{\frac{25m^3+15m^2+2m}{6}}^{(1)}, u_{\frac{25m^3+15m^2+2m}{6}}^{(2)}, u_{\frac{25m^3+15m^2+2m}{6}}^{(3)}, v_{\frac{25m^3+15m^2+2m}{6}}$ of length $4(\frac{m(5m+1)(5m+2)}{6})$. To prove $(\frac{m(5m+1)(5m+2)}{6})C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_n)$ where $n = 5m$, $m \in N$. We prove this theorem by using induction on 'm'. When $m = 1$, $q(\frac{m(5m+1)(5m+2)}{6}C_5) = \frac{5(5+1)(5+2)}{6}$. Clearly $\frac{5(5+1)(5+2)}{6}$ admits TPD $(P_1, P_2, \dots, P_5)$. Suppose the result is true when $m = k$. That is $(\frac{k(5k+1)(5k+2)}{6})C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_n)$ where $n = 5k$, $m \in N$. To prove the result is true for $m = k + 1$. That is to prove $(\frac{(k+1)(5(k+1)+1)(5(k+1)+2)}{6})C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_{5(k+1)})$. Now $q(\frac{(k+1)(5(k+1)+1)(5(k+1)+2)}{6}C_5) = (\frac{25k^3+75k^2+15k^2+75k+30k+25+15+2+2k}{6})5 = (\frac{25k^3+15k^2+2k}{6})5 + (\frac{75k^2+75k+30k+42}{6})5$ and the sum of edges of $(P_{5k+1}, P_{5k+2}, \dots, P_{5(k+1)})$ equal to $(\frac{75k^2+75k+30k+42}{6})5$ and hence $(\frac{(k+1)(5(k+1)+1)(5(k+1)+2)}{6})C_5$ can be decompose into $(P_1, P_2, P_3, \dots, P_{5(k+1)})$. We conclude by the induction principle that any pentagonal snake graphs $\frac{m(5m+1)(5m+2)}{6}C_5$ admits TPD $(P_1, P_2, P_3, \dots, P_n)$

if and only if $n = 5m$, $m \in \mathbb{N}$.

□

Table 3: $\left(\frac{(m)(5m+1)(5m+2)}{6}\right) C_5$

m	q	TPD $(P_1, P_2, P_3, \dots, P_n)$
1	35	$(P_1, P_2, P_3, P_4, P_5)$
2	220	$(P_1, P_2, P_3, \dots, P_{10})$
3	680	$(P_1, P_2, P_3, \dots, P_{15})$
4	1540	$(P_1, P_2, P_3, \dots, P_{20})$
5	2925	$(P_1, P_2, P_3, \dots, P_{25})$
6	4960	$(P_1, P_2, P_3, \dots, P_{30})$
7	7770	$(P_1, P_2, P_3, \dots, P_{35})$
8	11480	$(P_1, P_2, P_3, \dots, P_{40})$
9	16215	$(P_1, P_2, P_3, \dots, P_{45})$
10	22100	$(P_1, P_2, P_3, \dots, P_{50})$

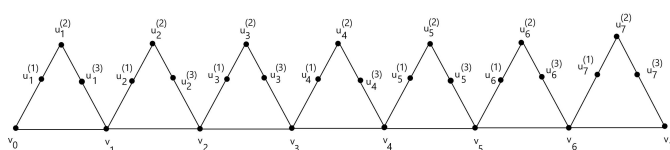


Figure 5: Pentagonal Snake Graph $7C_5$

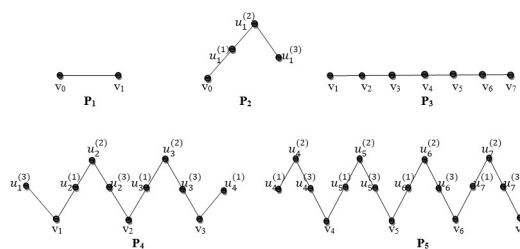


Figure 6: TPD $(P_1, P_2, P_3, \dots, P_5)$ of $7C_5$

4 Conclusion

In this paper we described above pentagonal cyclic snake graphs that accept Triangular path decomposition. Finally we conclude that this study can be extended to any odd cyclic snake graphs.

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