

An Investigation of Collaborative Deteriorating Inventory Models with Two-Level Trade Credit Policy and a Price Negotiation Scheme for Price-Sensitive, Stock-Dependent Demand

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ARTICLE INFO	ABSTRACT
Published Online: 20 January 2026	This paper presents a joint inventory policy that involves price negotiation, inventory-dependent demand, and deterioration under two-level trade credit in a coordinated model. The model allows realistic seller-buyer interactions through specification of the selling price, and thus reflects more closely supplier-buyer bargaining asymmetries in integrated supply chains. The demand is a function of the on-hand inventory and negotiated price and the inventory levels deplete continuously by customer occurrences and physical decay. The effect of cash flow pressure in the form of delayed (credit) payment is addressed via a two-level trade credit structure; an interest-free period and then an interest-incurring delay, consequently aligning inventory decisions with financial constraints. A user-friendly graphical interactive interface (GUI) is designed to obtain numerical solutions of the inventory differential equation, visualize trajectories of inventories and demands, and calculate the total collaborative cost of order quantity for holding, purchasing, ordering, deterioration and interest which is summed over both sides. Based on the simulation results, it is shown that the negotiation power and credit terms together with deterioration rate have a strong impact on both inventory behaviours and total system cost for numerical examples, significant advantage of cooperation decision rather than isolated decisions vendor or buyer. The model is highly suitable as a decision-making tool for enhancing and studying collaborative supply chain performance from integrated operational and financial perspectives.
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1. INTRODUCTION

In the competitive and globalised market environment, efficient inventory management is significantly central to the study of how supply chain performance can be improved upon and how firms can become sustainable. Traditional models that ignore the competition between vendor and buyer, such as independent decision-making inventory models, are no longer capable of solving the supply chain problem in modern industries consisting of uncertain demand which can become wilder, finite replenishment rate, financial limit and high price competition. To cope with these challenges, collaborative inventory management has been evolving as a strategic alternative where the several echelons of the supply chain synchronize operational and financial

decision making in order to generate mutual benefits, decrease costs and improve service. A main disadvantage of the classical inventory models, including EOQ (Economic Order Quantity), is the assumption essentially of constant demand and an instantaneous payment. In reality, demand is seldom constant and can be affected by many factors such as price of the items, visibility of its stock. Consumers buy more of a commodity when substitutes are abundant, since high stock levels communicate reliability and availability, and prices that are too high reduce consumption. This behaviour has motivated the development of stock-dependent demand functions and price-elastic demands, which are more realistic market models. By taking such demand structures into account in inventory models, the decision-maker can

evaluate how pricing and inventory policies work together to affect customer behaviour and sales. Another important feature of most real-world products is fading. Foodstuffs, medicines, chemicals, fashion goods, and high-technology components degrade in both value and use over time to spoilage, obsolescence, or innovation. Failure to account for decay can result in inefficient ordering and pricing, unnecessary waste, and additional operating cost exposure [1]. Decremental inventory models thus occupy a vital position in supply chain planning and decision making, for they capture the fact that on-hand inventory is spoiled on a continuous basis as well as its financial implications.

On the other hand, financial policies also have a remarkable effect on inventory decisions. Trade credit, in particular, is a common financial inducement in supply chains that enables buyers to postpone payments and free up working capital as well as additional opportunities for sellers to create sales. Two-level trade credit policies are commonly adopted in practice, which means there exists an interest-free period initially before interest charges begin accruing. It promotes purchasers to place larger orders, and it compensates suppliers for the time value of money by earning interest on delayed payments. However, long-term credit can also lead to increased risk in finance and interest expenses due to unsold inventory. The use of the two-echelon credit policy in inventory models, makes it possible to analyse not only optimal logistics systems trade-off regarding efficiency and financial cash flow aspects. Although deteriorating inventory systems, price-sensitive demand and trade credit policies have been studied separately by many researchers not much work has appeared jointly for the interaction of these three factors under GR-credit policy [2]. A small number of studies also include the price negotiation function that mirrors how sellers and buyers can haggle with each other. In actual

markets, prices tend not to be external since they are seldom, but rather instead negotiated influenced by market power and long-term relations and strategic motivations. The proposed pricing scheme is based on a negotiation between members of the supply chain, hence enables us to endogenously determine the selling price that reflects both partners' relative bargaining power. Such a mechanism has particular relevance in collaborative environments where pricing and revenue determination decisions also influence demand, inventory levels, and the cost of financing. Treating the vendor and buyer as unity rather than two individual parts, this model aims to minimize the total collaborative cost which is composed of holding cost, purchasing order cost, ordering cost (inventory), decayed product disposal cost, and interest penalty incurred in slow payment. This integrated perspective is consistent with modern supply chain methodologies that focus on coordination, information sharing and co-optimization. The application of a simulation-based GUI improves the practical relevance of the model [3]. The GUI allows decision-makers to test different parameter values, observe the inventory and demand trajectories over time, and instantly see how negotiation power, credit terms, and unsold stock profits also affect total cost. Such an interface brings together the theoretical model and its practical implementation, rendering the tool user-friendly for both practitioners and academics. Overall, this study enriches the literature by addressing a joint inventory system with deterioration, two-level trade credit, price protection and general demand pattern. Through such an integrated model with both operations and financials mixed mechanisms, the structured model can bring additional insights into supply chain dynamics and serve as a reliable reference to construct more effective and environmentally-friendly inventory policies in competitive markets.

II. FINDINGS FROM LITERATURE STUDY

Year	Author(s)	Type of Model	Key Assumptions & Features	Major Findings
2004	Balkhi & Benkherouf	Perishable inventory model	Demand varies with stock level and time; mathematical modeling of deterioration	Demonstrated the importance of realistic demand-decay modeling using simple mathematical formulations
2005	Teng & Chang	EOQ model for deteriorating items	Time-varying and promotion-sensitive demand; stock-level restriction due to shelf space	Identified sufficient conditions for profit maximization and analyzed pricing and production effects
2006	Jolai et al.	Enhanced EPLS model	Stock- and demand-dependent inflation; Whipple deterioration; shortages allowed	Showed sensitivity of optimal policies to inflation and deterioration parameters
2007	Alfares	Inventory model with time-varying holding cost	Inventory-dependent demand; carrying cost increases over time	Proved that time-varying holding cost assumptions are more realistic than constant-cost models
2009	Mirzazadeh et al.	Perishable inventory model under inflation	Stochastic inflation; restricted production; delivery delays	Demonstrated superiority of stochastic inflation modeling through numerical validation

2012	Lee & Dye	Inventory model with preservation investment	Inventory-dependent demand; deterioration controlled via conservation technology	Proved concavity of profit function and established optimal replacement and investment policy
2014	Alfares	EPQ model with relaxed assumptions	Restricted production rate; inventory-dependent demand; life-cycle-based storage cost	Validated the practicality of abandoning classical EPQ assumptions in industrial applications
2017	Mishra et al.	EOQ model for seasonal deteriorating items	Discount-dependent demand; inventory-level-sensitive price; preservation investment	Found profit function concave in price, order frequency, and preservation investment
2021	Sharma et al.	Pricing and inventory model for perishables	Time-dependent deterioration; demand-based production; no stockouts	Demonstrated feasibility of coordinated pricing and inventory control for short-lived products
2022	Sharma et al.	Time- and price-dependent inventory model	Demand transitions from time-based to price-based; perishability depends on time	Showed that single demand patterns are unrealistic; validated model with numerical examples

III. MATHEMATICAL MODEL

Let the system operate over a finite planning horizon T , where the buyer places a single order of size Q at the beginning of the cycle. The initial inventory level is therefore given by

$$I(0) = Q.$$

3.1 Negotiated Pricing Mechanism

The selling price in the collaborative framework is determined through a **negotiation process** between the vendor and the buyer. Let p_v denote the vendor's quoted price and p_b the buyer's acceptable price. A negotiation power parameter $\alpha \in [0,1]$ represents the relative bargaining strength of the vendor. The negotiated unit price p is defined as a convex combination:

$$p = \alpha p_v + (1 - \alpha)p_b.$$

A higher value of α implies stronger vendor dominance in the negotiation, while lower values favor the buyer.

3.2 Inventory-Dependent Demand Function

Demand is assumed to depend on the **available inventory level** as well as the negotiated price. Let a represent the base market demand, b the stock-dependency coefficient, and c the price sensitivity parameter. The instantaneous demand rate $D(t)$ is modeled as

$$D(t) = a + bI(t) - cp.$$

This formulation captures consumer behavior where higher visible inventory stimulates demand, while higher prices suppress it.

3.3 Inventory Dynamics with Deterioration

The inventory level decreases over time due to **customer demand** and **physical deterioration**. Let θ denote the constant deterioration rate. The inventory evolution is governed by the following differential equation

$$\frac{dI(t)}{dt} = -D(t) - \theta I(t).$$

Substituting the demand function, the governing equation becomes

$$\frac{dI(t)}{dt} = -(a + bI(t) - cp) - \theta I(t),$$

Which represents a linear first-order differential equation. This equation is numerically solved over the interval $[0, T]$ to obtain the inventory trajectory $I(t)$.

3.4 Cost Components of the Collaborative System

The final cost will depend on the all components of vendors and buyers of the inventory model.

Holding Cost: Let H_v and H_b denote the holding costs per unit per unit time for the vendor and buyer, respectively. The average inventory over the cycle is computed as

$$\bar{I} = \frac{1}{T} \int_0^T I(t) dt.$$

Thus, the total holding cost is

$$HC = (H_v + H_b)\bar{I}.$$

Purchasing Cost: The purchasing cost incurred for the order quantity is

$$PC = Qp.$$

Deterioration Cost: Let C_d represent the cost associated with each deteriorated unit. Since deterioration occurs continuously, the total deterioration cost is

$$DC = C_d \int_0^T \theta I(t) dt.$$

Ordering Cost: A fixed ordering cost S is incurred for placing the order, independent of order size.

3.5 Two-Level Trade Credit and Interest Cost

The model incorporates a **two-level trade credit policy**, where no interest is charged during the first credit period T_1 .

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Beyond this, interest is charged for an additional credit window $T2$ at an interest rate r . The interest cost is calculated over the interval $(T1, T1 + T2]$ as

$$IC = r \int_{T1}^{T1+T2} pI(t)dt.$$

This above formula gives the financial burden of delayed payment on the buyer and incentivizes optimal value under inventory model.

3.6 Total Collaborative Cost Function

Through aggregating all cost components, the **total collaborative cost** of the system is obtained as

$$TC = HC + PC + DC + S + IC.$$

The objective of the collaborative inventory system is to **minimize TC** through appropriate decisions on order quantity Q , negotiated price p , and credit policy parameters, while accounting for demand uncertainty, deterioration, and financial constraints.

Overall, the TC has been minimized for better result oriented inventory model. Through incorporating the interactions between operational actions and financial decisions, the model offers a practical and versatile instrument to analyse

coordinated supply chain policies under negotiations for price setting mechanisms and trade credits management [4].

IV. RESULT AND DISCUSSION

The GUI screenshot presented below shows the simulation windows. The Graphical User Interface (GUI) has been developed for simulating collaborative vendor-buyer inventory system featuring price negotiation, inventory-dependent demand, constant deterioration, and two-level trade credit policy. This interface has been developed in such a way as to enable an intuitive understanding of how operational, pricing, and financial parameters jointly impact inventory dynamics and total system cost. Left panel of the figure shows an orderly list of input parameters, each represented by a variable in the mathematical model. Together these parameters specify market, demand, pricing policy, stockholding dynamics, and cost. The Base Demand (a) is the smallest market demand that does not depend on inventory and price, while Stock Dependency (b) describes the positive impact of displayed inventory at a store on customer demand. The price sensitivity (c) represents the way in which demand falls as the negotiated price rises, thus directly connecting economic activity to pricing behaviour [5].

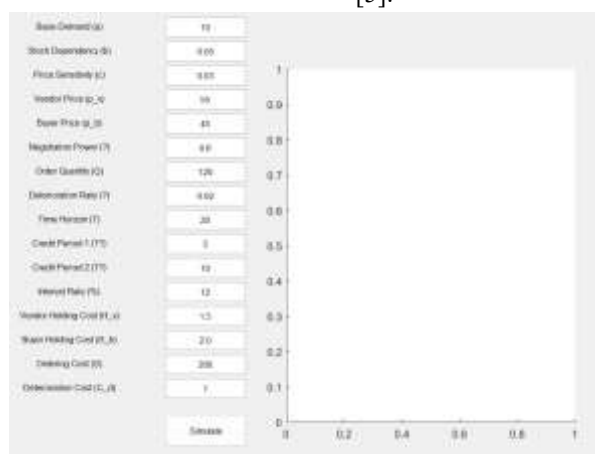


Fig. 1. Graphical User Interface (GUI)

Pricing-inputs the pricing inputs consist of the Vendor Price (pv) and Buyer Price (pb), which are bounds within which the haggling may take place. Negotiation Power (α) measures the weight of the vendor versus the buyer in determining at which price a deal ends up. If α is higher, the buyers' market power is weaker and therefore closer to p_{vp} p_{vp} , while lower values signify a stronger position of the buyer. This latter feature will be especially useful when examining cooperative vs. non-cooperative bargaining situations with the GUI of this toolbox. Stock-related ones are the Order Quantity (Q), which defines the initial inventory quantity, and the Deterioration Rate (θ), representing the physical decay of items. Time Horizon (T) determines the duration of time period for which the replenishment cycle is studied. Taken together these inputs determine the inventory path obtained through solving the inventory differential equation embedded in our model

[6]. The financial implication of the system is analysed based on Credit Period 1 (T1) and Credit Period 2 (T2). The first one is interest-free, the second carries an Interest Rate (%), this captures realistic payment delays in supply chains. Different cost elements as Vendor Holding Cost (Hv), Buyer Holding Cost (Hb), Setup/Ordering Cost (S) and Deterioration Cost allow proper assessment of the operational costs alongside financial costs shared between them. Simulate runs the numerical solution of inventory differential equation and computes the total collaborative cost. The graphical result (Inventory Level and Demand Rate versus time) of this realization is shown in the right panel of Figure 1. The drop of inventory curve captures both the impact of replenishment and deterioration, whereas demand curve explains how market consumption over the cycle is influenced by inventory availability and negotiated price. The figure caption

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automatically shows the Total Collaborative Cost and the Negotiated Price making it possible to quickly evaluate system behaviour under specific parameters. Figure 1 illustrates how the GUI becomes such a decision-support

tool, transforming a complex mathematical model of inventory into an intuitive visual tool for analysing joint supply chain strategies under negotiation and trade credit.

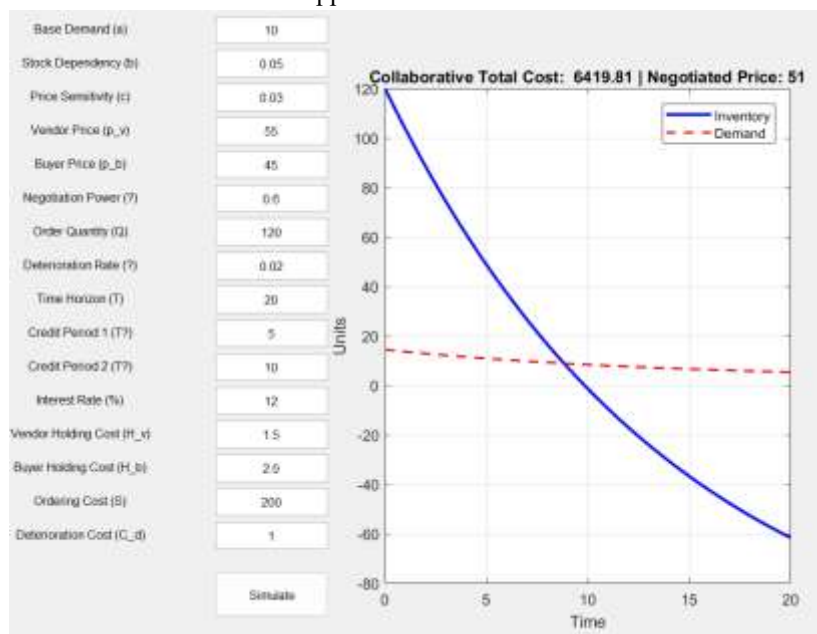


Figure 2: Simulation Results of the Collaborative Inventory Model Showing Inventory and Demand Dynamics

Figure 2 is the simulation result of the co-op inventory model (After running your system with parameter values you fed in using GUI). This graph visually illustrates how level of inventory and sales change through time for negotiated pricing, deterioration and trade credit terms as well reports the Collaborative Total Cost and the Negotiated Price at the top. At the start of the planning horizon ($t=0$), the inventory level starts at 120 units, which corresponds to the chosen order quantity $Q=120$. The blue solid curve represents the inventory trajectory over time. As the simulation progresses, inventory declines steadily due to the combined effects of customer demand and deterioration. The downward-sloping nature of the inventory curve reflects the continuous depletion modelled by the inventory differential equation. Initially, the inventory decreases rapidly because demand is relatively high when stock availability is large. Over time, as inventory diminishes, the rate of decline becomes smoother, although deterioration continues to exert pressure on the remaining stock. The dashed red curve is the demand rate (growth rate) that depends on inventory and price. Demand is bigger at the beginning of the cycle, since stock availability affects positively on customer preferences through the stock dependence parameter [7]. As time goes on and stock depletes, so does demand. This behaviour is exactly what can be expected from the demand function of the model, which involves a direct effect of inventory level on demand and an inverse effect of negotiated price. The downward slope of the demand line process indicates that the lower volume available in on-hand balance pool results in lack of visibility and therefore less consumption in the market. The negotiated

price displayed in the figure is 51, which results from the negotiation power parameter $\alpha=0.6$, the vendor price $p_v=55$, and the buyer price $p_b=45$. Since the vendor has a stronger bargaining position, the final price is closer to the vendor's quoted price. This negotiated price directly influences demand reduction and increases purchasing and interest-related costs, demonstrating the trade-off between pricing power and demand sustainability in collaborative supply chains. The Collaborative Total cost 6419.81 represents the sum of holding cost, purchasing cost, ordering cost deterioration and interest costs incurred by implementing two-level trade credit policy. The interest cost term starts from the first credit period ($T_1=5$) and spans second credit window ($T_2=10$), that is, interest accumulated on the value of unsold inventories. This reinforces the significance of considering credit terms in determining inventory policies and indicates more cost can be created by late payment. The interesting feature from Fig.2 is obviously the fact that for a certain order size the inventory curve crosses negatively near the end of the planning horizon, as it signalizes over-expansion of time horizon or over-demand-deterioration pressure with respect to this order amount. A Practical Implication From this observation we can infer that, for the system parameters considered, if non-negative inventory levels are to be maintained then the order size should be increased or the time horizon shortened or pricing and spoilage rates adjusted accordingly. Figure 2 successfully illustrates how the joint inventory model incorporates operational dynamics, negotiation results, and financial policies in a single simulation architecture. Well-defined

visual implications of inventory depletion and demand reduction for the decision-makers can help them to make informed decisions about controlling total collaborative costs, reducing system disruption and lead-time variability due to coordinated strategies such as pricing adjustments, order quantities, and credit policies [8-10].

V. CONCLUSION

In this research work, combination of price bargaining and inventory dependent demand is to employed with deteriorated products, Two-level trade credit (TLTC) into the basic model has been discussed. These components are integrated into an interactive GUI which enables the model to transform the complicated mathematical relations into a decision-support tool for coordinated vendor–buyer systems. The bargaining model presented in this paper explicates the effect of negotiation power on the selling price. Developed under a negotiated pricing rule, it is able to explain some everyday phenomena and illustrate that changes in relative negotiating strength impact on final market prices influencing demand, acquisition cost and expenses incurred. The inventory-based demand model is to reflect the market behaviour in a realistic manner where availability of products induces consumption, whereas spoilage represents an inevitable physical loss which speeds up the rate of depleting the inventory. Through simulation results some important trade-off relationships are emphasized in the collaborative supply chain. Higher negotiated prices, while favouring the provider, are a source of demand repression and financial costs, especially under longer accounts receivable periods. On the other hand, a larger order size provides better service levels and higher holding and deterioration costs. The proposed model is beneficial to illustrate that collaborative decision-making among pricing, inventory and finance operations can achieve lower total collaborative cost than isolated non-cooperative ones.

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