



In-Depth Analysis of Steady-State and Dynamic Modeling and Simulation of Multi-Effect Evaporators in Paper and Process Industries

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ARTICLE INFO	ABSTRACT
Published Online: 05 August 2025	Evaporators play a vital role in the chemical industry, particularly in concentrating dilute solutions and recovering solvents. Industries such as paper, sugar, and caustic soda production rely on evaporators to concentrate materials like black liquor, sugar cane juice, and caustic soda solutions. Multiple-effect evaporators (MEE) are commonly employed for this purpose. Notably, the evaporation stage is the most energy-intensive process in paper manufacturing, accounting for substantial steam consumption. In India's cellulosic fiber and filament industries, for instance, annual steam consumption for evaporation exceeds 1.75 million tons. Furthermore, studies have shown that multi-effect evaporator systems alone utilize approximately 24-30% of the total steam in large paper mills. Consequently, it is essential to conduct in-depth analyses and explore energy-saving strategies. Developing a mathematical model to predict the performance of the evaporation process would be highly beneficial in this regard.
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1. INTRODUCTION

The pulp and paper industry relies heavily on efficient chemical recovery from spent liquor. This process involves concentrating black liquor through evaporation, burning it to produce heat, and generating steam and electricity. Multiple-effect evaporators (MEEs) are commonly used for concentration, and their efficiency is influenced by design, liquor flow sequence, and handling methods.

Various configurations, such as forward, backward, mixed, and parallel feeds, are possible, but backward feeding is widely adopted for its balance between capacity and steam economy.

Long-tube vertical (LTV) and falling-film evaporators are popular designs, offering benefits like high heat transfer coefficients and compact designs.

Mathematical models for MEEs have been developed for various industries, including paper. These models differ in assumptions, approaches, and methodologies.

Research has focused on steady-state models, but dynamic modeling is essential for predicting system responses to changing conditions. Dynamic modeling enables the development of efficient control systems and optimizes performance.

Studies on dynamic behavior of MEE systems are limited, but researchers like Stefanov and Hoo (2003, 2004) and Kumar

et al. (2013) have made significant contributions to understanding MEE dynamics in the paper industry.

2. MODELING AND SIMULATION OF MULTI-EFFECT EVAPORATION: STEADY STATE CONDITION

Researchers have developed mathematical models for multiple-effect evaporator (MEE) systems, primarily focusing on steady-state conditions, across various process industries. Key contributions include:

- Lambert et al. (1987): Developed a nonlinear equation system for general MEE systems, addressing boiling point rise and nonlinear enthalpy relationships. They employed iterative linearization and Gauss elimination techniques to solve the system.
- Koko and Joye (1987): Compared linear and nonlinear methods, concluding that linear approaches offered faster computation, improved stability, and better convergence.
- Patel and Babu (1996): Created a computer program for designing and calculating single- and multiple-effect evaporators in general evaporation plants, facilitating practical applications of MEE systems.

2.1 Desalination/distillation Industries

Researchers have made significant contributions to modeling and simulating multiple-effect evaporator (MEE) systems for desalination and distillation processes. Key studies include:

- El-Dessouky et al. (1998): Developed a mathematical model for MEE desalination, analyzing the impact of design and operating variables on fresh water production costs.
- El-Dessouky et al. (2000): Conducted a performance analysis of parallel-feed MEE systems, examining the effects of heating steam temperature, seawater salinity, and number of effects.
- Al-Ansari et al. (2001): Modeled a single-effect evaporation desalination process integrated with an adsorption heat pump.
- Khademi et al. (2009): Conducted steady-state simulation and optimization of a six-effect evaporator for desalination.
- Zhou et al. (2019): Developed a comprehensive mathematical model for MEE desalination systems with thermal vapor compression.

Recently, Chandra et al. (2023) designed a vertical tube evaporator for small-scale, thermal-driven multiple-effect distillation systems, offering a low-cost, high-efficiency solution.

2.2 Various Process Industries:

Researchers have developed various steady-state models for evaporation plants across different industries, including food and juice processing. These models aim to improve the design, simulation, and control of multi-effect evaporators.

Several studies have contributed to this effort. For example:

- Cadet et al. (1999) developed an energy and mass balance-based model for a sugar plant, incorporating semi-empirical equilibrium functions.
- Ugrin and Urbicain (1999) proposed an algorithm for designing and simulating multi-effect evaporators with forward and backward feed.
- Bakker et al. (2006) demonstrated the effectiveness of cascade control architecture in enhancing control over total solids concentration in multi-pass, multi-effect falling film evaporators.
- Kaya and Sarac (2007) created a model for multi-effect evaporators with concurrent, countercurrent, and parallel flow options, examining the impact of pre-heating on energy economy.
- Karimi et al. (2007) applied inferential cascade control to a three-effect falling-film evaporator, improving control over total solid concentration.

More recent studies, such as those by Chen and Ruan (2016), Ganjeizadeh et al. (2020), and Castro Rubens et al. (2022), have continued to advance the development of mathematical models and algorithms for multi-effect evaporators.

Recent research has led to the development of more advanced models for multiple-effect evaporators (MEEs). Notably:

- Chen and Ruan (2016) created a rigorous mathematical model that integrated thermal vapor compression, vapor bleeding, and condensate flashing.

- Ganjeizadeh et al. (2020) simplified complex mass and energy balance equations into a matrix form, using MATLAB to solve them. Their study examined how increasing the number of effects impacts steam consumption and process efficiency.

- Castro Rubens et al. (2022) simulated MEEs, highlighting the challenges of integrating batch and continuous operations. Their model, based on fundamental principles, proved useful in the sugarcane industry. The study revealed that steam flow rate changes affect steam pressure, and that using relief and bypass valves to maintain constant pressure increases overall steam consumption.

2.3 Paper Industry

Researchers have extensively studied the modeling and simulation of evaporation plants in the paper industry. Studies by Ray and Singh (2000), Algehed and Berntsson (2003), and others have focused on designing and optimizing multiple-effect evaporators for improved steam economy and reduced energy consumption.

Various mathematical models have been developed to simulate and analyze different feed flow sequences, operating strategies, and energy reduction schemes. These models have been used to investigate the effects of input parameters on output variables, such as steam consumption and steam economy.

Some notable studies include:

- Ray and Singh (2000): Designed a sextuple-effect black liquor evaporator system and developed a system of nonlinear equations to model the process.
- Algehed and Berntsson (2003): Proposed a novel design for evaporation plants, reducing live steam demand by approximately 70%.
- Bhargava et al. (2008): Developed a nonlinear model for a sextuple-effect flat falling film evaporator system, incorporating different operating strategies.
- Khanam and Mohanty (2010, 2011): Proposed energy reduction schemes, including condensate-, feed-, and product-flashing, and vapor bleeding.
- Verma et al. (2017): Used the Interior-Point Method to solve a nonlinear optimal problem, maximizing steam efficiency in a multi-stage evaporator system.

These studies demonstrate the importance of mathematical modeling and simulation in optimizing evaporation plants and improving energy efficiency in the paper industry.

3. USES OF DIFFERENT TECHNOLOGIES IN EVAPORATION

Recent studies have explored the application of various technologies in evaporation processes. For instance:

- Higa et al. (2009) developed equations for thermal integration projects, including multiple-effect evaporators (MEEs), which can be used as references for sugar plant optimization.

- Lee et al. (2021) created a mathematical model for simulating droplet evaporation, incorporating the Cahn-Hilliard equation and interfacial evaporation mass flux. Their numerical solution and computational experiments investigated the effects of evaporation rate and contact angle on droplet behavior.
- Syrodoy et al. (2023) used mathematical modeling to study the evaporation of a single water droplet from a copper substrate. Their results showed that convective currents within the droplet had a negligible impact on heat and mass transfer processes. The model was validated using experimental data.

4. MODELING AND SIMULATION OF MULTI-EFFECT EVAPORATION: UNSTEADY STATE CONDITION

Although design and steady-state operation methods for single and multi-effect evaporators are well-documented, the dynamic behavior of these systems has been studied by only a few researchers, particularly in industries like sugar, food, desalination, and paper.

Initial interest in mathematical modeling focused on open-loop simulations for conventional control purposes. However, by the 1970s, researchers shifted their focus to developing state-space models for multivariable control and state estimation.

Studies by Johnson (1960), Andersen et al. (1961), Kropholler and Spikins (1965), and Andre and Ritter (1968) contributed to the development of empirical and theoretical models for evaporators. These models were used to simulate control systems, analyze dynamic responses, and predict behavior under various operating conditions.

Notably, Bolmstedt and Jernquist (1976, 1977) developed computer programs to simulate steady-state and dynamic behavior of multiple-effect evaporators. The DYNEFF program, developed by Bolmstedt (1977), used dynamic blocks to simulate complex plants and was combined with the INDUNS program for design and evaluation calculations.

4.1 Desalination/distillation industry

Researchers have developed various models to simulate and analyze the behavior of desalination and distillation plants. Key studies include:

- El-Nashar and Qamhiyeh (1990): Developed a simulation model to predict the transient behavior of multi-effect stack-type distillation plants, validated with actual plant data.
- Hanbury (1995): Presented a steady-state solution to the performance equations of a multi-effect distillation plant, successfully describing the dynamic behavior of MED systems.
- Aly and Marwan (1997): Developed a dynamic model for the multi-effect evaporation process, studying transient behavior during start-up, shutdown, load changes, and troubleshooting.

- Svandova et al. (2006): Conducted steady-state analysis and dynamic simulation of a methyl tertiary-butyl ether reactive distillation process, studying the effects of input parameters on system behavior.

These models have improved our understanding of desalination and distillation processes, enabling better design, operation, and control of these systems.

4.2 Various Process Industries

Researchers have developed various models for simulating and analyzing multiple-effect evaporators (MEEs) in different process industries. Key studies include:

- Tonelli et al. (1990): Created a computer package for simulating MEEs in liquid food concentration, demonstrating excellent agreement between simulated and experimental results.
- Cadet et al. (1999): Proposed a nonlinear model for control strategy development, emphasizing the importance of dynamic accuracy.
- Winchester and Marsh (1999): Analyzed a model of a falling film evaporator with mechanical vapor recompression to examine control loop coupling and controllability.
- Miranda and Simpson (2005): Developed a phenomenological model for MEEs, incorporating empirical knowledge and validating it using industrial plant data.
- Bojnourd et al. (2015): Compared distributed and lumped dynamic models for an industrial four-effect falling-film evaporator, finding that the distributed model provided slightly better predictions.
- Luyben (2018): Demonstrated how dynamic simulation can be developed using standard Aspen simulation models, highlighting design issues and heat-transfer constraints.

4.3 Paper Industry

Only a few studies have investigated the dynamic response of multiple-effect evaporator (MEE) systems in the paper industry. Key research includes:

- Stefanov and Hoo (2003, 2004): Developed distributed parameter models for black liquor falling film evaporators, describing evaporation, heating, and condensation phenomena. They studied dynamic responses for backward feed sequences.
- Kumar et al. (2013): Created a lumped parameter model for a sextuple-effect falling-film MEE system, incorporating unsteady-state energy and material balance equations. They analyzed transient behavior by disturbing input parameters.
- Ben-Ali, Samia (2018): Modeled a double-effect evaporator system (DEE) using the bond graph approach, describing heat transfer and other physical phenomena. A dynamic model was developed, and simulation data were compared with experimental results.

These studies contribute to understanding the dynamic behavior of MEE systems in the paper industry.

5. RESULTS AND DISCUSSION

Analysis of existing multiple-effect evaporator (MEE) models reveals that:

- Most models rely on material and energy balance equations.
- Steady-state simulations are common, using various techniques.
- Some studies examine dynamic behavior under input parameter disturbances.
- Models differ mainly in incorporated heuristic knowledge and simulation techniques.
- A significant research gap exists in dynamic modeling under unsteady-state conditions.

6. CONCLUSION

Mathematical modeling and simulation are crucial in process industries, particularly under steady-state conditions. Researchers have used steady-state solutions to study the impact of various parameters on steam economy and optimize feeding sequences. However, dynamic modeling is essential for understanding process behavior under transient conditions, which is vital for designing effective control systems.

The process industry needs advanced control technologies that are easy to use and affordable. Traditional control methods are often ineffective, leading to productivity, quality, and competitiveness losses. Dynamic modeling can help develop better control systems by providing insight into process behavior and variable interactions.

Despite its importance, few researchers have explored dynamic modeling of multiple-effect evaporator (MEE) systems. Therefore, there is significant scope for further research on dynamic modeling of MEE systems to improve control and understanding of their complex behavior.

7. FUTURE RESEARCH DIRECTIONS BASED ON THE COMPREHENSIVE STUDY

- Optimize evaporator design with various feed sequences and economy measures to minimize energy consumption.
- Develop a cost optimization function that includes pumping and labor costs, in addition to steam economy.
- Investigate the use of unequal heating surface areas in evaporator design for improved concentration of liquor in multiple-effect evaporators (MEEs).
- Conduct detailed studies on modeling physico-chemical properties of black liquor using experimental data from Indian pulp mills.
- Explore dynamic modeling using both distributed and lumped parameter approaches, and compare the results.

- Utilize mathematical numerical methods and software packages for simulating dynamic responses.
- Analyze the economic impact of optimized operating times using unsteady-state simulation.

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