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Gaps in Successive Palindromic Primes till 1 Trillion

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Abstract:

Successive palindromic primes are analyzed here by considering gaps between them. The range-wise distinct gap values, maximum gap, its occurrence frequency, the minimum gap, its occurrence frequency, gap occurring more and lesser times are analyzed for all palindromic primes in the range of 1 to 1 trillion.

Keywords: Palindromic prime numbers, successive palindromic primes, gaps**.**

Introduction:

A numeral palindrome or a palindromic number is a number that remains the same after reversing its digits.

All single digits numbers.

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9$$

are naturally palindromic numbers. All discussion in this work is with respect to common base 10.
As far as double digit numbers are concerned, all repeated digit numbers

$$11, 22, 33, 44, 55, 66, 77, 88, 99$$

and only these are palindromic numbers.
Three digit palindromic numbers are

$$101, 111, 121, 131, 141, 151, 161, 171, 181, 191,$$
$$202, 212, 222, 232, 242, 252, 262, 272, 282, 292,$$
$$909, 919, 929, 939, 949, 959, 969, 979, 989, 999.$$

With increasing digit count, their count keeps on increasing.
Interesting properties regarding palindromic numbers are found in [1].
Prime numbers are domain of vast study in number theory. They have been exhaustively analyzed both computationally within large ranges [2]-[15] and theoretically asymptotically [16].
We combine both concepts here and consider palindromic primes, first few of which are

$$2, 3, 5, 7, 11, 101, 131, 151, 181, 191, 313, 353, 373, 383, 727, 757, 787, 797, \cdots$$

Its an open problem to prove their infinitude but they are conjectured to be infinite. One of the largest known palindromic prime

$10^{320236} + 10^{160118} + \dfrac{\left(137\times10^{160119} + 731\times10^{159275}\right) \times \left(10^{843} - 1\right)}{999} + 1$ has as many as 3,20,237 decimal digits in it!

The palindromic primes are comparatively quite rare. It has been proved that the ratio of composite palindromic numbers to prime palindromic numbers tends to 1 [17].

1. Distinct Gaps in Palindromic Primes in Ranges of 1 – 10^n

Successive primes have been analyzed in detail for gaps between them [18]. On same lines, here we consider palindromic primes. As many as 633 different gaps are there amongst successive palindromic primes in the range of 1 to 1 trillion.

Table 1: Distinct Gaps between Successive Palindromic Primes in Increasing Ranges

Sr. No.	Range of Numbers	Number of Distinct Gaps
1.	1 – 10	3
2.	1 – 100	4
3.	1 – 1,000	11
4.	1 – 10,000	11
5.	1 – 100,000	33
6.	1 – 1,000,000	33
7.	1 – 10,000,000	98
8.	1 – 100,000,000	98
9.	1 – 1,000,000,000	253
10.	1 – 10,000,000,000	253
11.	1 – 100,000,000,000	633
12.	1 – 1,000,000,000,000	633

The number of distinct gaps that appear in successive primes keeps increasing with halts with increasing 10 power ranges.

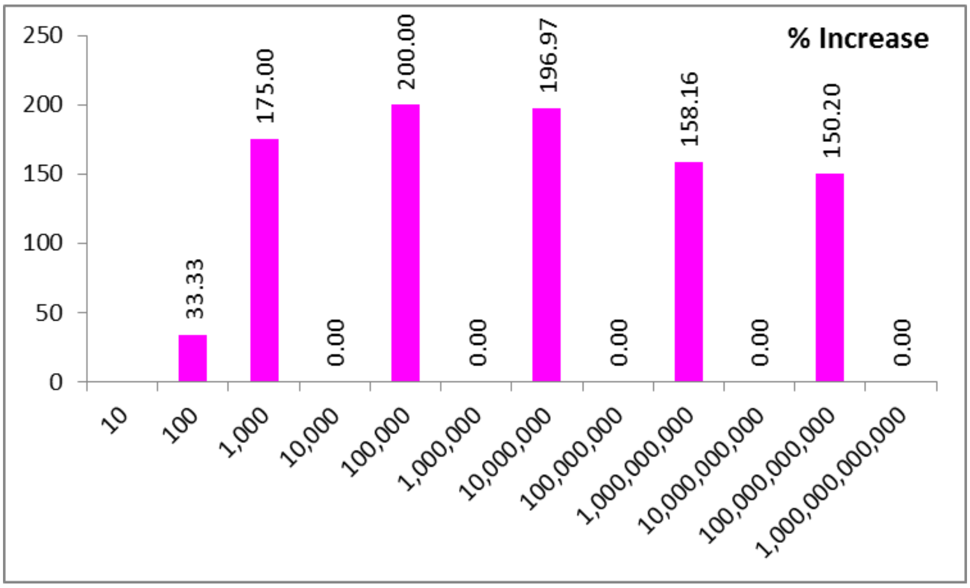

Figure 1: Percentage Increase in Number of Distinct Gaps between Successive Palindromic Primes in Increasing Ranges

Actually, except for 1,000, these increases can be taken to be half only as they remain stable and increase alternatively.

In these calculations, the first palindromic prime of the successive pair is considered even if the next one falls in higher range.

2. Early Appearance of Large Distinct Gaps than Small Ones

Here also we consider distinct gap values between pairs of successive palindromic primes in range of 1 – 10^{12}.

Out of 633 distinct gaps 610 gaps have appeared before first appearance of their smaller values till 1 trillion.

The first gap to appear before smaller values is 90, which came before 2 gaps of 10, 20, 30 and 40 smaller than it. Gap 90 comes first between primes 11 & 101; while smaller gaps of 10, 20, 30 and 40 come first between successive palindromic prime pairs 181 & 191, 131 & 151, 101 & 131 and 313 & 353, respectively.

The list of the gap values that have preceded the smaller gaps is given ahead but is sorted out magnitude-wise.

20	30	90	122	200	210	300	310
344	400	410	500	510	600	610	700
710	810	910	1,000	1,010	1,100	1,110	2,000
2,100	2,120	3,000	3,100	3,110	4,000	4,100	4,110
5,000	5,100	5,110	6,000	6,100	6,110	7,000	7,100
7,110	8,000	8,100	8,110	9,000	9,100	9,110	9,372
10,100	10,110	10,112	10,712	11,100	11,110	12,100	12,110
12,200	12,210	13,100	13,110	13,200	13,210	14,100	14,200
14,210	15,100	15,200	15,210	16,100	16,110	16,200	17,100
17,200	18,200	18,210	19,200	20,000	20,200	20,210	21,000
21,200	22,200	24,200	24,300	28,310	30,000	30,914	31,000
31,100	38,400	40,000	40,110	41,000	41,100	50,000	51,000
51,100	60,000	60,110	61,000	61,100	70,000	70,110	71,000
71,100	80,000	80,110	81,000	81,100	90,000	91,000	91,100
101,000	101,100	102,100	111,000	111,100	112,000	112,100	121,000
121,100	121,110	122,000	122,100	130,110	131,000	131,100	131,110
132,000	132,100	141,000	141,100	142,000	142,100	151,000	151,100
151,110	152,000	152,100	161,000	161,100	162,000	162,100	171,000
171,100	172,000	172,100	181,000	181,100	181,110	182,000	182,100
191,000	191,110	192,000	192,100	200,000	201,110	202,000	202,100
210,000	211,000	212,000	212,100	213,000	213,100	221,110	222,000
223,000	223,100	232,000	233,000	233,100	241,110	242,000	242,100
242,110	243,000	243,100	251,110	252,000	252,110	253,000	253,100
262,000	262,100	262,110	263,000	263,100	273,000	273,100	282,000
282,100	283,000	283,100	292,000	293,000	293,100	300,000	303,000
303,100	310,000	311,000	313,000	313,100	314,100	323,000	323,100
324,000	333,000	333,100	334,000	334,100	343,100	344,000	344,100
353,000	354,100	363,000	364,000	364,100	374,000	374,100	384,000
394,000	400,000	401,100	404,000	410,000	411,000	414,000	414,100
435,100	485,000	500,000	501,100	510,000	511,000	525,000	546,100
554,110	556,100	566,100	600,000	601,100	610,000	611,000	626,100
636,100	700,000	700,110	701,100	710,000	711,000	800,000	800,110
801,100	810,000	811,000	900,000	900,110	901,100	904,312	910,000
911,000	1,000,110	1,001,100	1,002,012	1,005,012	1,010,000	1,011,000	1,011,100
1,021,000	1,101,100	1,110,000	1,111,000	1,111,100	1,120,000	1,121,000	1,200,110
1,201,10	1,210,000	1,211,000	1,211,100	1,220,00	1,221,000	1,300,110	1,301,100

0				0			
1,310,000	1,311,000	1,311,100	1,320,000	1,321,000	1,401,100	1,410,000	1,410,110
1,411,000	1,411,100	1,420,000	1,421,000	1,501,100	1,510,000	1,510,110	1,511,000
1,511,100	1,520,000	1,521,000	1,610,000	1,611,000	1,611,100	1,620,000	1,621,000
1,710,000	1,711,000	1,711,100	1,720,000	1,721,000	1,810,000	1,810,110	1,811,000
1,811,100	1,820,000	1,821,000	1,910,000	1,911,100	1,920,000	1,921,000	2,011,100
2,020,000	2,021,000	2,021,100	2,031,000	2,111,100	2,120,000	2,121,000	2,121,100
2,130,000	2,131,000	2,210,110	2,211,100	2,220,000	2,221,000	2,221,100	2,230,000
2,231,000	2,311,100	2,320,000	2,320,110	2,321,000	2,321,100	2,330,000	2,331,000
2,411,100	2,420,000	2,421,000	2,421,100	2,430,000	2,431,000	2,511,100	2,520,000
2,521,000	2,521,100	2,530,000	2,531,000	2,611,100	2,620,000	2,621,000	2,621,100
2,630,000	2,631,000	2,710,110	2,711,100	2,720,000	2,721,000	2,721,100	2,730,000
2,731,000	2,820,000	2,821,000	2,821,100	2,830,000	2,831,000	2,920,000	2,921,100
2,930,000	2,931,000	3,015,114	3,021,100	3,030,000	3,031,000	3,031,100	3,041,000
3,121,100	3,130,000	3,131,000	3,131,100	3,140,000	3,141,000	3,221,100	3,230,000
3,231,000	3,240,000	3,241,000	3,321,100	3,330,000	3,331,000	3,340,000	3,341,000
3,420,110	3,421,100	3,430,000	3,431,000	3,431,100	3,440,000	3,441,000	3,521,100
3,530,000	3,531,000	3,540,000	3,541,000	3,621,100	3,630,000	3,631,000	3,631,100
3,640,000	3,641,000	3,730,000	3,731,000	3,731,100	3,740,000	3,741,000	3,830,000
3,831,000	3,831,100	3,840,000	3,841,000	3,930,000	3,931,100	3,940,000	3,941,000
4,030,110	4,031,100	4,040,000	4,041,000	4,041,100	4,051,000	4,140,000	4,141,000
4,150,000	4,151,000	4,240,000	4,241,000	4,250,000	4,251,000	4,340,000	4,341,000
4,350,000	4,351,000	4,440,000	4,441,000	4,441,100	4,450,000	4,451,000	4,530,110

4,540,000	4,541,000	4,541,100	4,550,000	4,551,000	4,631,100	4,640,000	4,641,000
4,650,000	4,651,000	4,740,000	4,750,000	4,751,000	4,840,000	4,840,110	4,841,000
4,841,100	4,850,000	4,851,000	4,950,000	4,951,000	5,050,000	5,051,000	5,061,000
5,141,100	5,150,000	5,151,000	5,241,100	5,250,000	5,251,000	5,261,000	5,350,000
5,350,110	5,351,000	5,361,000	5,440,110	5,450,000	5,451,000	5,460,000	5,461,000
5,550,000	5,551,000	5,551,100	5,560,000	5,561,000	5,650,000	5,651,000	5,651,100
5,661,000	5,750,000	5,751,100	5,760,000	5,761,000	5,850,000	5,851,000	5,861,000
5,960,000	5,961,000	6,060,000	6,061,000	6,160,000	6,171,000	6,260,000	6,261,000
6,270,000	6,471,000	6,561,000	6,570,000	6,571,000	6,660,000	6,771,000	6,860,110
6,971,000	7,061,100	7,181,000	7,281,000	7,371,000	7,470,000	7,481,000	7,581,000
7,770,000	7,881,000	8,391,000	8,481,000	8,490,000	8,491,000	8,591,000	8,681,100
8,991,000	9,801,000	10,091,100	10,101,000	10,512,000	10,611,000	11,721,000	11,811,000
12,021,000	12,132,000	12,222,000	12,321,000	13,131,000	13,242,000	13,341,000	13,431,000
13,542,000	15,252,000	15,462,000	15,861,000	16,062,000	16,151,100	16,161,000	16,572,000
16,662,000	16,971,000	18,192,000	18,272,100	18,282,000	18,381,000	18,482,100	19,802,000
19,892,100	19,902,000	20,913,000	21,822,000	22,323,000	24,642,000	26,973,000	28,083,000
28,182,000	28,583,100	29,292,000	30,803,100	32,523,000	36,063,000	37,064,100	58,676,100
59,696,100	72,327,000	90,040,102	100,020,012	100,050,012	300,141,014	9,000,772,002	10,001,610,012
10,009,690,012	30,007,060,014						

As many as 150 different number of precedences are found with different frequencies, for which we give graphical representation.

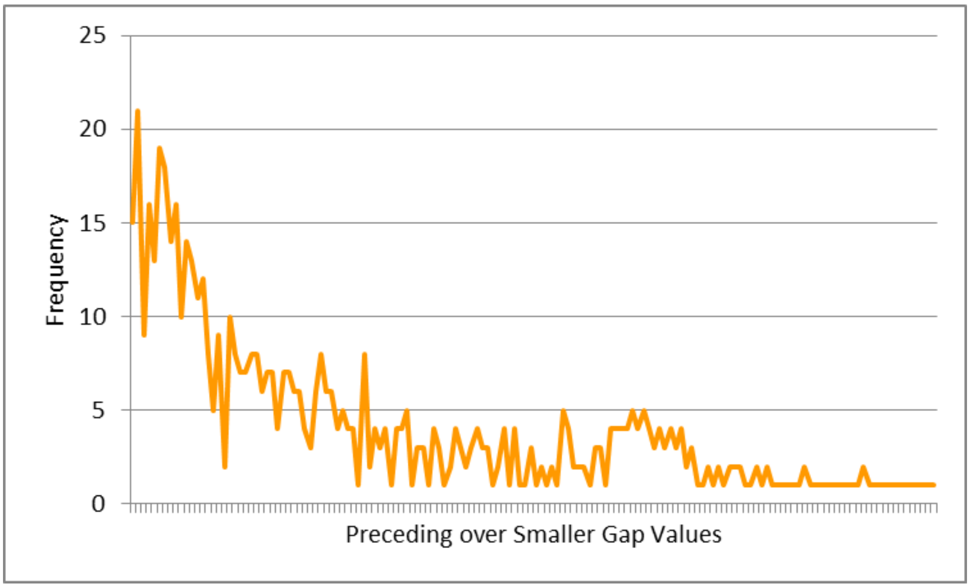

Figure 2: Frequency of Larger Gaps between Successive Palindromic Primes Preceding over Smaller Gaps

3. Minimum & Maximum Gaps between Successive Palindromic Primes

4.1 Minimum Gap between Successive Palindromic Primes

Out of these gaps, gap 1 is really unique universally. No other palindromic prime pair except the first one of 2 and 3 has gap of 1 and it is solo case. The gap of 1 cannot occur again between successive palindromic primes or any successive primes as gap of 1 means successive integers, out of successive integer, one is even and other is odd and even integer other than 2 cannot be prime.

4.2 Maximum Gap between Successive Palindromic Primes

The maximum gap between successive palindromic primes till one trillion is above 30 billion Few higher gaps are

30,007,060,014
10,009,690,012
10,001,610,012
9,000,772,002
300,141,014
100,050,012
100,020,012

These are exceptionally high gap values owing to simple fact that in number range 40,000,000,000 to 69,999,999,999 there is no palindromic prime as the leading digit gives of composite number in digit reversal. Same explanation is for the next higher gaps due to non-availability of palindromic primes between 20,000,000,000 & 29,999,999,999 and 80,000,000,000 & 89,999,999,999. Similar reasons are for lesser digit peaks in gaps. There is no 12 digit palindromic prime, in fact, except 11, there cannot be palindromic primes with any even-number of digits.

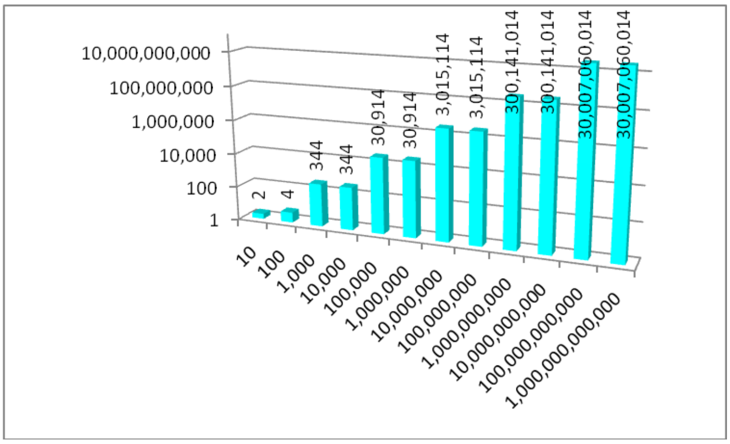

Figure 3: Maximum Gaps between Successive Palindromic Primes in Increasing Ranges

The peaks in all ranges are actually exceptional and are due to non-availability of palindromic primes beginning with digits 4, 5, 6 in particular. If we omit these, still there are some higher peaks due to non-availability of palindromic primes beginning with digits 2 and 8.

4. Gap Counts between Successive Palindromic Primes

Amongst as many as 633 distinct gaps between successive palindromic prime pairs till one trillion, 189 gaps appear only once. They are not unique universally, but within our range they come only once.
As except first case, there cannot be odd gaps between successive primes; there cannot be odd gaps between successive palindromic primes as well. So only even gaps appear between them.
In the graph below, on the horizontal axis, only those gap values are taken in increasing order which do occur and others are omitted.

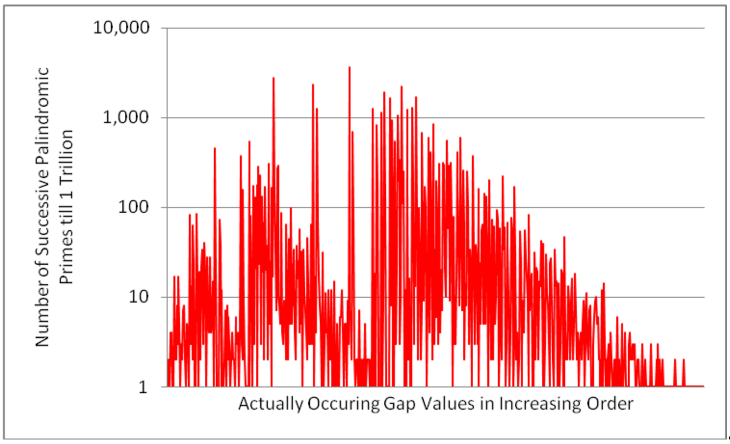

Figure 4: Number of Palindromic Prime Pairs till 10^{12} with Different Gaps in between Them.

The maximum times occurring gap is 300000. It occurs 3646 times till one trillion.
Frequencies of distinct gaps in different ranges are also determined.

Table 3: Number of Distinct Gaps between Successive Palindromic Primes in Different Frequency Ranges till 10^{12}.

Sr. No.	Frequency Range	Number of Distinct Gaps Values
1.	1 – 10	230
2.	1 – 100	444
3.	1 – 1,000	633

5. First Palindromic Primes having Different Gaps with Successive Palindromic Primes

Except gap values not appearing till 1 trillion, the first palindromic prime in the first palindromic prime pair with different gaps have following pattern.

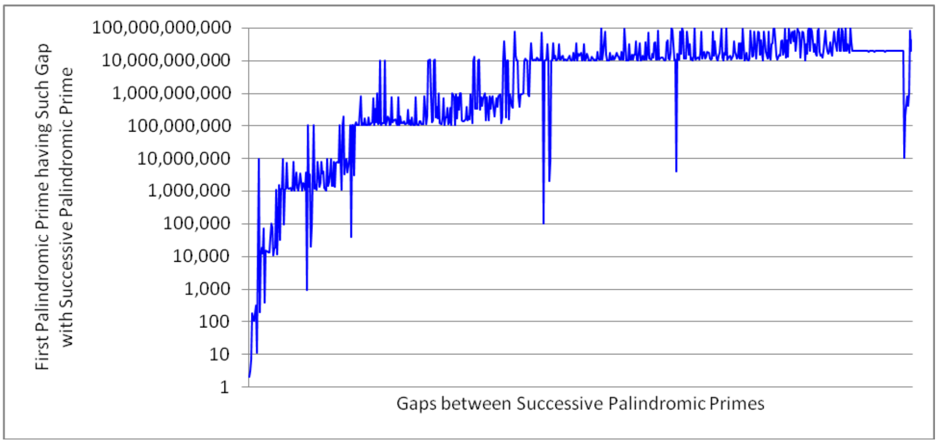

Figure 5: First Palindromic Primes till 10^{12} with Different Gaps with their Successor Palindromic Primes.

6. Last Palindromic Primes with Different Gaps with Successive Palindromic Primes

Similarly exempting gap values which don't come till 1 trillion, the first palindromic prime in the last palindromic prime pair with different gaps have following pattern.

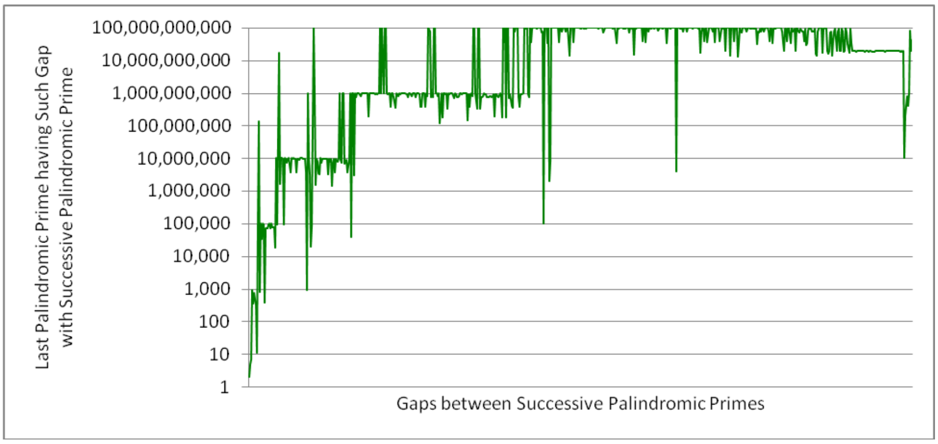

Figure 6: Last Palindromic Prime with Different Gaps with their Successor Palindromic Primes.

This work shows trends in distribution of palindromic primes with respect to gaps between.

Acknowledgements:

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