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Analysis of Successive Occurrence of Digit 3 in Prime Numbers till 1 Trillion

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Abstract:
All primes less than 1 trillion are analyzed for successive occurrence of 3 in their digits. Multiple successive occurrences of 3's are also searched. Within all first 12 ranges of increasing powers of 10, the first and last prime numbers with all possible multiple successive digit 3's are also determined.Comparison with number of prime numbers having multiple successive 1's and 2's in their digits with those many 3's in them is presented. The smallest and the largest primes with multiple successive 3's in their digits are determined.

Keywords: Prime numbers, digit 3, successive occurrences**.**

Mathematics Subject Classification 2010: 11Y35, 11Y60, 11Y99**.**

1. Introduction:

The concept of numbers is very basic in mathematics and known from ancient times [2]. Amongst them, prime numbers have special status.
Inherent lack of regularity in distribution of primes among the natural numbers forces mathematicians to study them both ways asymptotically [1] as well as in huge ranges exhaustively [4].
The trends of general, successive and non-successive occurrences of 0 [5], [6], [7] and any non-zero digit like 3 [11], [12], [13] in all natural numbers are already analyzed. The occurrence of digit 3 in prime numbers is also analyzed [20]. Similar analysis of all types of occurrences of digit 0, 1 and 2 in primes are available [8], [9], [10], [14]-[19].
Here successive 3's in digits of primes are considered in detail.

2. Occurrence of Single Successive Digit 3 in Prime Numbers

Single occurrence of any digit in numbers is by default considered successive. So, values determined in [20] for occurrences of single 3 in prime numbers are just occurrences of single successive 3 in them!

Sr. No.	Range	Number of Primes with Single Successive(!) 3
1.	$1 - 10^1$	1
2.	$1 - 10^2$	9
3.	$1 - 10^3$	57
4.	$1 - 10^4$	457
5.	$1 - 10^5$	3,693
6.	$1 - 10^6$	30,928
7.	$1 - 10^7$	264,820
8.	$1 - 10^8$	2,296,417
9.	$1 - 10^9$	20,065,110
10.	$1 - 10^{10}$	176,290,694
11.	$1 - 10^{11}$	1,555,436,420
12.	$1 - 10^{12}$	13,767,790,131

Table 1[20] : Number of Prime Numbers in Various Ranges with Single 3 in Their Digits

A host of computers were executing a special programme for long time to make these determinations

possible with implementation of choosy algorithms [3].

3. Occurrence of Multiple Successive Digit 3 in Prime Numbers

Successive property has real meaning for two or more instances. The count of all positive integers containing double, triple and higher number of successive 3's in them within the ranges of $1 - 10^n$, $1 \leq n \leq 12$ can be inferred from [12] by generalizing the cases of non-zero digit 1. In this work, the number of primes in identical ranges containing multiple number of successive digit 3's is determined.

Table 2 : Number of Prime Numbers in Various Ranges with Multiple Successive 3's in Their Digits

Sr. No.	Number Range<	Number of Primes with 2 Successive 3's	Number of Primes with 3 Successive 3's	Number of Primes with 4 Successive 3's
1.	10^3	5	0	0
2.	10^4	43	4	0
3.	10^5	345	42	2
4.	10^6	2,951	314	21
5.	10^7	26,224	2,550	240
6.	10^8	229,749	22,624	2,198
7.	10^9	2,026,784	202,098	20,018
8.	10^{10}	17,957,407	1,812,972	181,475
9.	10^{11}	159,557,042	16,255,912	1,642,363
10.	10^{12}	1,420,733,864	145,719,295	14,841,537

Table 2 : Continued …

Sr. No.	Number Range<	Number of Primes with 5 Successive 3's	Number of Primes with 6 Successive 3's	Number of Primes with 7 Successive 3's
1.	10^6	3	0	0
2.	10^7	24	1	0
3.	10^8	216	19	2
4.	10^9	1,940	186	21
5.	10^{10}	17,795	1,722	170
6.	10^{11}	163,684	16,192	1,560
7.	10^{12}	1,498,649	149,717	14,635

Table 2 : Continued …

Sr. No.	Number Range<	Number of Primes with 8 Successive 3's	Number of Primes with 9 Successive 3's	Number of Primes with 10 Successive 3's	Number of Primes with 11 Successive 3's
1.	10^{10}	11	0	0	0
2.	10^{11}	149	15	1	0
3.	10^{12}	1,386	134	14	0

The number of prime numbers with multiple successive digits 3's in ranges of 10^n is plotted where vertical axis is on logarithmic scale.

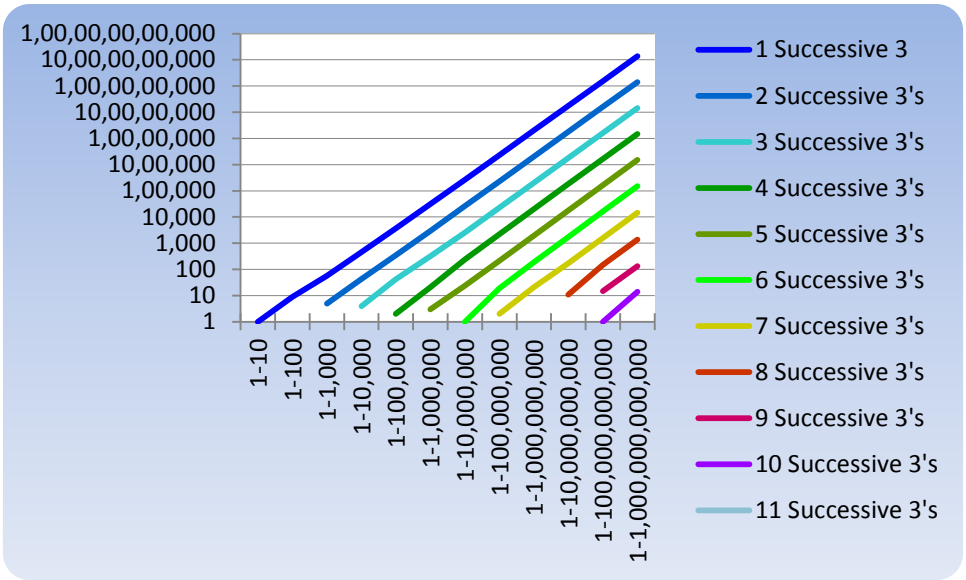


Figure 1: Number of Primes in Different Ranges Containing Multiple Successive 3's in Their Digits

The percentage of number of primes with respect to number of all integers with equal number of successive 3's in corresponding ranges is also plotted.

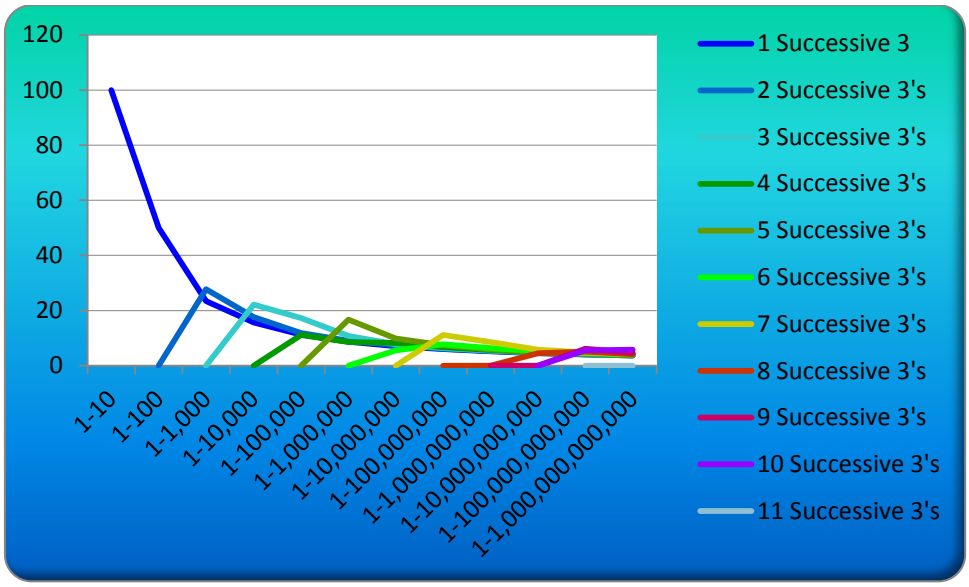


Figure 2: Percentage of Primes in Different Ranges with Multiple Successive 3's in Their Digits with Respect To All Such Integers in Respective Ranges

Now follows the comparisonof differences of number of multiple successive occurrences of digits 1 and 2 in primes with those of 3 in them in our ranges. Digit 0 is dropped in these comparisons as it can't occupy two places, units and leading n^{th}in any n digit prime.

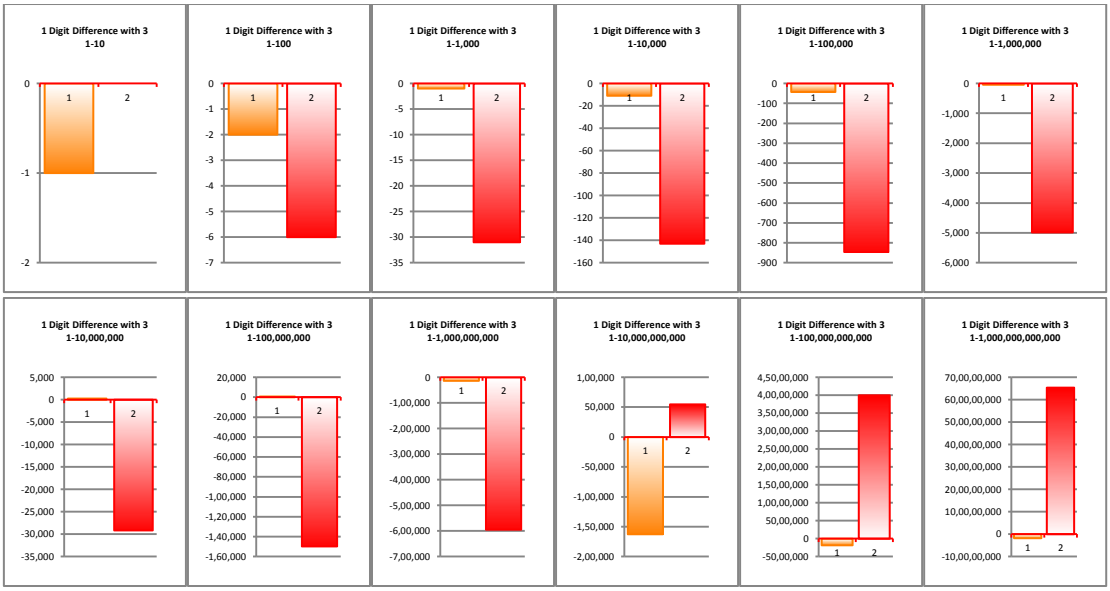


Figure 3: Differences of Number of Primes having Single (Successive) 1 and Single (Successive) 2 in their Digits with those having Single (Successive) 3 in them in Ranges of $1 – 10^n$.

Due to the remark in Section II above, all graphs in Figure 3 resemble in toto with those in [20].

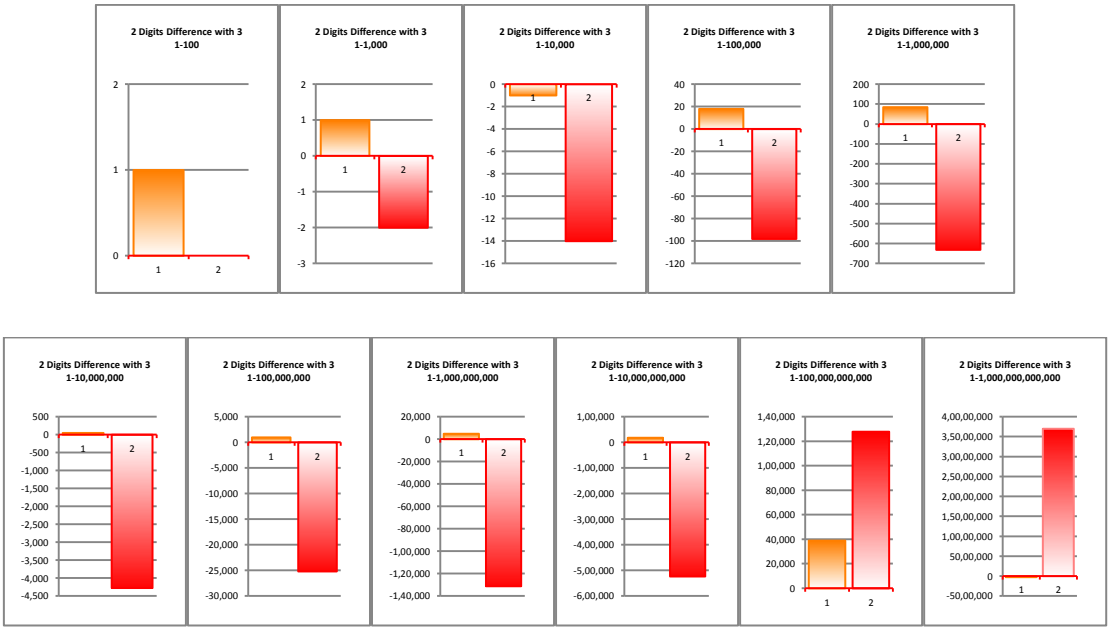
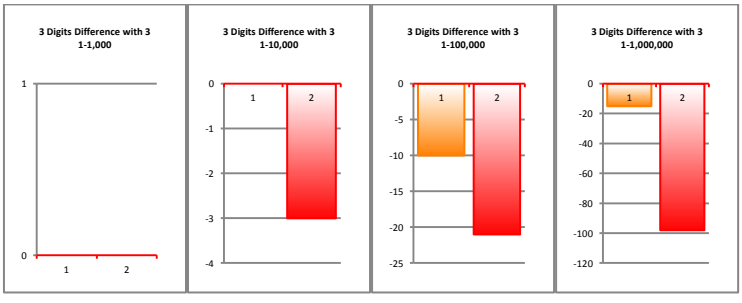


Figure 4: Differences of Number of Primes having Two Successive 1's and Two Successive 2's in their Digits with those having TwoSuccessive 3's in them in Ranges of $1 – 10^n$.



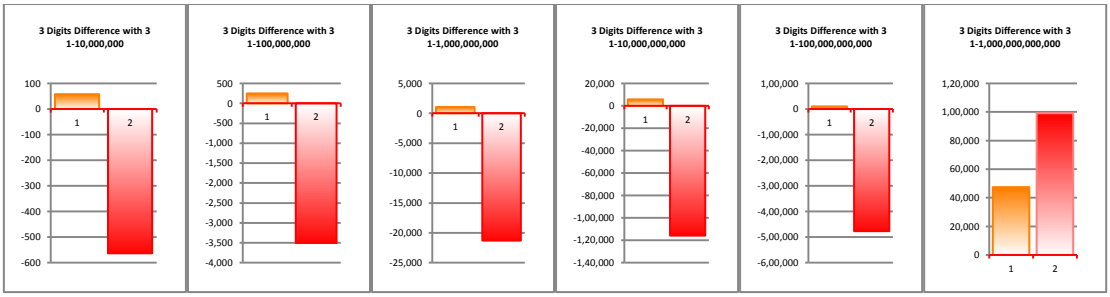


Figure 5: Differences of Number of Primes having Three Successive 1's and Three Successive 2's in their Digits with those having ThreeSuccessive 3's in them in Ranges of $1 - 10^n$.

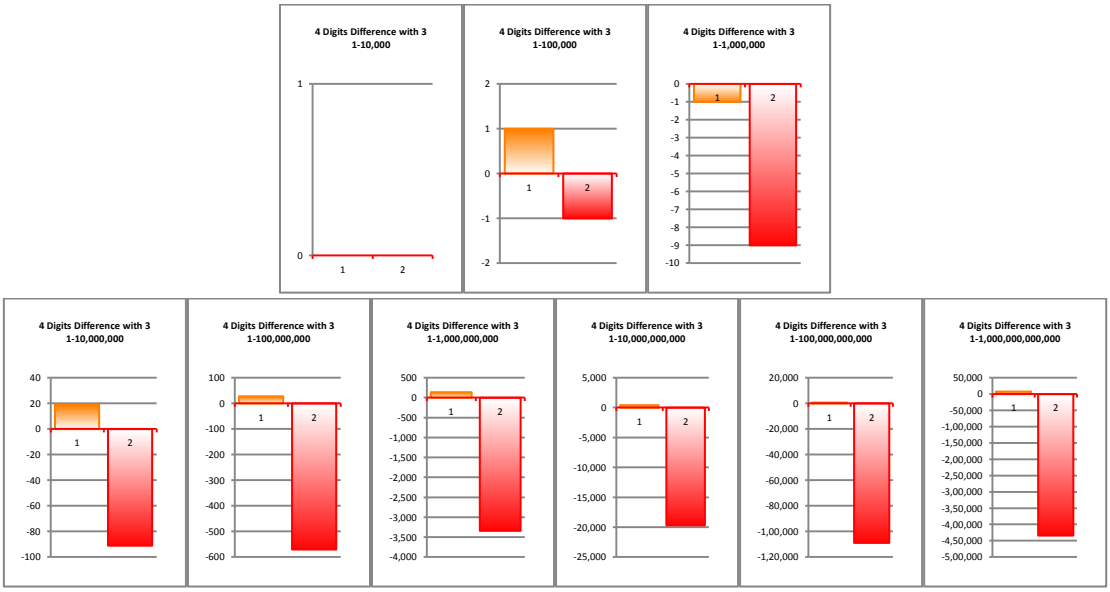


Figure 6: Differences of Number of Primes having Four Successive 1's and Four Successive 2's in their Digits with those having FourSuccessive 3's in them in Ranges of $1 - 10^n$.

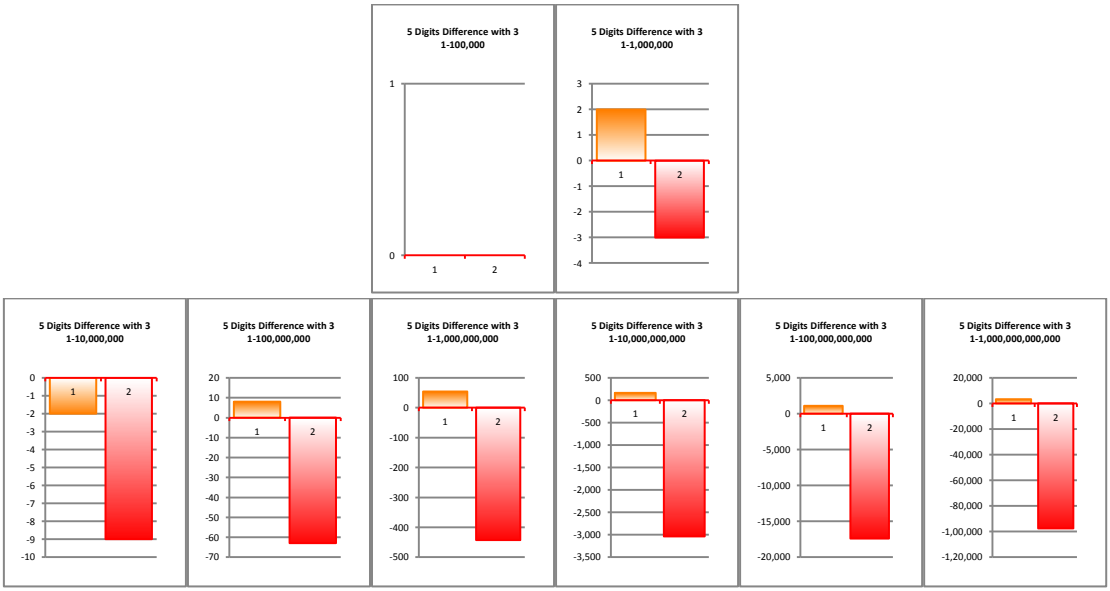


Figure 7: Differences of Number of Primes having Five Successive 1's and Five Successive 2's in their Digits with those having FiveSuccessive 3's in them in Ranges of $1 - 10^n$.

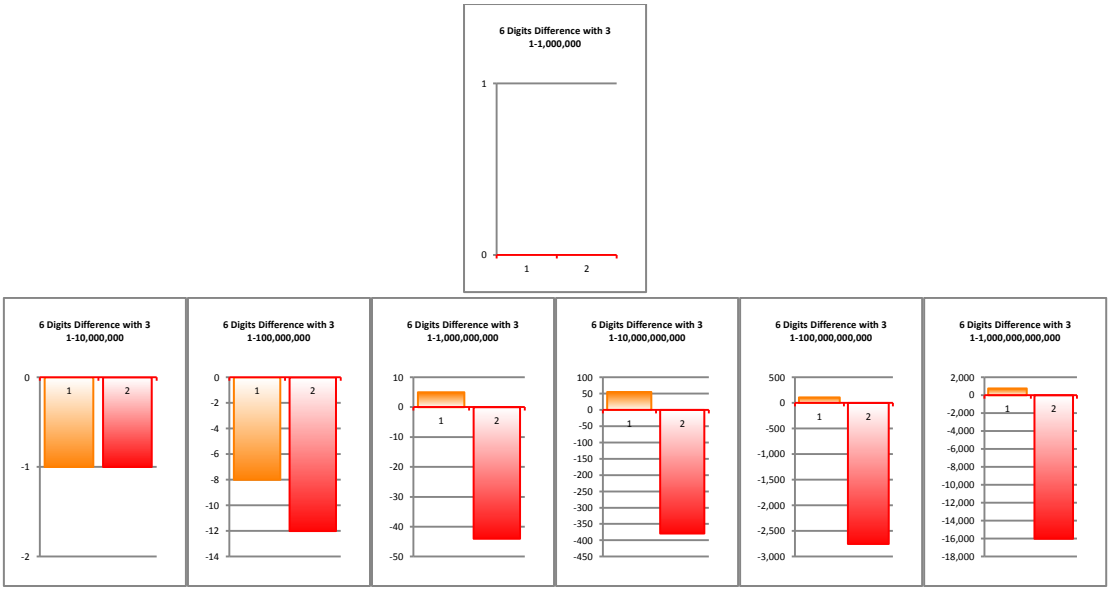


Figure 8: Differences of Number of Primes having Six Successive 1's and Six Successive 2's in their Digits with those having SixSuccessive 3's in them in Ranges of $1 - 10^n$.

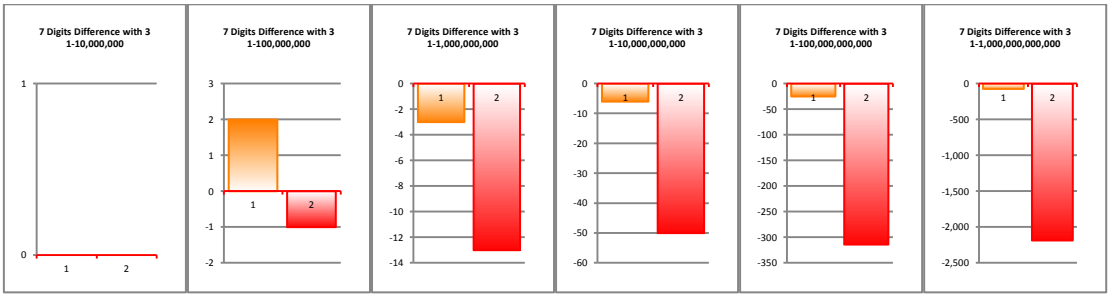


Figure 9: Differences of Number of Primes having Seven Successive 1's and Seven Successive 2's in their Digits with those having SevenSuccessive 3's in them in Ranges of $1 - 10^n$.

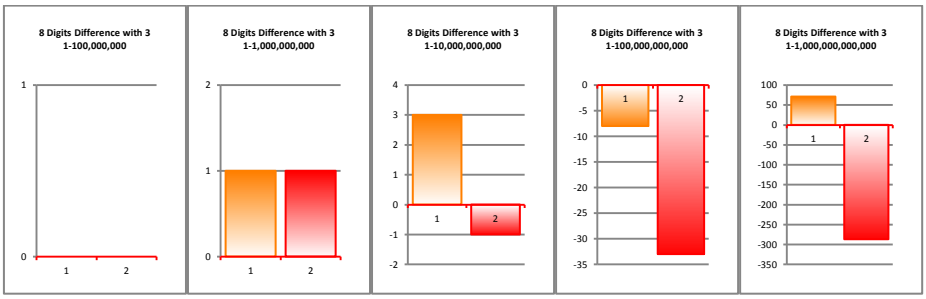


Figure 10: Differences of Number of Primes having Eight Successive 1's and Eight Successive 2's in their Digits with those having EightSuccessive 3's in them in Ranges of $1 - 10^n$.

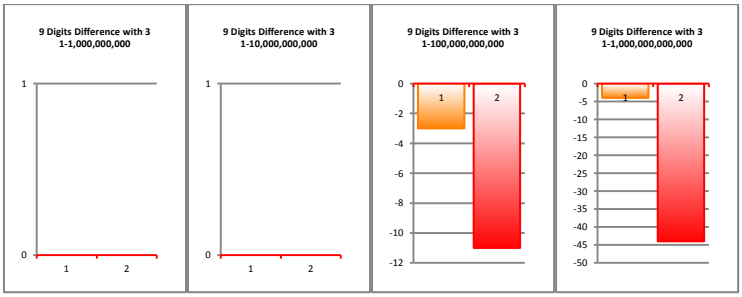


Figure 11: Differences of Number of Primes having Nine Successive 1's and Nine Successive 2's in their Digits with those having NineSuccessive 3's in them in Ranges of $1 - 10^n$.

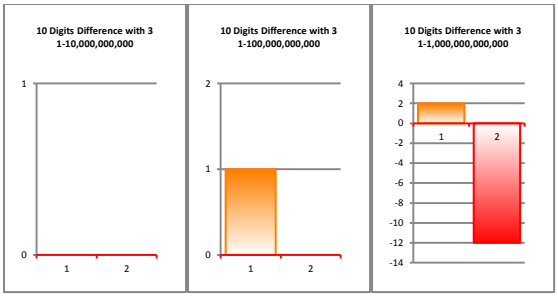


Figure 12: Differences of Number of Primes having Ten Successive 1's and Ten Successive 2's in their Digits with those having TenSuccessive 3's in them in Ranges of $1 - 10^n$.

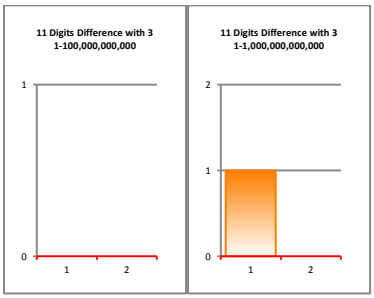


Figure 13: Differences of Number of Primes having Eleven Successive 1's and Eleven Successive 2's in their Digits with those having ElevenSuccessive 3's in them in Ranges of $1 - 10^n$.

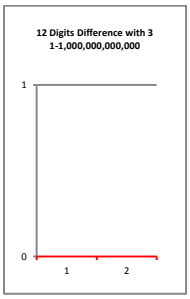


Figure 14: Differences of Number of Primes having Twelve Successive 1's and Twelve Successive 2's in their Digits with those having TwelveSuccessive 3's in them in Ranges of $1 - 10^n$.

4. First Occurrence of Successive Digit 3's in Prime Numbers

The first positive integer containing 3 is 3 itself. For higher ranges, first occurrence of 2 3's is in 33, of 3 is in 333 and so on. The very first occurrence of multiple 3's happens to be just that of successive 3's.

Formula 1 [12] : If n and r are natural numbers, then the first occurrence of r number of successive 3's in numbers in range $1 \leq m < 10^n$ is

$$f = \begin{cases} - & , \text{ if } r > n \\ \sum_{j=0}^{r-1} \left(3 \times 10^j\right), & \text{if } r \leq n \end{cases}.$$

Unfortunately, no formula is available yet for such occurrences in prime numbers and they have been searched by actual computations.

Table 3 : First Prime Numbers in Various Ranges with Multiple Successive 3's in Their Digits

Sr. No.	Range	First Prime Number in Range with					
		1 Successive 3	2 Successive 3's	3 Successive 3's	4 Successive 3's	5 Successive 3's	6 Successive 3's
1.	$1 - 10^1$	3	-	-	-	-	-
2.	$1 - 10^2$	3	-	-	-	-	-
3.	$1 - 10^3$	3	233	-	-	-	-
4.	$1 - 10^4$	3	233	2,333	-	-	-
5.	$1 - 10^5$	3	233	2,333	23,333	-	-
6.	$1 - 10^6$	3	233	2,333	23,333	333,331	-
7.	$1 - 10^7$	3	233	2,333	23,333	333,331	3,333,331
8.	$1 - 10^8$	3	233	2,333	23,333	333,331	3,333,331
9.	$1 - 10^9$	3	233	2,333	23,333	333,331	3,333,331
10.	$1 - 10^{10}$	3	233	2,333	23,333	333,331	3,333,331
11.	$1 - 10^{11}$	3	233	2,333	23,333	333,331	3,333,331
12.	$1 - 10^{12}$	3	233	2,333	23,333	333,331	3,333,331

Table 3 : Continued …

Sr. No.	Range	First Prime Number in Range with				
		7 Successive 3's	8 Successive 3's	9 Successive 3's	10 Successive 3's	11 Successive 3's
1.	$1 - 10^1$	-	-	-	-	-
2.	$1 - 10^2$	-	-	-	-	-
3.	$1 - 10^3$	-	-	-	-	-
4.	$1 - 10^4$	-	-	-	-	-
5.	$1 - 10^5$	-	-	-	-	-
6.	$1 - 10^6$	-	-	-	-	-
7.	$1 - 10^7$	-	-	-	-	-
8.	$1 - 10^8$	33,333,331	-	-	-	-
9.	$1 - 10^9$	33,333,331	-	-	-	-
10.	$1 - 10^{10}$	33,333,331	1,033,333,333	-	-	-
11.	$1 - 10^{11}$	33,333,331	1,033,333,333	13,333,333,339	23,333,333,333	-
12.	$1 - 10^{12}$	33,333,331	1,033,333,333	13,333,333,339	23,333,333,333	-

On many occasions it so happens that first occurrences of multiple 3's are successive.

5. Last Occurrence of Successive Digit 3's in Prime Numbers

There is a formula giving last occurrence of r number of successive non-zero digits in natural numbers in ranges $1 - 10^n$, $1 \leq n \leq 12$, which is applicable to 3.

Formula 2 [12] : If n and r are natural numbers, then the last occurrence of r successive 3's in numbers in range $1 \leq m < 10^n$ is

$$l = \begin{cases} - & , \text{if } r > n \\ \sum_{j=0}^{r-1}\left(3\times 10^{j}\right) + \begin{cases} 0 & , \text{if } r = n \\ \sum_{j=r}^{n-1}\left(9\times 10^{j}\right), & \text{if } r < n \end{cases} \end{cases}.$$

As such simple way is not available for prime numbers, the last prime numbers with r number of successive 3's in them in these ranges needed be worked out.

Table 4 : Last Prime Numbers in Various Ranges with Multiple Successive 3's in Their Digits

Sr. No.	Number of Successive 3's	Last Prime Number in Range 1 –							
		10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
1.	1	3	83	983	9,973	99,923	999,983	9,999,973	99,999,931
2.	2	-	-	733	9,833	99,833	999,433	9,999,533	99,998,933
3.	3	-	-	-	7,333	93,337	997,333	9,998,333	99,993,331
4.	4	-	-	-	-	33,331	973,333	9,943,333	99,983,333
5.	5	-	-	-	-	-	733,333	9,533,333	99,333,337
6.	6	-	-	-	-	-	-	3,333,331	89,333,333
7.	7	-	-	-	-	-	-	-	83,333,333
8.	8	-	-	-	-	-	-	-	-
9.	9	-	-	-	-	-	-	-	-
10.	10	-	-	-	-	-	-	-	-
11.	11	-	-	-	-	-	-	-	-

Table 4 : Continued …

Sr. No.	Number of Successive 3's	Last Prime Number in Range 1 –		
		10^9	10^{10}	10^{11}
1.	1	999,999,937	9,999,999,943	99,999,999,943
2.	2	999,999,733	9,999,999,833	99,999,999,833
3.	3	999,986,333	9,999,993,331	99,999,989,333
4.	4	999,913,333	9,999,933,337	99,999,923,333
5.	5	999,333,337	9,999,533,333	99,996,433,333
6.	6	995,333,333	9,995,333,333	99,994,333,333
7.	7	983,333,333	9,943,333,333	99,833,333,339
8.	8	-	9,433,333,333	99,433,333,333
9.	9	-	-	97,333,333,333
10.	10	-	-	23,333,333,333
11.	11	-	-	-

The remark made for general occurrences of 3's also applies to successive occurrences of 3's.

Remark : The maximum number of successive 3's in any prime number in the range 1 - 10^n, for $n > 1$, is at most $n - 1$, with well-known exception for $n = 1$.

The numbers coming up in all sections of this work give important integer sequences deserving separate analysis.

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The author could compute within and analyze huge ranges by use of Java Programming Language, NetBeans IDE & Microsoft Office Excel and is thankful to their Development Teams.

The computer systems of the Mathematics & Statistics Department of the author's institution have been used extensively on hardware side. Additionally, the uninterrupted power supply facility offered by the Department of Electronics could materialize long executions without disruption. Both have acknowledgement due.

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