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On The Structure Equation $F^{2k+1}+F=0$

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Abstract:
In this paper, we have studied various properties of the *F*- sturcture manifold satisfying $F^{2k+1}+F=0$ where *k* is positive integer. Nijenhuis tensor *F*-structures and kernel have also been discussed.

Keywords: Differnetiable manifold, projection operators, Nijenhuis tensor, metric and kernel.

1. Introduction:

Let M^n be a differentiable manifold of class C^∞ and F be a (1,1) tensor of class C^∞, satisfying

$$(1.1)\quad F^{2K+1}+F=0$$

we define the operators *l* and *m* on M^n by

$$(1.2)\quad l=-F^{2k},\qquad m=I+F^{2k}$$

From (1.1) and (1.2), we have

$$(1.3)\quad l+m=I,\qquad l^2=l,\ m^2=m,\quad lm=ml=0$$

$$lF=Fl=F,\quad Fm=mF=0,$$

where *I* denotes the identity operator.

Theorem (1.1): Let the (1,1) tensors *p* and *q* be defined by

$$(1.4)\quad p=m+F^k,\quad q=m-F^k$$

Then *p* and *q* are invertible operators satisfying

$$(1.5)\quad p^{-1}=q=p^3,\quad q^{-1}=p=q^3,\quad p^2=q^2.\quad p^2-p-q+I=0$$

$$= q^2 - p - q + I,\ pl = -ql = F^k,\ \ pm = qm = p^2 m = q^2 m = m,$$

$$p^2 l = -l = q^2 l$$

Proof: Using (1.2), (1.3) and (1.4), we have

(1.6) $pq = qp = I$, Thus

(1.7) $p^{-1} = q, \quad q^{-1} = p$

Also, using (1.2), (1.3) and (1.4), we get

(1.8) $p^3 = q, \quad q^3 = p$

From (1.7) and (1.8) we have $p^{-1} = q = p^3$. Other results follow similarly.

Theorem (1.2): Let the (1,1) tensors α and β be defined by

(1.9) $\alpha = l + F^k, \quad \beta = l - F^k$, then

(1.10) $\alpha^2 + \beta^2 = 0, \quad \alpha^3 + 2\beta = 0, \ \beta^3 + 2\alpha = 0$

Proof: Using (1.2), (1.3) and (1.9), we get

$\alpha^2 = 2F^k, \quad \beta^2 = -2F^k$ Thus we get $\alpha^2 + \beta^2 = 0$

The other results follow similarly.

Theorem (1.3): Define the (1,1) tensors γ and δ by

(1.11) $\gamma = m + F^{2k}, \ \delta = m - F^{2k}$, then

(1.12) $\gamma^{-1} = \gamma$ and $\delta = I$

Proof: Using (1.2), (1.3) and (1.11), we get

(1.13) $\gamma = m - l, \quad \gamma^2 = I$ thus $\gamma^{-1} = \gamma$ and $\delta = m + l = I$

Theorem (1.4): Define the (1.1) tensors ξ and η by

(1.14) $\xi = m + F, \quad \eta = m - F$, then

(1.15) $\xi^n = m + F^n, \quad \eta^n = m + \left(-1\right)^n F^n$

Proof: Using (1.3) and (1.14), we have

$$\eta^2 = m + F^2, \quad \eta^3 = m - F^3 \ldots\ldots \eta^n = m + (-1)^n F^n$$

The other results follow similarly

2. NIJENHUIS TENSOR:

The Nijenhuis tensors corresponding to the operators *F, l, m* be defined as

$$(2.1) \quad N(X,Y) = [FX,FY] + F^2[X,Y] - F[FX,Y] - F[X,FY]$$

$$(2.2) \quad \underset{l}{N}(X,Y) = [lX,lY] + l^2[X,Y] - l[lX,Y] - l[X,lY]$$

$$(2.3) \quad \underset{m}{N}(X,Y) = [mX,mY] + m^2[X,Y] - m[mX,Y] - m[X,mY]$$

Theorem (2.1): Let *F, l, m* satisfy (1.1) and (1.2), then

$$(2.4) \quad \text{(i)} \quad N(mX,mY) = F^2[mX,mY]$$

$$\text{(ii)} \quad mN(mX,mY) = 0$$

$$\text{(iii)} \quad \underset{l}{N}(mX,mY) = l[mX,mY]$$

$$\text{(iv)} \quad \underset{m}{N}(lX,lY) = m[lX,lY]$$

$$\text{(v)} \quad \underset{l}{N}(lX,mY) = 0$$

$$\text{(vi)} \quad \underset{m}{N}(mX,lY) = 0$$

Proof: With proper replacements of *X* and *Y* in (2.1), (2.2) and (2.3), and using (1.3) we get the reuslts.

3. METRIC F-STRUCTURE:

Let the Riemannian metric *g* be such that

(3.1) $`F(X,Y) = g(FX,Y)$ is skew- symmetric. Then

$$(3.2) \quad g\left(FX,Y\right)=-g\left(X,FY\right), \text{ and}$$

$\{F,g\}$ is called metric *F*-structrure.

Theorem (3.1): On the metric structrure-*F*, satisfying (1.1) we have

$$(3.3) \quad g\left(F^{k}X,F^{k}Y\right)=\left(-1\right)^{k+1}\left[g\left(X,Y\right)-\grave{m}\left(X,Y\right)\right]$$

where

$$(3.4) \quad \grave{m}\left(X,Y\right)=g\left(mX,Y\right)=g\left(X,mY\right).$$

Proof: From (1.2), (1.3) and (3.2), (3.4)

$$g\left(F^{k}X,F^{k}Y\right)=\left(-1\right)^{k}g\left(X,F^{2k}Y\right)$$

$$=\left(-1\right)^{k}g\left(X,lY\right)$$

$$=\left(-1\right)^{k+1}g\left(X,\left(I-m\right)Y\right),$$

$$=\left(-1\right)^{k+1}\left[g\left(X,Y\right)-\grave{m}\left(X,Y\right)\right]$$

4. KERNEL:

Let *F* be a (1,1) tensor, we define

$$(4.1) \quad Ker\left(F\right)=\left\{X:FX=0\right\}$$

Theorem (4.1): For the (1,1) tensor *F* satisfying (1.1), we have

$$(4.2) \quad Ker\ F=Ker\ F^{2}=.....=Ker\ F^{2k+1}$$

Proof: Let $X\in Ker\ F$

$$\Rightarrow FX=0$$

$$\Rightarrow F^{2}X=0$$

$$\Rightarrow X\in Ker\ F^{2}$$

Thus

(4.3) $\quad Ker\ F \subseteq Ker\ F^2$

Now let $X \in Ker\ F^2$

(4.4) $\quad F^2 X = 0$

$\Rightarrow F^3 X = 0$

(4.5) $\quad F^{2K+1} X = 0$ Using (1.1) in (4.5), we have

(4.6) $\quad FX = 0 \Rightarrow X \in Ker\ F$ Thus

(4.7) $\quad Ker\ F^2 \subseteq Ker\ F$

From (4.3) and (4.7), we get

(4.8) $\quad Ker\ F = Ker\ F^2$

Proceeding similarly, we get (4.2)

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