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Matrix Sieve – New Algorithm for Finding Prime Numbers

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Abstract

A new deterministic sieving algorithm based on derived "matrix definition" of prime numbers is proposed. The algorithm allows to calculate indexes$P$ of prime numbers in two sequences:$S_1(P) = 5 + 6P = 5,11,17, \dots;\ P = 0,1,2,3, \dots$and $S_2(P) = 7 + 6P = 7,13,19, \dots;\ P = 0,1,2,3, \dots$ in a given range of natural numbers ($N_1;N_2$). Also generalprimality criteria and twin-prime criteria are formulated. C++ program for finding primes in given range (N_1,N_2) and C++ program for primality testing of given natural number N are presented in Attachments 1 and 2.

1. Matrix generalexpression for composite numbers

Consider asequence of natural numbers from which members divisible by 2 and 3 are removed

$$S(p) = 5,7,11,13,17,19,23,25,29, \dots = \begin{cases} 3p + 5, p = 0,2,4,6,8, \dots \\ 3(p-1) + 7, p = 1,3,5,7, \dots \end{cases} \quad (1)$$

Sequence$S(p)$ can be divided into two sequences $S_1(i)$and $S_2(i)$:

$$S_1(i) = 5 + 6i = 5,11,17, \dots;\ i = 0,1,2,3, \dots \quad (2)$$
$$S_2(i) = 7 + 6i = 7,13,19, \dots;\ i = 0,1,2,3, \dots \quad (3)$$

Sequence $S(p)$contains all primes (except 2 and 3) and, using definitions of $S_1(i)$and $S_2(i)$, all composite numbers (except divisible by 2 and 3) of four different types:

$$FF(i,j) = S_1(i) * S_1(j) = (5 + 6i)(5 + 6j); \qquad SS(i,j) = S_2(i) * S_2(j) = (7 + 6i)(7 + 6j);$$
$$SF(i,j) = S_1(i) * S_2(j) = (5 + 6i)(7 + 6j); \ FS(i,j) = S_1(j) * S_2(i) = (5 + 6j)(7 + 6i);$$

which constitute four 2-dimensional arrays:

$$FF(i,j) = 25 + 30(i+j) + 36ij = \begin{pmatrix} 25 & 55 & 85 & 115 & 145 & \dots \\ 55 & 121 & 187 & 253 & 319 & \dots \\ 85 & 187 & 289 & 391 & 493 & \dots \\ 115 & 253 & 391 & 529 & 667 & \dots \\ 145 & 319 & 493 & 667 & 841 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}; i,j =$$

0,1,2,3,4 ....(4)

$$SS(i,j) = 49 + 42(i+j) + 36ij = \begin{pmatrix} 49 & 91 & 133 & 175 & 217 & \dots \\ 91 & 169 & 247 & 325 & 403 & \dots \\ 133 & 247 & 361 & 475 & 589 & \dots \\ 175 & 325 & 475 & 625 & 775 & \dots \\ 217 & 403 & 589 & 775 & 961 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}; i,j =$$

0,1,2,3,4 .... (5)

$$FS(i,j) = 35 + 6(5i+7j) + 36ij = \begin{pmatrix} 35 & 77 & 119 & 161 & 203 & \dots \\ 65 & 143 & 221 & 299 & 377 & \dots \\ 95 & 209 & 323 & 437 & 551 & \dots \\ 125 & 275 & 425 & 575 & 725 & \dots \\ 155 & 341 & 527 & 713 & 899 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}; i,j =$$

0,1,2,3,4 ....(6)

.

$$SF(i,j) = 35 + 6(7i+5j) + 36ij = \begin{pmatrix} 35 & 65 & 95 & 125 & 155 & \dots \\ 77 & 143 & 209 & 275 & 341 & \dots \\ 119 & 221 & 323 & 425 & 527 & \dots \\ 161 & 299 & 437 & 575 & 713 & \dots \\ 203 & 377 & 551 & 725 & 899 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}; i,j =$$

0,1,2,3,4 .... (7)

.

Substituting numbers in the above matrices with their corresponding indexes p in accordance with equation (1), the following four matrices can be obtained:

$$ff(i,j) = \begin{pmatrix} 7 & 17 & 27 & 37 & \dots & & \dots \\ 17 & 39 & 61 & 83 & \dots & & \dots \\ 27 & 61 & 95 & 129 & \dots & & \dots \\ 37 & 83 & 129 & 175 & \dots & & \dots \\ 47 & 105 & 163 & 221 & \dots & & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \dots \end{pmatrix} (8)$$

$$ss(i,j) = \begin{pmatrix} 15 & 29 & 43 & 57 & \dots & & \dots \\ 29 & 55 & 81 & 107 & \dots & & \dots \\ 43 & 81 & 119 & 157 & \dots & & \dots \\ 57 & 107 & 157 & 207 & \dots & & \dots \\ 71 & 133 & 195 & 257 & \dots & & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \dots \end{pmatrix} \quad (9)$$

$$fs(i,j) = \begin{pmatrix} 10 & 24 & 38 & 52 & \dots & & \dots \\ 20 & 46 & 72 & 98 & \dots & & \dots \\ 30 & 68 & 106 & 144 & \dots & & \dots \\ 40 & 90 & 140 & 190 & \dots & & \dots \\ 50 & 112 & 174 & 236 & \dots & & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \dots \end{pmatrix} (10)$$

$$sf(i,j) = \begin{pmatrix} 10 & 20 & 30 & 40 & \dots & & \dots \\ 24 & 46 & 68 & 90 & \dots & & \dots \\ 38 & 72 & 106 & 140 & \dots & & \dots \\ 52 & 98 & 144 & 190 & \dots & & \dots \\ 66 & 124 & 182 & 240 & \dots & & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \dots \end{pmatrix} \quad (11)$$

In general form these arrays can be expressed as

$ff(i,j) = 12ij - 2(i+j) - 1; i,j = 1,2,3,4 ....$ (12)
$ss(i,j) = 12ij + 2(i+j) - 1;\ i,j = 1,2,3,4 ....$(13)
$fs(i,j) = 12ij - 2(i-j) - 2; i,j = 1,2,3,4 \ldots$ (14)
$sf(i,j) = 12ij + 2(i-j) - 2;\ i,j = 1,2,3,4 ....$(15)

2. "Matrix definition" of prime numbers

As it follows from expressions (12)-(15), arrays$ff(i,j)$ and $ss(i,j)$ are symmetric and are comprised of odd integers (i.e. odd indexes of elements of $S(p)$);arrays$sf(i,j)$ and $fs(i,j)$ are transposes of each other and are comprised of even integers (i.e. even indexes of elements of $S(p)$). Thus, all elements of arrays $ff(i,j)$ and$ss(i,j)$ are indexes of members of $S_2(p)$and all elements of arrays $sf(i,j)$and $fs(i,j)$ are indexes of members of$S_1(p)$. It means that all composite members of the sequence $S_1(p)$ correspond to formula

$$S_1(P) = 6P + 5 = (5+6i)(7+6j); i,j = 0,1,2,3,4 ....$$

And all composite members of the sequence $S_2(p)$ correspond to formulae

$S_2(P) = 6P + 7 = (5+6i)(5+6j); i = 0,1,2,3,4 \dots, j \geq i$or

$$S_2(P) = 6P + 7 = (7+6i)(7+6j); i = 0,1,2,3,4 \dots, j \geq i$$

. Let us now consider sequences $S_1(P)$ and $S_2(P)$ separately as defined below

$S_1(P) = 6P + 5;\quad P = 0,1,2,3, \dots = p/2$ (16)
$S_2(P) = 6P + 7;\quad P = 0,1,2,3, \dots = (p-1)/2$(17)

Indexes $P_1(i,j)$, $P_2(i,j)$ and $P_3(i,j)$, $P_4(i,j)$ corresponding to composite numbers in $S_1(P)$ and $S_2(P)$ respectively are then defined as:
for $S_1(P)$

$$P_1(i,j) = 6ij - i + j - 1 = \begin{pmatrix} 5 & 12 & 19 & 26 & 33 & 40 & \\ 0 & 23 & 36 & 49 & 62 & 75 & \ldots \\ 0 & 0 & 53 & 72 & 91 & 110 & \ldots \\ 0 & 0 & 0 & 95 & 120 & 145 & \ldots \\ 0 & 0 & 0 & 0 & 149 & 180 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ldots & 215 & \ldots \end{pmatrix};\ i = 1,2,3,4\ldots;\ j \geq i \quad (18)$$

$$P_2(i,j) = 6ij + i - j - 1 = \begin{pmatrix} 0 & 10 & 15 & 20 & 25 & 30 & \ldots \\ 0 & 0 & 34 & 45 & 56 & 67 & \ldots \\ 0 & 0 & 0 & 70 & 87 & 104 & \ldots \\ 0 & 0 & 0 & 0 & 118 & 141 & \ldots \\ 0 & 0 & 0 & 0 & 0 & 178 & \ldots \\ . & . & . & . & . & 0 & \ldots \end{pmatrix};\ i = 1,2,3,4\ldots;\ j \geq i + 1 \quad (19)$$

for $S_2(P)$

$$P_3(i,j) = 6ij - i - j - 1 = \begin{pmatrix} 3 & 8 & 13 & 18 & 23 & 28 & \ldots \\ 0 & 19 & 30 & 41 & 52 & 63 & \ldots \\ 0 & 0 & 47 & 64 & 81 & 98 & \ldots \\ 0 & 0 & 0 & 87 & 110 & 133 & \ldots \\ 0 & 0 & 0 & 0 & 139 & 168 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ldots & 203 & \ldots \end{pmatrix};\ i = 1,2,3,4\ldots;\ j \geq i \quad (20)$$

$$P_4(i,j) = 6ij + i + j - 1 = \begin{pmatrix} 7 & 14 & 21 & 28 & 35 & 42 & \ldots \\ 0 & 27 & 40 & 53 & 66 & 79 & \ldots \\ 0 & 0 & 59 & 78 & 97 & 116 & \ldots \\ 0 & 0 & 0 & 103 & 128 & 153 & \ldots \\ 0 & 0 & 0 & 0 & 159 & 190 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ldots & 227 & \ldots \end{pmatrix};\quad i = 1,2,3,4\ldots;\ j \geq i \quad (21)$$

Thus, we have obtained a matrix representation of indexes of all composite numbers, which allows us to determine indexes of all prime numbers; so **"matrix definition" of prime numbers"** can be formulated as follows:
Natural numbers not contained in arrays $P_1(i,j)$ and $P_2(i,j)$are indexes P of all prime numbers in sequence$S_1(P)$ and natural numbersnot contained in arrays$P_3(i,j)$ and $P_4(i,j)$are indexes P of all prime numbers in sequence$S_2(P)$.

Additionally,equations (18) – (21) provide algebraic relationships connecting integers i and $j$with indexes of composite numbers in $S_1(P)$ and $S_2(P)$. Now, for sieving on an interval purposes, we need to establish a relationship between our range of interest (N_1,N_2) and the corresponding area of change of integers i and j:

Since $j \geq i$, maximum value of i will be when $i = j$and it approximately equals

$$i\ max = \frac{\sqrt{N_2}}{6} \quad (22)$$

For array $P_1(i,j)$ we have: $jmin = \frac{\frac{N_1}{6}+\mathrm{i}+1}{6\mathrm{i}+1}$; $jmax = \frac{\frac{N_2}{6}+\mathrm{i}+1}{6\mathrm{i}+1}$;

(23)

For array $P_2(i,j)$: $jmin = \frac{\frac{N_1}{6}-\mathrm{i}+1}{6\mathrm{i}-1}$; $jmax = \frac{\frac{N_2}{6}-\mathrm{i}+1}{6\mathrm{i}-1}$;

(24)

For array $P_3(i,j)$: $jmin = \frac{\frac{N_1}{6}+\mathrm{i}+1}{6\mathrm{i}-1}$; $jmax = \frac{\frac{N_2}{6}+\mathrm{i}+1}{6\mathrm{i}-1}$; (25)

For array $P_2(i,j)$: $jmin = \frac{\frac{N_1}{6}-\mathrm{i}+1}{6\mathrm{i}+1}$; $jmax = \frac{\frac{N_2}{6}-\mathrm{i}+1}{6\mathrm{i}+1}$; (26)

Matrix sieving algorithm can be formulated as follows:

In order to find all prime numbers in the range from N_1 to N_2 it is necessarily to remove from sequence $S_1(P)$members with indexes $P_1(i,j)$, $P_2(i,j)$:

$$S_1(P) = 6P + 5; \ \ P = 0,1,2,3, \dots$$

$$P_1(i,j) = 6ij - i + j - 1; \ \ i = 1,2, \dots, \left\lceil \frac{\sqrt{N_2}}{6} \right\rceil; \ j = \left\lceil \frac{\frac{N_1}{6}+\mathrm{i}+1}{6\mathrm{i}+1} \right\rceil, \left\lceil \frac{\frac{N_1}{6}+\mathrm{i}+1}{6\mathrm{i}+1} \right\rceil + 1, \dots, \left\lceil \frac{\frac{N_2}{6}+i+1}{6i+1} \right\rceil; j \geq i \quad (27)$$

$$P_2(i,j) = 6ij + i - j - 1; \ \ i = 1,2, \dots, \left\lceil \frac{\sqrt{N_2}}{6} \right\rceil; \ \ j = \left\lceil \frac{\frac{N_1}{6}-\mathrm{i}+1}{6\mathrm{i}-1} \right\rceil, \left\lceil \frac{\frac{N_1}{6}-\mathrm{i}+1}{6\mathrm{i}-1} \right\rceil + 1, \dots, \left\lceil \frac{\frac{N_2}{6}-i+1}{6i-1} \right\rceil; \ j \geq i+1 \quad (28)$$

And remove from sequence $S_2(P)$ members with indexes$P_3(i,j)$, $P_4(i,j)$:

$$S_2(P) = 6P + 7; \ \ P = 0,1,2,3, \dots$$

$$P_3(i,j) = 6ij - i - j - 1; \ \ i = 1,2, \dots, \left\lceil \frac{\sqrt{N_2}}{6} \right\rceil; \ j = \left\lceil \frac{\frac{\mathrm{N}_1}{6}+\mathrm{i}+1}{6\mathrm{i}-1} \right\rceil, \left\lceil \frac{\frac{\mathrm{N}_1}{6}+\mathrm{i}+1}{6\mathrm{i}-1} \right\rceil + 1, \dots, \left\lceil \frac{\frac{N_2}{6}+i+1}{6i-1} \right\rceil; j \geq i \quad (29)$$

$$P_4(i,j) = 6ij + i + j - 1; \ \ i = 1,2, \dots, \left\lceil \frac{\sqrt{N_2}}{6} \right\rceil; \ \ j = \left\lceil \frac{\frac{\mathrm{N}_1}{6}-\mathrm{i}+1}{6\mathrm{i}+1} \right\rceil, \left\lceil \frac{\frac{\mathrm{N}_1}{6}-\mathrm{i}+1}{6\mathrm{i}+1} \right\rceil + 1, \dots, \left\lceil \frac{\frac{N_2}{6}-i+1}{6i+1} \right\rceil; \ j \geq i \quad (30)$$

3. C++ program based on matrix sieve algorithm

In programming code were used following notations:

pr1= $P_{\min}$; pr2= $P_{\max}$;

R1[q], R2[r], S1[q], S2[r]-additional arrays corresponding to the range of P (pr1;pr2).

q- index of the arrays R1[q], S1[q].

r – index of the arrays R2[r], S2[r].

i2=$imax$; j1= $jmin$; j2= $jmax$;

P1=$P_1(i,j)$; P2=$P_2(i,j)$; P3=$P_3(i,j)$; P4=$P_4(i,j)$;

Expressions (18)-(21) for programming code can be rewritten as:

P1[i;j]=5+5*(i-1) + (7+ 6*(i-1))*(j-1) =(18a)

P2[i;j]=5+7*(i-1)+(5+6*(i-1))*(j-1)(19a)

P3[i;j]=3+5*(i-1) + (5+6*(i-1))*(j-1) (20a)

P4[i;j]=7+7*(i-1) + (7+6*(i-1))*(j-1) (21a)

Using above equations (18a)-(21a) with area of change of i and j determined as before (22)-(26), the C++ program was developed to find prime numbers in the range (N_1;N_2) (Appendix 1).Program was successfully tested on the ordinary notebook up to N= 2 000 000 000 000 000 000, for this value run time equals 10 seconds.

4. Primality criteria

Taking into account above stated considerations following criteria can be formulated:
Natural number N=6P+5; P=0, 1, 2, 3,…is a prime if and only if there is no solution for Diophantine equation

$$P = 6xy - x + y - 1; x \geq 1; \ y \geq 1 \quad (27)$$

Natural number N=6P+7; P=0, 1, 2, 3,… is a prime if and only if no one of two Diophantine equations

$$P = 6xy - x - y - 1; \ x \geq 1; y \geq x \quad (28)$$

$$P = 6xy + x + y - 1; x \geq 1; y \geq x \quad (29)$$

Has solution.

To prove primality criteria suppose thatnatural number N=6P+5is a prime and integer solution for equation (27) does exist.This means that there is corresponding member$P$ in arrays $P_1(i,j)$ or $P_2(i,j)$, i.e. natural number N=6P+5 is not prime, this is a contradiction.

So natural number N=6P+5; P=0, 1, 2, 3,… is a prime if and only if there is no integer solution for equation (27). The same is true for N=6P+7 (28) and (29).

C++ program for primality testing of N is presented in Appendix 2.

Employment of Diophantine equation for primality testing is illustrated on the pagehttps://www.wolframcloud.com/objects/f8816adc-cf73-452a-90ab-29a14b763f3f.

5. Twin-prime criteria

Obviously, twin primes N1 and N2 (N2-N1=2) correspond to formulae

N1=6P+5 and N2=6P+7

So twin-prime criteria can be formulated as follows:

Natural numbers$N1$=6P+5 and N=6P+7,P=0, 1, 2, 3, are twin primes if and only if no one of three Diophantine equation

$$P = 6xy - x + y - 1; x \geq 1;\ y \geq 1 \quad (28)$$

$$P = 6xy - x - y - 1;\ x \geq 1; y \geq x \quad (29)$$

$$P = 6xy + x + y - 1; x \geq 1; y \geq x \quad (30)$$

has solution.

6. Conclusions

We have shownthat all composite numbers in the sequence$S_1(P)$=6P+5; P=0, 1, 2, 3,…correspond to formula

$$S_1(P) = (5 + 6i)(7 + 6j);$$

andall composite numbers in the sequence$S_2(P)$=6P+7; P=0, 1, 2, 3,…correspond to formulae

$S_2(P) = (5 + 6i)(5 + 6j)$or

$$S_2(P) = (7 + 6i)(7 + 6j);$$

Deterministic **"matrix definition" for prime numbers** was derived:

Natural numbers that are **not** contained in arrays

$$P_1(i,j) = 6ij - i + j - 1 = \begin{pmatrix} 5 & 12 & 19 & 26 & 33 & 40 & \\ 0 & 23 & 36 & 49 & 62 & 75 & \dots \\ 0 & 0 & 53 & 72 & 91 & 110 & \dots \\ 0 & 0 & 0 & 95 & 120 & 145 & \dots \\ 0 & 0 & 0 & 0 & 149 & 180 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & 215 & \dots \end{pmatrix} ;\; i = 1,2,3,4 \dots ;\; j \ge i$$

$$P_2(i,j) = 6ij + i - j - 1 = \begin{pmatrix} 0 & 10 & 15 & 20 & 25 & 30 & \dots \\ 0 & 0 & 34 & 45 & 56 & 67 & \dots \\ 0 & 0 & 0 & 70 & 87 & 104 & \dots \\ 0 & 0 & 0 & 0 & 118 & 141 & \dots \\ 0 & 0 & 0 & 0 & 0 & 178 & \dots \\ . & . & . & . & . & 0 & \dots \end{pmatrix} ;\; i = 1,2,3,4 \dots ;\; j$$

$$\ge i + 1$$

are indexes $P$of **all primes** in the sequence $S_1(P)$=6 P+5.
Natural numbers that are **not** contained in arrays

$$P_3(i,j) = 6ij - i - j - 1 = \begin{pmatrix} 3 & 8 & 13 & 18 & 23 & 28 & \dots \\ 0 & 19 & 30 & 41 & 52 & 63 & \dots \\ 0 & 0 & 47 & 64 & 81 & 98 & \dots \\ 0 & 0 & 0 & 87 & 110 & 133 & \dots \\ 0 & 0 & 0 & 0 & 139 & 168 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & 203 & \dots \end{pmatrix} ;\; i = 1,2,3,4 \dots ;\; j \ge \text{i}$$

$$P_4(i,j) = 6ij + i + j - 1 = \begin{pmatrix} 7 & 14 & 21 & 28 & 35 & 42 & \dots \\ 0 & 27 & 40 & 53 & 66 & 79 & \dots \\ 0 & 0 & 59 & 78 & 97 & 116 & \dots \\ 0 & 0 & 0 & 103 & 128 & 153 & \dots \\ 0 & 0 & 0 & 0 & 159 & 190 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & 227 & \dots \end{pmatrix} \quad ; \quad i = 1,2,3,4 \dots ;\; j$$

$$\ge i$$

are indexes P of **all primes** in the sequence $S_2(P)$=6 P +7.
General criteria of primality and twin-prime criteria were formulated.
C++ program for finding primes in given range (N1;N2), (N2<2*10^18, N2-N1<1 000 000) was developed and successfully tested, confirming theoretical background. For N=2*10^18 run time on ordinary notebook equals 10 s.
C++ program for testing primality of given natural number N was developed and successfully tested. For N=2*10^18 run time on ordinary notebook equals 7 s,

Attachment 1

```
#include <cstdlib>
```

```
#include <iostream>
#include <math.h>
#include <ctime>

using namespace std;
main( )
{
                                /*  3S MATRIX  SIEVE*/
                               /*FINDING PRIMES IN THE RANGE (N1;N2)*/

                      /* N1>23;    N2<2 000 000 000 000 000 000; N2-N1<500 000*/

unsigned long longint N1=111222333444555000; unsigned long longint N2
=111222333444555600;
unsigned long longint pr1=floor(N1/6); unsigned long longint pr2=ceil( N2/6);
int r=84000; int R2[r]; intrm=pr2-pr1;unsigned long longint S2[r];int r3, r4;
int q=84000; int R1[q] ; intqm=rm; unsigned long longint S1[q] ; int q2, q1;
for (q=1;q<qm;q++)
R1[q] =1;
for (r=1;r<rm;r++)
                R2[r] =1;
unsigned long longinti, j, P1, P2, P3, P4, B, K;

unsigned long longint i2= sqrt( pr2/6)+2;
longlongint j1, j2;
int l1=0;int l2=0;
for ( i=1;i<i2;i++)
{ j2=(pr2+i+1)/( 6*i+1)+1;j1=(pr1+i+1)/( 6*i+1);
B=5+5*( i-1);  K=7+6*( i-1);
if ( i>j1) j1=i;
for(j=j1; j<j2;j++)
{ P1=B+K*( j-1);
if(( P1>pr1)&&( P1<pr2))
{  q1=P1-pr1; R1[ q1] =0; } }
   j2=(pr2-i+1)/( 6*i-1)+1;j1=(pr1-i+1)/( 6*i-1);
if (j1<1) j1=1;
   B=5+7*( i-1);  K=5+6*( i-1);
if ( i>j1-1) j1=i+1;
for(j=j1; j<j2;j++)
```

```
{P2=B+K*( j-1);
         if(( P2>pr1)&&( P2<pr2))
   {      q2=P2-pr1; R1[ q2] =0;
  } }
        j2=(pr2+i+1)/( 6*i-1)+1;j1=(pr1+i+1)/( 6*i-1);
       B=3+5*( i-1);  K=5+6*( i-1);
if ( i>j1) j1=i;
for(j=j1;  P3=B+K*( j1);
if(( P3>pr1)&&( P3<pr2))
   {     r3=P3-pr1; R2[ r3]=0;
  }  }
      j2=(pr2-i+1)/( 6*i+1)+1;j1=(pr1-i+1)/( 6*i+1);
      B=7+7*(i-1);  K=7+6*(i-1);
if ( i>j1) j1=i;
for(j=j1; j<j2;j++)
{ P4=B+K*( j-1);
if(( P4>pr1)&&( P4<pr2))
{ r4=P4-pr1; R2[ r4] =0; } } }
cout<<"\ni2="<<i2<<";pr2="<<pr2<<"    \n";
cout<<"\nP=pr1+q; pr1="<<pr1<<"   \n";
    for ( q=1;q<qm;q++) { S1[q] =R1[q]*((pr1+q)*6+5);  if (S1[q]%5==0) continue;
l1=l1+1;
cout<<"q="<<q<<"; Prime in S1[P]=6*P+5="<<S1[ q]<<"  \n";}
cout<<"\nP=pr1+r; pr1="<<pr1<<"   \n";
for ( r=1;r<rm;r++) { S2[r] =R2[r]*((pr1+r)*6+7);if (S2[r]%5==0) continue;l2=l2+1;
cout<<"r="<<r<<"; Prime in S2[P]=6*P+7="<<S2[ r]<<"  \n";}
cout<<"l1="<<l1<<"  \n";
cout<<"l2="<<l2<<"  \n";
cout<<"number of Primes in the range (N1;N2) =l1+l2\n";
cout<<"  run time (ms)=";
cout<<clock();
system("PAUSE");
return EXIT_SUCCESS;

}
```

Attachment 2

```
#include <cstdlib>
#include <iostream>
```

```
#include <math.h>
#include <ctime>

using namespace std;

                                    /*TESTING PRIMALITY OF N<2*10^18*/
main(  )
{longlonginti;unsigned long longint N=1111222333444555249;
longlongint P1;long  double y1; long double y2;
long double mod1;long double mod2;int mod3N;
longlongint i2=sqrt(N)/6+2; mod3N=((long double)N/3-N/3)*3+0.4;
cout<<"\n  mod3N="<<mod3N<<"  \n";
if (mod3N==2)
{P1=(N-5)/6;
cout<<"\n  P1="<<P1<<"  \n";
for ( i=1;i<i2; i++)
  {   y1=(long double)(P1+i+1)/(6*i+1);
   mod1=(long double)(y1-(long longint)y1);
     y2=(long double)(P1-i+1)/(6*i-1);
  mod2=(long double)(y2-(long longint)y2);
if ((mod1==0)||(mod2==0))
{
cout<<"N is not prime"<<"\nN="<<N<<"  \n";
break;}
if (i==i2-1)
{
cout<<"N is prime"<<"\nN="<<N<<"  \n";
}}}
if (mod3N==1)
{ P1=(N-7)/6;
cout<<"\n  P1="<<P1<<"  \n";
for ( i=1;i<i2; i++)
  {   y1=(long double)(P1+i+1)/(6*i-1);
   mod1=(long double)(y1-(long longint)y1);
     y2=(long double)(P1-i+1)/(6*i+1);
  mod2=(long double)(y2-(long longint)y2);
if ((mod1==0)||(mod2==0))
{
cout<<"N is not prime"<<"\nN="<<N<<"  \n";
```

```
break;}
if (i==i2-1)
cout<<"N is prime"<<"\nN="<<N<<"  \n";
}}
if (mod3N==0)
cout<<"N is not prime"<<"\nN="<<N<<"  \n";
cout<<clock();
system("PAUSE");
return EXIT_SUCCESS;

}
```