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Empowering Discovery Learning and Direct Instruction with Computer-Assisted to Enhance Mathematical Performance

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ABSTRACT

In the current educational context, the low academic performance of students in mathematics indicates the need for the implementation of more effective teaching methods to support the understanding of complex concepts. The effectiveness of learning can be enhanced through computer-assisted instruction, particularly in discovery learning (DL-CA) and direct instruction (DI-CA) utilizing web-based media. This study employs a quantitative approach with statistical analysis to examine the significant impact of these teaching methods on students' mathematical performance. Discovery learning and direct instruction were applied to two groups studying numerical methods, focusing on the topic of nonlinear equation solutions using the bisection method, with each group consisting of 33 students. The statistical test results indicated that there was no significant difference in mathematical performance between the DL-CA and DI-CA groups; both methods showed comparable effectiveness. However, both methods had a significant effect on the N-Gain of the learning outcomes in both groups. The improvement in mathematical performance for the DL-CA group was better than that for the DI-CA group, with more students showing high improvement (11 students) compared to the DI-CA group (1 student). Additionally, in the low improvement category, the DL-CA group had fewer students (2 students) than the DI-CA group (8 students). This suggests that both discovery learning and direct instruction have almost equivalent reliability in achieving students' mathematical abilities, but DL-CA is more effective in enhancing performance compared to DI-CA.

KEYWORDS: Computer assisted, Discovery, Direct Instruction, Mathematics Performance

I. INTRODUCTION

Numerous studies have highlighted the importance of effective teaching methods in enhancing students' mathematics learning outcomes. Instructional practices play a critical role in student achievement, particularly in mathematics [1]. They argue that methods that promote active engagement, such as discovery learning, encourage students to explore and understand mathematical concepts deeply. This active engagement not only improves students' problem-solving skills but also fosters a more positive attitude toward mathematics, ultimately leading to better academic performance [1].

In contrast, direct instruction has also been shown to yield significant improvements in mathematics learning outcomes. Direct instruction provides a structured approach that helps students grasp complex mathematical concepts more efficiently[2]. This method involves clear explanations and guided practice, which can lead to higher retention rates and understanding of the material. Research further supports this, indicating that students exposed to direct instruction often demonstrate improved performance in standardized assessments[3]. The combination of these structured approaches with technological tools can amplify their effectiveness in a classroom setting.

Furthermore, the integration of technology in mathematics education has been shown to enhance learning outcomes significantly. Computer-assisted learning tools can increase student engagement and motivation, leading to improved academic results[4]. These tools often provide interactive and personalized learning experiences that cater to individual student needs, making the learning process more effective [5]. As such, employing both discovery learning and direct instruction, particularly when supported by technology, can create a comprehensive approach that addresses various learning styles and improves overall mathematics performance.

For that reason, this study aims to answer the following question: 1)How to empower discovery learning and direct instruction with computer assissted to mathematics performance? 2) Is there a significant difference in mathematics learning outcomes between students who use computer-assisted discovery learning and those who use computer-assisted direct instruction? 3)Is there a significant difference of N-Gain from mathematics learning between students who use computer-assisted discovery learning and those who use computer-assisted direct instruction? With the purpose of this study is to examine the impact of computer-assisted discovery learning and computer-assisted direct instruction on students' mathematics learning outcomes and improvements. By using a quantitative approach and statistical analysis, this research is expected to provide deeper insights into the effectiveness of both methods in the field of mathematics. Thus, the findings of this study can assist in the development of more effective and efficient learning strategies.

II. THEORITICAL REVIEW

Results and Improvements in Mathematics Learning

Mathematics learning outcomes can be defined as the level of achievement students attain in understanding and applying the mathematical concepts that have been taught. Learning outcomes encompass various aspects, such as knowledge, comprehension, application, analysis, synthesis, and evaluation[6]. In the context of mathematics education, learning outcomes are often measured through formal assessments, such as exams and assignments, aimed at evaluating students' abilities to solve mathematical problems and understand fundamental principles of mathematics [1]. Therefore, learning outcomes not only reflect academic capabilities but also the readiness of students to face real-world challenges related to mathematics.

On the other hand, improvements in mathematics learning refer to the progress that students make over time in mastering mathematical concepts and skills. This improvement can be measured by comparing students' learning outcomes before and after the implementation of specific teaching methods[7]. For instance, the implementation of more interactive teaching methods, such as discovery learning, can encourage students to participate more actively in the learning process and contribute to improving their learning outcomes [8]. Hence, it is important to understand the factors that influence learning improvements to design effective teaching strategies.

In the context of mathematics education, measuring learning outcomes and improvements also involves analyzing how students can apply mathematical skills in different situations. Good learning outcomes are not only measured by final grades but also by students' ability to use their mathematical knowledge flexibly and creatively[9]. Thus, successful mathematics education focuses not only on achieving high scores but also on developing deep understanding and critical thinking skills. This forms an essential foundation for designing curricula and teaching methods that can enhance both learning outcomes and improvements in mathematics comprehensively.

Computer-Assisted Discovery Learning

Computer-assisted discovery learning integrates technology with active learning methods, where students are encouraged to discover concepts and knowledge through exploration and problem-solving using digital tools[10][11]. In this method, students are given opportunities to interact with various simulations, educational games, and software designed to facilitate independent concept discovery [12]. This approach aims to increase student engagement and help them understand the material more deeply and intuitively through active learning experiences [13].

Characteristics or indicators of computer-assisted discovery learning include the use of interactive digital tools, such as simulations and educational software, that allow students to conduct virtual experiments and observe the results directly [11]. Another indicator is the presence of challenging tasks that require critical thinking, as well as opportunities for students to collaborate in problem-solving [12]. Furthermore, technological support allows for immediate and adaptive feedback that helps students understand mistakes and correct their understanding in real-time [4].

Research shows that computer-assisted discovery learning can significantly enhance students' mathematical performance and motivation [4]. This is primarily because this method provides students with greater control over their learning process, which can increase their sense of responsibility and engagement in learning [12]. Additionally, the use of technology allows for more engaging and interactive content delivery, which can help students understand abstract and complex concepts more easily [11].

Computer-Assisted Direct Instruction

Computer-assisted direct instruction is an approach where structured direct instruction is delivered through digital platforms, enabling systematic and consistent presentation of material [2] [14]. This method often employs videos, presentations, and interactive modules to convey complex concepts in an easily understandable manner [3]. Computer-assisted direct instruction is designed to provide clear and detailed explanations while guiding students through the necessary steps to understand and master the material [15].

Characteristics or indicators of computer-assisted direct instruction include the use of digital media to deliver clear and structured instructions, such as tutorial videos and interactive presentations [14]. Other indicators include the presence of systematic learning steps broken down into manageable parts for students, along with integrated practice and assessment to ensure student understanding [2].

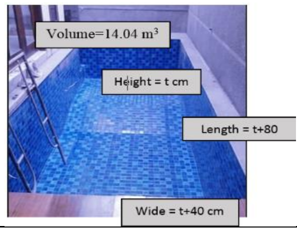
Additionally, this method often provides immediate feedback and automated assessments that help students gauge their progress and identify areas needing improvement [3].

Research indicates that computer-assisted direct instruction is effective in enhancing students' understanding of the material being taught, especially in contexts where detailed and structured explanations are required [15]. This method allows for consistent and repetitive delivery of content, which is crucial for reinforcing students' understanding of the taught concepts [14]. Thus, computer-assisted direct instruction can be a highly effective tool for teaching complex material and preparing students to face challenging academic tasks [3].

III. REFERENCES

This study adopts a quantitative approach with a quasi-experimental design to compare the effectiveness of two computer-assisted learning methods: computer-assisted discovery learning (DL-CA) and computer-assisted direct instruction (DI-CA). The main objective of this research is to evaluate the impact of both methods on students' mathematical abilities, particularly in the topic of solving non-linear equations using the bisection method. The population targeted in this research consists of all students enrolled in a numerical methods course at a university. The sample was taken using purposive sampling, comprising 66 students divided into two groups, each consisting of 33 individuals. The first group was taught using the DL-CA method, while the second group utilized the DI-CA method, allowing for a clear comparison between the two approaches. The research procedure involves several stages, starting with the development of instruments designed to measure students' mathematical abilities.

Table 1: Instrument of Mathematics Reasoning Test

No	Problem	Expectation
1	A non-linear equation $$x^2 - 3x - 5 = 0$$ Determine solutions to the equation using two approaches: analytical method and numerical method, specifically employing the bisection method with the selection of bounds that contain the solution.	Memorize Plausible reasoning Applied bisection algorithm Understanding solution of non-linear equation
2	Given an equation $$x^{0,3} + x^2 - 1 = 0$$ Find the solution to the equation using the bisection method by determining the initial bounds, constructing the algorithm, and computing each bound until obtaining the solution	Plausible reasoning Understanding bisection method Applied bisection algorithm
3	A pool with a volume of 14.04 m³, its height, length, and width are unknown. Find the surface area of the pool to be covered with tiles. 	Mathematics foundation Algorithmic reasoning Applied bisection algorithm Mathematics foundation

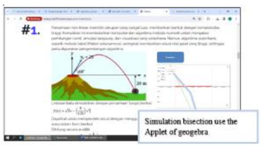
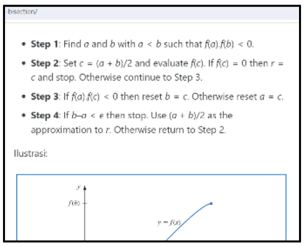
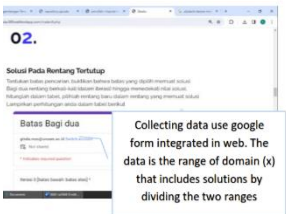
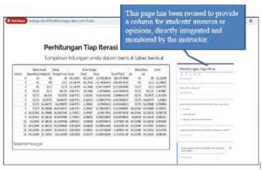
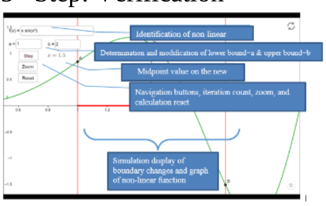
Prior to implementing the test instrument, validity and reliability assessments were conducted, as emphasized[16]. The content validation approach was chosen to ensure that the test instrument effectively measures students' understanding of numerical methods for solving non-linear equations, particularly the bisection method. Furthermore, the reliability assessment analyzed how the contextual information presented influenced students' performance in answering the questions [17]. The learning media used have also undergone a development and validation process before being implemented. After the instruction was conducted according to the established methods, data were collected through tests administered to both groups. This data was then analyzed using inferential statistical techniques, including independent t-tests to compare the average mathematical performance between the two groups, as well as N-Gain tests to measure the improvement in abilities from pre-test to post-test.

IV. RESULT

The first step in this research is to design a learning model that is integrated with computer technology. This model aims to create an interactive learning environment that supports more effective learning. The development of the model includes the creation and programming of learning materials, such as simulations, tutorial videos, and interactive modules that can be accessed by students through digital platforms [11]. In discovery learning and direct instruction, each step is accompanied by web-based media, as follows:

Table 2 : Computer assisted to empower DL and DI Steps

Empowering Discovery & Direct Intruction to CAL	
Discovery Learning	Direct Instruction
1st Step: Stimulation  Fig.1: Interface of Stimulation Students are prepared in advance by understanding the material, learning objectives,	1st Step: Preparation and Introduction  Fig.2: Interface head of website

and the teaching methods that will be applied. Stimuli are provided through the application of non-linear equations in various fields through several journals.	Students are informed about the teaching material, learning methods, learning objectives, and the expected learning outcomes through the initial display of the web interface.
2nd Step: Problem Statement  Figure 3: Visualization problem statement Identification problem of nonlinear equation from trajectory of rocks from a volcanic eruption which cannot solved by analytic way	2nd Step: Presentation  Figure 4: Interface of material in website page
3rd Step: Data Collection  Figure 5: Students are asked to determine the bounds (domain), then divide them into two and identify new bounds that approach the solution.	3rd Step: Guided Practice  Fugure 6: The process of answering is guided by the instructor and can be simulated through the GeoGebra applet.
4th Step:Data Processing  Figure 7: Calculate the solution from student responses The data provided by students is processed by implementing the bisection method steps.	4th Step:Independet Practice Students are asked to solve the non-linear equation without guidance from the instructor. 1. Find solution of $x^3 - 5 = 0$ Determine its range of domain to find solution 2. Find solution from the equation: $x + e^x = 0$ In range [-1:0] using bisection method
5th Step: Verification  Figure 8: GeoGebra to find the bounds The steps or results provided by the students are verified using the GeoGebra applet to determine the bounds and divide the two bounds until a solution is obtained.	5th Step: Evaluation Students' understanding is directly analyzed by the instructor through their written answers and the reasoning provided while attempting to obtain solutions to the non-linear equation in Step 4.
6th Step:Generalization Students are asked to summarize the steps of the bisection method and to inquire about the convergence of the iterations of the bisection algorithm. The instructor provides a generalization for the correct steps or answers.	6th Step:Closure The instructor summarizes the learning, provides an opportunity for questions and answers, and responds to the students' reflection results.

Acquisition of Students' Mathematical Ability through (DL-CA) and (DI-CA)

Based on descriptive statistical analysis using SPSS software, resulting in output on the acquisition of mathematical ability for those learning with DL-CA and DI-CA as follows:

Table 3: Descriptive Analysis Output of Acquisition

Statistics Aspect	**Learning**	
	DL-CA	**DI-CA**
Mean	63,8003	61,2124
Std. Deviation	18,02469	13,04529
Minimum	16,36	30,91
Maximum	87,27	96,36
Skewness	-1,157	-,222
Kurtosis	1,231	1,024

Based on the analysis, learning through computer-assisted discovery learning (DL-CA) has an average score of 63.80, which is higher than direct instruction (DI) at 61.21. DL-CA learning allows students to actively engage in the exploration of mathematical concepts, contributing to a deeper understanding. Although both methods show a tendency for higher scores, the DL-CA class exhibits a wider distribution of scores and a greater tendency to cluster at high scores compared to the DI-CA class.

The Influence of DL-CA and DI-CA on the Acquisition of Mathematical Ability

Based on the analysis of pretest and posttest scores, the following output was obtained:

Table 4: Paired sample test DL-CA and DI-CA

		Mean	Std. Deviation	Std. Error Mean	Corre-lation	Sig.	Paired Sample Test	
							Mean	Sig.
Pair 1	Pre_Tes_ DL-CA	15.37	8.97	1.56	.609	.000	-48.429	.000
	Pos_Tes_ DL-CA	63.80	18.02	3.14				
Pair 2	Pre_Tes_ DI-CA	32.67	6.62	1.15	.284	.109	-28.539	.000
	Pos_Tes_ DI-CA	61.21	13.04	2.27				

The results of the test indicate that learning through Computer Assisted Discovery Learning (DL-CA) and Direct Instruction with Computer Assistance (DI-CA) has a significant impact on students' mathematical reasoning ability. In the DL-CA group, the average pre-test score is 15.37 and the post-test score reaches 63.80, with a significant positive correlation of 0.609, where 37% of the post-test scores are influenced by the pre-test scores. Although the impact of DL-CA on mathematical reasoning ability is categorized as low, it remains significant. For the DI-CA group, the average pre-test score is 32.67 and the post-test score is 61.21, with a positive correlation of 0.609, indicating that only 8.1% of the post-test scores are influenced by the pre-test scores. Overall, both methods show a significant effect but are categorized as having a low impact on mathematical reasoning ability. The differences in the effects of DL-CA and DI-CA on learning outcomes can be demonstrated through the following analysis:

Table 5. Anova Analysis for the effect of DL-CA and DI-CA to mathematics N-Gain

Source	**Type III Sum of Squares**	**df**	**F**	**Sig.**
Corrected Model	110.502a	1	.446	.506
Intercept	257865.003	1	1041.734	.000
Learning	110.502	1	.446	.506
Error	15842.208	64		
Total	273817.713	66		
Corrected Total	15952.710	65		
a. R Squared = .007 (Adjusted R Squared = -.009)				
Tabel 5: Tests of Between-Subjects Effects DL-CA and DI-CA for test result				

The results of the ANOVA analysis indicate that DL-CA and DI-CA do not have a significant effect or difference. From the Tests of Between-Subjects Effects table, the F value for the Learning factor is 0.446 with a p-value of 0.506. This indicates that there is no significant difference between DL-CA and DI-CA. Additionally, the R Squared value of 0.007 shows that only 0.7% of the variation in post-test scores can be explained by this model. The negative Adjusted R Squared (-0.009) indicates that this model is not even able to adequately explain the variance in the data. In other words, the learning methods do not contribute significantly to the acquisition of reasoning ability; both methods have almost the same effect.

Improvement (N-Gain) in Mathematical Ability in DL-CA and DI-CA Learning

The scores from the students' answers on both tests, namely the pre-test and post-test, serve as the basis for calculating the improvement in students' mathematical reasoning ability through normalized gain scores or N-Gain. The clustering data obtained is shown in the table:

Table 6. Frequency of N-Gain Categories Based on Learning

N-Gain	DL-CA			DI-CA			All
	Hi	Med	Low	Hi	Med	Low	
Amount	11	20	2	1	24	8	66
Mean	0.759	0.529	0.125	0.93	0.36	0.15	0.50
Median	0.74	0.54	0.125	0.93	0.505	0.15	0.53
Minimum	0.7	0.31	0.12	0.93	0.37	0.03	0.03
Maximum	0.83	0.66	0.13	0.93	0.66	0.28	0.93

Table 6 shows the frequency of N-Gain categories based on two learning methods, DL-CA and DI-CA. In the high N-Gain category, DI-CA has a higher mean value (0.93) compared to DL-CA (0.74), although it only involves one student. In the moderate N-Gain category, DL-CA shows a higher mean value (0.54) compared to DI-CA (0.3655), but DI-CA involves more students (24 and 20, respectively). For the low N-Gain category, DI-CA has a higher mean value (0.15) compared to DL-CA (0.125). Overall, DL-CA demonstrates better performance than DI-CA in terms of the number of students in the high and moderate N-Gain categories.

The Influence of DL-CA and DI-CA on the N-Gain of Mathematical Ability

The effect of DL-CA and DI-CA on the N-Gain of mathematics learning outcomes was tested using analysis of variance (ANOVA), as indicated by the following results.

Table 7. Anova test influence DL-CA and DI-CA to mathematics N-Gain

N-Gain	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.404	1	.404	11.605	.001
Within Groups	2.228	64	.035		
Total	2.632	65			

The results of the ANOVA analysis in the table show a significant difference between the two learning groups (DL-CA and DI-CA) in terms of N-Gain. The Sum of Squares between groups is 0.404, while the Sum of Squares within groups is 2.228, resulting in a total Sum of Squares of 2.632. The degrees of freedom (df) for between groups is 1 and for within groups is 64, giving a total df of 65. The Mean Square between groups is 0.404, while the Mean Square within groups is 0.035. The obtained F value is 11.605, with a significance (Sig.) of 0.001. Since this significance value is less than 0.05, it can be concluded that there is a significant difference in N-Gain between the two learning methods.

Table 8. ANOVA effect size to N-Gain of DL-CA & DI-CA

		Point Estimate	95% Confidence Interval	
			Lower	Upper
N_ Gain	Eta-squared	.153	.027	.310
	Epsilon-squared	.140	.012	.299
	Omega-squared Fixed-effect	.138	.011	.296
	Omega-squared Random-effect	.138	.011	.296

The results of the ANOVA analysis showing the effect size for N-Gain provide an interesting perspective on the influence of the independent variable. The largest effect size is found in eta-squared (η^2) with a point estimate of 0.153, meaning that approximately 15.3% of the variance in N-Gain can be explained by the analyzed factor. Epsilon-squared (ε^2) shows an estimate of 0.140, indicating that the group differences contribute 14.0% to that variance. Meanwhile, omega-squared (ω^2) for fixed and random effects both have the same estimate of 0.138, indicating that the explained variance remains around 13.8%. All these effect sizes indicate a moderate influence, although with a significant degree of uncertainty. The 95% confidence interval for each effect size provides a varying range, from 2.7% to 31.0% for η^2, suggesting that the effect may be smaller or larger than estimated.he heading of the References section must not be numbered.

V. Discussion

The acquisition of mathematical abilities among students who underwent Computer-Assisted Discovery Learning (DL-CA) and Computer-Assisted Direct Instruction (DI-CA) shows distinct differences. The average mathematical test scores in the discovery learning group (DL-CA) is higher, with a distribution that is more concentrated around the highest values compared to the scores of students in the direct instruction group. This is because discovery learning allows students to actively engage in exploring mathematical concepts, which fosters deeper understanding. Previous studies have shown that discovery learning aids students in grasping mathematical concepts in a more meaningful manner, enabling them to analyze information, evaluate solutions, and make decisions, as opposed to direct instruction, which emphasizes less on developing mathematical skills [18], [19].

Computer-Assisted Discovery Learning (DL-CA) using the web significantly influences students' mathematical abilities. The computer assistance through web pages in discovery learning can capture students' attention, making learning more engaging and helping them visualize abstract and complex concepts that are difficult to understand. Studies conducted by [20] and [21] indicate that students are more engaged when they can explore concepts independently and make their own discoveries. Additionally, computer-assisted discovery learning has the potential to be more effective in enhancing students' mathematical abilities[22].

The group of students who received Computer-Assisted Direct Instruction (DI-CA) also showed a significant influence on their mathematical abilities. The instructions presented via computer media help in the visual and interactive presentation of information, assisting students in better understanding abstract concepts. Additional key learning resources, such as instructors and computer media, support the delivery of instructions. The effects of computer-based direct instruction can enhance engagement and motivation in learning, provide clear and interactive visualizations of mathematical concepts, and offer opportunities for more interactive practice and learning[23]. Therefore, it can be concluded that DI-CA is a promising method for enhancing mathematical abilities. Both DL-CA and DI-CA provide significant impacts on the acquisition of students' mathematical abilities.

The influence of Computer-Assisted Discovery Learning (DL-CA) and Computer-Assisted Direct Instruction (DI-CA) on the acquisition of students' mathematical abilities does not show a significant difference in effect. The research results indicate that both learning methods have nearly similar reliability in enhancing students' mathematical abilities. Each method has its shortcomings; for instance, computer-assisted discovery learning requires a longer duration compared to the more efficient duration of computer-assisted direct instruction. Both methods, DL-CA and DI-CA, have the potential to enhance mathematical abilities, and the choice of the appropriate method depends on the characteristics of the students, the material being taught, and the instructor's skills. Discovery learning is one model of a constructivist approach that has the potential to fail in knowledge construction [15], [25], [28]. Additionally, direct instruction can be an effective method for improving students' understanding of mathematical concepts by providing clear and structured instructions, followed by practice and feedback, and has proven successful for students from diverse backgrounds and abilities [26], [27], [29].

Regarding the improvement in students' mathematical abilities, measured by normalized gain (N-Gain), there is a difference between the groups of students who received Computer-Assisted Discovery Learning (DL-CA) and Computer-Assisted Direct Instruction (DI-CA). The mathematical abilities improved more significantly in the group receiving Computer-Assisted Discovery Learning compared to those receiving Computer-Assisted Direct Instruction [24]. This is consistent statement that students who learned mathematics using discovery learning showed greater N-Gain than those who learned through direct instruction, and research by [13], which indicates that discovery learning can help students develop skills in obtaining solutions to more complex mathematical problems.

Both discovery learning and direct instruction can serve as effective methods for enhancing students' mathematical abilities. The improvement in mathematical abilities observed among students in both the DL-CA and DI-CA groups shows significant differences, with DL-CA having a more favorable impact than DI-CA. Discovery learning guides students to explore concepts, discover patterns, and build generalizations as knowledge used in solving problems, whereas direct instruction employs structured and directed instructions by presenting information directly to students and providing clear guidance in completing tasks. These two concepts yield differences in enhancing students' mathematical abilities, with DL-CA proving to be superior to DI-CA in improving students' mathematical reasoning skills. The influence of DL-CA on mathematical improvement is supported by the characteristics of discovery learning, which aids students in better understanding mathematical concepts and retaining information longer than those learning through direct instruction [30]

VI. CONCLUTION

Computer-assisted learning, whether through Discovery Learning (DL-CA) or Direct Instruction (DI-CA), has a significant impact on enhancing students' mathematical abilities. Although both methods demonstrate effectiveness in improving these skills, DL-CA has proven to be superior to DI-CA in terms of score improvement and conceptual understanding. Discovery Learning encourages students to actively engage in the exploration of mathematical concepts, allowing for deeper understanding and the ability to retain information longer. Meanwhile, Direct Instruction is effective in delivering information in a clear and structured manner but falls short in developing students' analytical skills and decision-making abilities. These results also imply that instructors should consider using the Discovery Learning method (DL-CA) to enhance student engagement and achieve a deeper understanding of mathematical concepts. Additionally, integrating both DL-CA and Direct Instruction (DI-CA) approaches into the curriculum can help maximize learning potential by catering to the needs and characteristics of students.

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