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An Ear Decomposition of a Graph

A. N. Bhavale[1], S. N. Gajul[2]

[1]Head, Department of Mathematics, P.E.S. Modern College of Arts, Science and Commerce (Autonomous), Shivajinagar, Pune-411005, India.

[2]Department of Mathematics, P.E.S. Modern College of Arts, Science and Commerce (Autonomous), Shivajinagar, Pune-411005, India.

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Corresponding Author:
A. N. Bhavale

ABSTRACT

In 1932 Whitney posed the problem that find an ear decomposition of a graph starting with an ear which is a cycle. In 1939 Robbins proved this result for 2-edge connected graphs. In 1986 Laszlo et al. have given an efficient parallel algorithm for constructing ear decomposition of various types of graphs. In this paper, we survey an ear decomposition of several classes of graphs in particular, covering graphs of posets. We will also discuss in detail about the various types of ear decompositions. In the end, we provide a list of some open problems related to ear decomposition of graphs.

KEYWORDS: Graph, Ear decomposition, Posets, Cover graph.

2020 Mathematics Subject Classification: 06A05, 06A06, 06A07.

1. INTRODUCTION

In 1932 Whitney [36] posed the problem that find an ear decomposition of a graph starting with an ear which is a cycle. Whitney found an open ear decomposition for 2-vertex connected graphs, and Robbin [32] found the ear decomposition of 2-edge connected graph. After then Whitney generalizes this result for 3-vertex connected graph as a graph $G = (V,E)$ with $|V| \geq 2$ is a 3-vertex connected if and only if G has non separating ear decomposition. In 1939 Robbin [32] have answered this problem for 2-edge connected graphs, and state that a graph is 2-edge connected if and only if it has an ear decomposition.

In 1939 Robbin [32] introduced the ear decomposition of 2-edge connected graphs a tool for proving the Robbin's theorem that these are exactly the graphs that may be given a strongly connected orientations. Because of pioneering work of Whitney and Robbin on ear decomposition, an ear decomposition is sometimes also called WhitneyRobbin's Synthesis[11]. Lovasz, Maon Schieber, Vishkin, Miller, and Ramchandran [23] provided an efficient parallel algorithm for constructing an ear decomposition of various types of graphs.

Ear decomposition have number of uses, in particular, in computing the connectivity of graphs, strong connectivity of directed graphs. Ear decomposition has the flavor of general search technique in graphs. It arranges the vertices of the graph by partitioning them into paths. This enables further exploration of the graph in an orderly manner. Such a search technique is called an Ear-Decomposition Search(EDS). It is known that Depth-First Search(DFS) and Breadth-First Search(BFS) are main techniques for searching graphs. Ear decomposition may be used to characterize several important graph classes and as a part of efficient graph algorithms. They may also be generalized from graphs to matroid. For more details see [34] . Several important classes of graphs may be characterized as the graphs having certain types of ear decomposition.

On a classical computers ear decomposition of 2-edge connected graphs and open ear decomposition of 2-vertex connected graphs may be found by greedy algorithm that find each ear one at time. The decomposition of ears allows us to systematically remove the nodes of degree greater than two. It is helpful in the context of parallel graph algorithm to increase the available parallelism in the computation and decrease the work required.

2. PRILIMINARIES

The following definitions can be found in Wilson [37] .

Definition 2.1. A *graph* is $G = (V,E)$ is a mathematical structure consisting of two sets V and E. The elements of V are called vertices, and the elements of E are called edges.

A graph G is said to be a *connected* if every two vertices in G is connected by a path.

Definition 2.2. A graph is said to be *2-vertex connected* if minimum 2 vertices are needed to delete in order disconnect the graph.

A graph is said to be *3-vertex connected* if minimum 3 vertices are needed to delete in order disconnect the graph. A graph is said to be *2-edge connected* if minimum 2 edges are needed to delete in order disconnect the graph.

A simple graph on n vertices is said to be a *complete graph* if each pair of distinct vertices are adjacent. It is denoted by K_n. The *degree* of a vertex is the number of edges with that vertex as an end point.

Definition 2.3. A *Planar Graph* is a graph that can be drawn in the plane without crossing, that is no two edges intersect geometrically except at a vertex with which both are incident.

Definition 2.4. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. A subset M of $E(G)$ is called *matching* of G if no two of the edges in M are adjacent.

Definition 2.5. A *perfect matching* in a graph is a subset of its edges with the property that each of its vertices is the endpoint of exactly one of the edges in the subset.

Definition 2.6. A *matching covered graph* is a connected graph in which every edge belongs to at least one perfect matching.

Definition 2.7. *A matroid M* consists of a non-empty finite set E and a non-empty collection B of subsets of E, called *bases*, satisfying the following properties:

(1) no base properly contains another base

(2) if B_1 and B_2 are bases and if e is any element of B, then there is an element f of B_2 such that $(B_1 - \{e\}) \cup \{f\}$ is also a base.

The following definitions are due to Bhavale and Waphare [2] .

Definition 2.8. An *ear* of a loopless connected graph G is a subgraph of G such that it is a maximal path in which all internal vertices are of degree two in G or it is a cycle in which all but one vertex have degree two in G. If G is a cycle itself then that cycle is the only ear of G.

An ear of of a graph G is called an *open ear* if the two end points do not coincide in G.

Definition 2.9. Let G be a loopless connected graph. An *ear decomposition* of G is a partition of its set of edges into a sequence of ears $E_0,E_1,...,E_k$ such that

1) For each i, E_i is a cycle or a path of G

2) $E_0 \cup E_1 \cup ... \cup E_i$ is a connected and having E_i as an ear of $E_0 \cup E_1 \cup ... \cup E_i$ for all $i = 1,2,...,k$. If E_i is a cycle then it is attached to $E_0 \cup E_1 \cup ... \cup E_{i-1}$ by exactly one vertex. If E_i is a path then it is attached to $E_0 \cup E_1 \cup ... \cup E_{i-1}$ by atleast one end vertex. Clearly $G = \cup_{i=0}^{k} E_i$.

An ear decomposition is called *open ear decomposition* if two end points of each ear after the first are distinct from each other. The following definition is due to Whitney [36] .

Definition 2.10. A *non-separating ear decomposition* is an open ear decomposition such that for each vertex V with one exception V has a neighbour whose first appearance in the decomposition is in a later ear than the first appearance of V .

The following definitions are due to Eppistein [6] .

Definition 2.11. *2-terminal series parallel graph* is a graph that may be constructed by a sequence of series and parallel composition starting from a set of a single edge graph K_2 with assigned terminals.

Definition 2.12. A graph is called *series graph* if it is 2-terminal series parallel graph when some two of vertices are regarded as a source and sink.

The following definition is due to Lovasz [19] .

Definition 2.13. A *factor critical graph* is a graph with n vertices in which every induced subgraph of $n - 1$ vertices has a perfect matching.

The following definitions are due to Khuller [14] .

Definition 2.14. A *tree ear decomposition* is a proper ear decomposition $E_0,E_1,...,E_k$ in which each ear E_i should have both its endpoints on the same ear E_j, $j < i$.

Definition 2.15. *Nested ear decomposition* is a tree ear decomposition such that within each ear the set of pairs of end points of other ears that lie within form a set of nested intervals.

The following definition is due to Smith [35] .

Definition 2.16. A graph is of *bounded genus* if you can always find some surface with a finite number of holes on which the graph can be drawn without edge crossings.

3. APPLICATIONS OF EAR DECOMPOSITION

In this section we will discuss in detail about some applications of ear decompositions like searching techniques in graphs, and connectivity of a graph.

3.1. Serching Techniques. The linear time serial algorithm of Lempel et al. [18] for testing planarity of graphs uses the linear time serial algorithm of Even and Tarjan [7] for st-numbering. This st-numbering algorithm is based on depth-first search (DFS). A known conjecture states that DFS, which is a key technique in designing serial algorithms, is not manageable to poly-log time parallelism using around linearly (or even polynomially) many processors. The first contribution of this paper is a general method for searching efficiently in parallel undirected graphs, called ear decomposition search (EDS). The second contribution demonstrates the applicability of this search method. They present an efficient parallel algorithm for st-numbering in a biconnected graph. The algorithm runs in logarithmic time using a linear number of processors on a concurrent-read concurrent-write (CRCW) PRAM.

In 1992 Vijaya Ramchandran [33] deals with a parallel algorithmic technique that has proved to be very useful in the design of efficient parallel algorithms for several problems on undirected graphs. She describe this method for searching undirected graphs, called open ear decomposition, and she relate this decomposition to graph biconnectivity. She present an efficient parallel algorithm for finding this decomposition and relate it to a sequential algorithm based on depth-first search. She then apply open ear decomposition to obtain an efficient parallel algorithm for testing graph triconnectivity, and for finding the triconnnected components of a graph.

In 2016 Pachorkar et al. [31] present a new GPU algorithm for obtaining an ear decomposition of a graph. Their implementation of the proposed algorithm on an NVidia Tesla K40c improves the state-of-the-art by a factor of 2.3x on average on a collection of real-world and synthetic graphs. The improved performance of their algorithm is due to our proposed characterization that identifies edges of the graph as redundant for the purposes of an ear decomposition. Then they study an application of the ear decomposition of a graph in computing the betweenness-centrality values of nodes in the graph. They use an ear decomposition of the input graph to systematically remove nodes of degree two. The actual computation of betweenness-centrality is done on the remaining nodes and the results are extended to nodes removed in the previous step. They show that this approach improves the state-of-the-art for computing betweenness-centrality on an NVidia K40c GPU by a factor of 1.9x on an average over a collection of real-world graphs.

In 2019 Havet and Nisse [12] studied the complexity of deciding whether a graph admits an ear decomposition with prescribed ear lengths. They proved that deciding whether a graph admits an ear decomposition with all ears of length at most l is polynomial-time solvable for all fixed positive integer l. On the other hand, deciding whether a graph admits an ear decomposition without ears of length in F is NP complete for any finite set F of positive integers. They also proved that, for any $k \geq 2$, deciding whether a graph admits an ear decomposition with all ears of length 0 (mod k) is NP complete.

3.2. Connectivity. In 1986 Miller and Ramachandran [28] present and efficient parallel algorithm for ear decomposition and a triconnectivity test based on it. Their algorithm runs in $O(\log m)$ parallel time using $O(n + m)$ processors, where n is the number of vertices and m is number of edges in a graph.

In 1994 Kavvadias et al. [16] shows that how to decompose efficiently in parallel any graph into a number $\bar{\gamma}$ of outerplanar subgraphs (called hammocks) satisfying certain separator properties. They achieve this decomposition in $O(\log n \log \log n)$ time using $O(n + m)$ CREW PRAM processors for an n-vertex m-edge graph. This decomposition provides a general framework for solving graph problems efficiently in parallel. Its value is demonstrated by using it to improve previous bounds for shortest paths and related problems in the class of sparse graphs which includes planar and bounded genus graphs.

In 2000 Kazmierczak and Radhakrishnan [15] present an asynchronous distributed algorithm to determine an ear decomposition of an arbitrary, connected, bidirectional network containing n-nodes and m-links which uses O(m) messages and which can be completed in O(n) time. Using the ear decomposition, they obtain two results for a distributed network as the distributed ear decomposition algorithm can be used to test biconnectivity, determine biconnected components, find cutpoints and bridges using $O(m)$ messages in $O(n)$ time. Also the distributed ear decomposition algorithm can be used to test if a biconnected network is outerplanar using $O(n)$ messages in $O(n)$ time, and if the network is outerplanar, the embedding is also given using the same message and time complexity.

In 2017 Dutta et al. [5] studied an ear decomposition of a graph G is a partition of the edge set of G into a sequence of edge-disjoint paths, such that only the end vertices of each path appear in earlier paths. For a graph on n vertices and m edges, the state-of-art algorithm for obtaining an ear decomposition by Schmidt takes $O(m + n)$ time. They design and implement a new algorithm to obtain an ear decomposition for a biconnected graph, whose running time $O(m + n)$. In practice, however, their experiments reveal that the proposed algorithm runs at least 2 times faster than Schmidts algorithm. The speed increases as the graph gets denser.

In 2017 Dutta et al. [8] studied the applicability of an ear decomposition of graphs to problems such as all-pairs-shortest paths and minimum cost cycle basis. Through experimentation they show that the resulting solutions are scalable in terms of both memory usage and also their speedup over best known current implementations. They believe that their techniques have the potential to be relevant for designing scalable solutions for other computations on large sparse graphs.

In 1999 Franzblau [10] proved that the existence of two natural variations on ear decomposition can be tested in polynomial time. The first is a weak long-ear decomposition in which every ear (path) is either disconnecting or is at least as long as a given bound B. The second is a strong short-ear decomposition, in which every ear is non-disconnecting and has length at most B.

4. TYPES OF EAR DECOMPOSITION

In this section we will discuss in detail about some types of ear decomposition like canonical ear decomposition, optimal ear decomposition etc.

4.1. Canonical Ear Decomposition. In 1983 Lovasz [20] study the technique of ear decomposition of matching covered graphs. Also he proved that a non-bipartite matching covered graph contains K_4 or $K_2 \oplus K_3$ (the triangular prism). Using this result they give a new characterization of those graphs whose matching and covering numbers are equal. Then he apply this results to the theory of 2-critical graphs. Also he proved the canonical ear decomposition theorem stated as Every non-bipartite matching covered graph G has a canonical ear decomposition.

In 1998 Carvalho et al. [26] shows that every matching covered graph G different from K_2 has atleast Δ edge-disjoint removable ears, where Δ is the maximum degree vertex of G. This shows that any matching covered graph G has atleast Δ! different ear decompositions,and thus is a generalization of the fundamental theorem of Lovasz and Plummer [23] establishing the existence of ear decompositions. They also show that every brick G different from K_4 and C_6 has $\Delta - 2$ edges, each of which is a removable edge in G, that is, an edge whose deletion from G results in a matching covered graph. This generalizes a well known theorem of Lovasz. Using this theorem, they give a simple proof of another theorem due to Lovasz, which says that every non-bipartite matching covered graph has a canonical ear decomposition, that is, one in which either the third graph in the sequence is an odd-subdivision of K_4 or the fourth graph in the sequence is an odd subdivision of C_6. This method in fact shows that every non-bipartite matching covered graph has a canonical ear decomposition which is optimal, that is, one which has as few double ears as possible.

In 2005 Carvalho et al. [24] gives main result is an $O(nm)$-time (deterministic) algorithm for constructing an ear decomposition of a matching-covered graph, where n and m denote the number of nodes and edges. The improvement in the running time comes from new structural results that give a sharpened version of Lovsz and Plummer's [22] Two-Ear theorem. Their algorithm is based on $O(nm)$-time algorithms for two other fundamental problems in matching theory, namely, finding all the allowed edges of a graph, and finding the canonical partition of an elementary graph.

In 2020 Hailun Zheng [39] found the first non-octahedral balanced 2-neighborly 3-sphere and the balanced 2-neighborly triangulation of the lens space $L(3,1)$. Each construction has 16 vertices. He proved that there exists a balanced 3-neighborly non-spherical 5-manifold with 18 vertices. Also he proved that the rank-selected subcomplexes of a balanced simplicial sphere do not necessarily have an ear decomposition.

4.2. Optimal Ear decomposition. In 2002 Marcelo et al. [27] studied about optimal ear decomposition. A Petersen brick is a graph whose underlying simple graph is isomorphic to the Petersen graph. For a matching covered graph G, $b(G)$ denote the number of bricks of G, and $p(G)$ denote the number of Petersen bricks of G. An ear decomposition of G is optimal if among all ear decompositions of G, it uses the least possible number of double ears. Marcelo et al. proved that the number of double ears in an optimal ear decomposition of a matching covered graph G is $b(G) + p(G)$. In particular, if G is a brick that is not a Petersen brick, then there is an ear decomposition of G with exactly one double ear. They give an alternative proof of L. Lovsz' matching lattice characterization theorem.

In 2005, Lee et al. [17] studied about WDM-based network as n WDM-based network, a single fiber abortion may cause many logical lightpaths failures such that embedded logical topology of a WDM network may become disconnected. And a huge amount of data of a single link carried is lost by this link errors. For this reason Lee et al. propose an efficient survivable routing approach, which is based on the technique of ear-decomposition, to create protected routing of the embedded logical topology that can withstand a physical link failure. Their approach divides the logical topology into several ears, and restricts the lightpaths of the same ear are routed by using disjoint physical links. To solve the survivable problem is NP-complete, and they formulate the survivable routing problem as an ILP problem based on the results of ear decomposition. And their experiments has shown that the solution generated from their ILP achieves the two results namely a high performance in terms of the survivable routing and a better performance than previous research results for the balance of traffic loads.

4.3. **Odd ear decomposition.** In 2006 B. Szegedy and C. Szegedy [34] studied about matroids. Matroids admitting an odd ear-decomposition can be viewed as natural generalizations of factor-critical graphs. They proved that a matroid representable over a field of characteristic 2 admits an odd ear-decomposition if and only if it can be represented by some space on which the induced scalar product is a non-degenerate symplectic form. They also show that, for a matroid representable over a field of characteristic 2, the independent sets whose contraction admits an odd ear-decomposition form the family of feasible sets of a representable Δ-matroid.

4.4. Convex Ear decomposition. In 2007 R Woodroofe [38] consider the problem of constructing a convex ear decomposition for a poset. The usual technique, introduced by Nyman and Swartz, starts with a CL-labeling and uses this to shell the ears of the decomposition. He axiomatize the necessary conditions for this technique as a CL-ced or EL-ced. He found an EL-ced of the d-divisible partition lattice, and a closely related convex ear decomposition of the coset lattice of a relatively complemented finite group. Along the way, he construct new EL-labelings of both lattices. The convex ear decompositions so constructed are formed by face lattices of hypercubes. He then proceed to show that if two posets P_1 and P_2 have convex ear decompositions (CL-ceds), then their products $P_1 \times P_2$ also have a convex ear decompositions (CL-ceds).

In 2008 Marcelo et al. [25] introduce the concept of combed graphs and present an ear decomposition theorem for this class of graphs. This theorem includes the well known ear decomposition theorem for matching covered graphs proved by Lovasz and Plummer [23]. Then they use the ear decomposition theorem to show that any two edges of a 2-connected combed graph lie in a balanced circuit of an equivalent combed graph. This result generalises the theorem that any two edges in a matching covered graph with at least four vertices belong to an alternating circuit.

4.5. Nested ear decomposition. In 1992 David Eppstein [6] proved that a 2-vertex connected graph is series parallel if and only if it has nested ear decomposition. Also he gives an algorithm for recognizing directed series parallel graphs based on a structural characterization of series parallel graphs in terms of their ear decompositions. This algorithm can recognize undirected as well as directed series parallel graphs.

In 1996 Coullard et al. [3] gives an algorithm for computing an e-based ear decomposition (that is, an ear decomposition of every circuit of which contains element) of a matroid using only a polynomial number of elementary operations and port oracle calls. In the case that matroid M is binary, the incidence vectors of the circuits in the ear decomposition form a matrix representation for M. Thus, this algorithm solves a problem in computational learning theory; it learns the class of binary matroid port (BMP) functions with membership queries in polynomial time.

In 1989 Samir Khuller [14] gives a definition of tree ear decomposition. For more details see[14] . In 1993, Frank [9] proved that it is possible to find the ear decomposition with fewest even ears in any graph. For more details see [9].

In 2024, Tibor Jordn [13] prove that if the two-dimensional rigidity matroid of a graph G on at least seven vertices is connected, and G is minimal with respect to this property, then G has at most $3n - 9$ edges. This bound, which is best possible, extends Dirac's bound [4] on the size of minimally 2-connected graphs to dimension two. The bound also sharpens the general upper bound of Murty [29] for the size of minimally connected matroids in the case when the matroid is a rigidity matroid of a graph. Their proofs rely on ear-decompositions of connected matroids and on a new lower bound on the size of the largest circuit in a connected rigidity matroid, which may be of independent interest. They use these results to determine the tight upper bound on the number of edges in a minimally redundantly rigid graph in two dimensions. Furthermore, as an application of their proof methods, they gives a new proof for Murty's theorem.

5. POSET DISMANTLABLE BY DOUBLY IRREDUCIBLE

In 2020, Bhavale and Waphare [1] introduced the concept of poset dismantlable by doubly irreducible. Also they gives the following definitions.

Definition 5.1. *A partially ordered set* (in short poset) is a set P of elements together with a binary relation $\leq$ on P which is reflexive, antisymmetric and transitive.

Definition 5.2. An element $x \in P$ is an *upper bound* for a subset $S \subset P$ if $s \leq x$ for all $s \in S$.

An *upper cone of* S denoted by S^u is defined as $S^u = \{x \in P \mid s \leq x, \forall s \in S\}$.

Definition 5.3. The least element of S^u is called join of S, denoted by $\vee S$.

An *lower cone of* S denoted by S^l is defined as $S^l = \{x \in P \mid x \leq s, \forall s \in S\}$.

Definition 5.4. The greatest element of S^l is called meet of S, denoted by $\wedge S$.

In particular, $\vee\{a,b\}$ and $\wedge\{a,b\}$ are respectively denoted by $a \vee b$ and $a \wedge b$.

Definition 5.5. A *lattice* is a poset in which every pair of elements has the meet and the join.

Definition 5.6. An element x in a lattice L is called *join-reducible (meet-reducible)* in L, if there exist $y,z \in L$ both distinct from x such that $y \vee z = x (y \wedge z = x)$.

Definition 5.7. An element x in a lattice L is called *join-irreducible (meet-irreducible)* if it is not join-reducible (meet-reducible).

Definition 5.8. An element x in a lattice L is called *doubly irreducible* if it is both join-irreducible and meet-irreducible.

Definition 5.9. An element a of a poset P is called *doubly irreducible* in P if a has at most one upper cover and at most one lower cover in P.

Definition 5.10. A finite lattice L of order n is called *dismantlable* if there exists a chain $L_1 \subset L_2 \subset \ldots \subset L_n (= L)$ of sublattices of L such that $|L_i| = i$ for all i.

Thus a dismantlable lattice is a lattice which can be completely dismantled by removing one element at each stage.

Definition 5.11. An element a of a poset P is called *irreducible* in P if a is an isolated element or a has precisely one upper cover or precisely one lower cover in P.

Definition 5.12. An n-element poset P is called *dismantlable by irreducibles* if there exists a chain $P_1 \subset P_2 \subset \ldots \subset P_n (= P)$ of subposets of P such that P_1 has one element and $P_{i-1} = P_i$ x, where x is an irreducible element in P_i, for all i.

In 2014 Bhavale and Waphare [2] found an ear decompsition of a covering graph of poset by doubly irreducible element. Also gives the following results.

Theorem 5.13. [2] *Let P be a poset dismantlable by doubly irreducibles and G be a covering graph of P which is a tree. Let E be a maximal path in G. Then G has an ear decomposition* $E_0, E_1, \ldots, E_k$ *such that* $E_0 = E$ *and* $G = E_0 \cup E_1 \cup \ldots \cup E_k$.

Theorem 5.14. [2] *Let P be a poset dismantlable by doubly irreducibles and G be a covering graph of P which is connected, loopless and contains a cycle C such that* $E_0 = C$ *and* $G = E_0 \cup E_1 \cup \ldots \cup E_k$.

6. OPEN PROBLEMS

(1) Find an ear decomposition of graph starting with an ear which is a cycle.

(2) Find an ear decomposition of graph starting with an ear which is a path.

(3) Find an ear decomposition of digraph starting with an ear which is a cycle.

(4) Find an ear decomposition of digraph starting with an ear which is a path.

(5) Find an ear decomposition of a covering graph of poset dismantlable by irreducibles starting with an ear which is a cycle.

(6) Find an ear decomposition of a covering graph of poset dismantlable by irreducibles starting with an ear which is a path.

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