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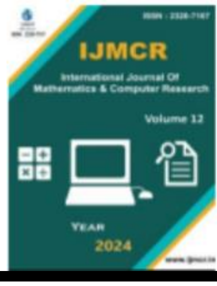

Cordial Labeling on Families of N-Antiprism Graphs

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ABSTRACT
In this paper, the findings contribute to the understanding of cordial labeling in the context of n-antiprism graphs, opening avenuesfor further research in graph theory and combinatorics.

KEYWORDS: Graph labeling, Cordial labeling, Path union, Cycle of graphs, n-antiprism graphs.

I. INTRODUCTION

Over the last 60 years, numerous labeling methods have been developed. The cordial labeling technique, first presented by Cahit[3] in 1987, is one of these labeling methods. Many graphs were proved as cordial. At its core, cordial labeling seeks to assign labels to the vertices and edges of a graph in such a way that promotes balance and harmony. The labels are craftedin a manner that minimizes disturbances and irregularities within the graph, fostering a sense of equilibrium. Consequently, cordial labeling offers an exclusive perspective that enables us to investigate the connections and exchanges that are intrinsic to intricate networks. Cordial labelings of the degree splitting graph of pathways, shells, helms, and gears are provided by Vaidya and Shah [6]. Andar *et al.* [1],[2] demonstrated that the closed helms, flowers, sunflower graphs, and numerous shells are cordial. The cordial nature of torch graphs, their path union, and their cycle was demonstrated by Jeba Jesintha and Subashini [5]. Gallian survey [4] has a comprehensive survey on cordial labeling.

II. PRELIMINARIES

Definition 1. The n - *antiprism graph* is obtained by using two cycles C_n, one is placed inside another in such a way that each vertex of the outer cycle C_n is connected to two adjacent vertices of the inner cycle C_n using edges. See Figure 1.

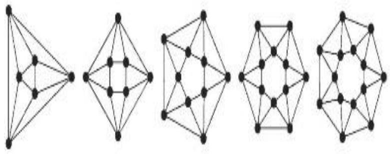

Figure 1: n-antiprism graphs

Definition 2. Let G be a graph and let $G_1, G_2, \dots, G_n, (n \geq 2)$ be n copies of the graph G. Then the new graph obtained by adding an edge between a vertex of G_i and a vertex of G_{i+1}, for $i = 1, 2, \dots (n-1)$ is called the *path union* of $G_1, G_2, \dots, G_n$.

Definition 3. Let $G_1, G_2, \dots, G_n$ be given connected graphs. Then *the cycle of graphs* $C(G_1, G_2, \dots, G_n)$ is the graph obtained by adding an edge joining G_i to G_{i+1}, for $i = 1, 2, \dots, (n-1)$ and an edge joining G_n to G_1. When the n graphs are isomorphic to G then it is denoted as $C(n \cdot G)$.

III. PATH UNION OF N-ANTIPRISM GRAPHS

Theorem 1. The path union of n-antiprism graphs is cordial.

Proof. Let D be a n-antiprism graph. Let $D_1, D_2, \dots, D_t$ be t copies of D. Each copy of the graph is connected by an edge and thus forms the path union of $D_1, D_2, \dots, D_t$ which is dis-played in Figure 2.

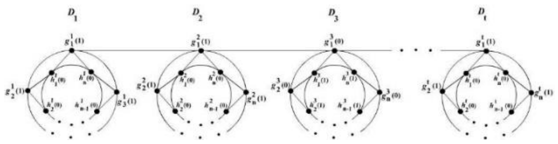

Figure 2: Path union of n - antiprism graph

Let the outer vertices of D_1 be represented as g_1^1, g_2^1, … , g_n^1 the inner vertices of D_1 be represented as h_1^1, h_2^1, … , h_n^1. Let the outer vertices of D_2 be represented as $g_1^2, g_2^2, \ldots, g_n^2$ and the inner vertices of D_2 be represented as $h_1^2, h_2^2, \ldots, h_n^2$. Similarly, let the outer vertices of D_t be represented as $g_1^t, g_2^t, \ldots, g_n^t$ and the inner vertices of D_t be represented as $h_1^t, h_2^t, \ldots, h_n^t$. Thus the outer vertices of the p^{th} copy are denoted as g_i^p and the inner vertices of the p^{th} copy are denoted as h_i^p , for $1 \leq p \leq t$ and $1 \leq i \leq n$.

The following labels are applied to the vertices g_i^p for $1 \leq p \leq t$ and $1 \leq i \leq n$.

$$f(g_i^p) = \begin{cases} 1, & p \equiv 1, 2 \ (mod 4) \\ 0, & p \equiv 0, 3 \ (mod 4) \end{cases}$$

The following labels are applied to the vertices h_i^p for $1 \leq p \leq t$ and $1 \leq i \leq n$.

$$f(h_i^p) = \begin{cases} 1, & p \equiv 1, 2 \ (mod 4) \\ 0, & p \equiv 0, 3 \ (mod 4) \end{cases}$$

Case 1: When odd number of copies of n - antiprism graphs are connected.	**Case 2:** When odd number of copies of n - antiprism graphs are connected.
$v_f(0) = \frac{p}{2}$ and $v_f(1) = \frac{p}{2}$ $\lvert v_f(0) - v_f(1)\rvert = \lvert\frac{p}{2} - \frac{p}{2}\rvert = 0$ $e_f(0) = \frac{q}{2}$ and $e_f(1) = \frac{q}{2}$ $\lvert e_f(0) - e_f(1)\rvert = \lvert\frac{q}{2} - \frac{q}{2}\rvert = 0$	$v_f(0) = \frac{p}{2}$ and $v_f(1) = \frac{p}{2}$ $\lvert v_f(0) - v_f(1)\rvert = \lvert\frac{p}{2} - \frac{p}{2}\rvert = 0$ $e_f(0) = \lceil\frac{q}{2}\rceil$ and $e_f(1) = \lfloor\frac{q}{2}\rfloor$ $\lvert e_f(0) - e_f(1)\rvert = \lvert\lceil\frac{q}{2}\rceil - \lfloor\frac{q}{2}\rfloor\rvert = 1$

From the above two cases we have satisfied the cordiality conditions are satisfied and thereforewe have proved that the path union of n - antiprism graphs is cordial. An illustration is shown in Figure 3.

Ilustration:

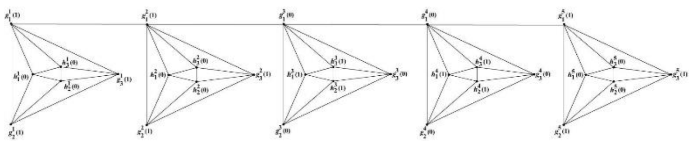

Figure 3: Cordial labeling of graph D when n=3 and t = 5

Corollary 1. The Cycle of n - antiprism graphs is cordial.

Proof. Let D be a n-antiprism graph. Let $D_1, D_2, \ldots, D_t$ be t copies of D. Each copy of the graph is connected by an edge and thus form a cycle of $D_1, D_2, \ldots, D_t$ which is shown in Figure 4. The outer vertices of the p^{th} copy are denoted as g_i^p and the inner vertices of the p^{th} copy are denoted as h_i^p for $1 \leq p \leq t$ and $1 \leq i \leq n$.

The following labels are applied to the vertices g_i^p for $1 \leq p \leq t$ and $1 \leq i \leq n$.

$$f(g_i^p) = \begin{cases} 1, & p \equiv 1, 2 \ (mod 4) \\ 0, & p \equiv 0, 3 \ (mod 4) \end{cases}$$

The following labels are applied to the vertices h_i^p for $1 \leq p \leq t$ and $1 \leq i \leq n$.

$$f(h_i^p) = \begin{cases} 1, & p \equiv 1, 2 \ (mod 4) \\ 0, & p \equiv 0, 3 \ (mod 4) \end{cases}$$

Case 1 : $t \equiv 0(mod\ 4)$	**Case 2:** $t \equiv 1(mod\ 4)$	**Case 3:** $t \equiv 3(mod\ 4)$
$v_f(0) = \frac{p}{2}$ and $v_f(1) = \frac{p}{2} \Rightarrow \lvert v_f(0) - v_f(1)\rvert = \lvert\frac{p}{2} - \frac{p}{2}\rvert = 0$		
$e_f(0) = \frac{q}{2}$ and $e_f(1) = \frac{q}{2}$ $\lvert e_f(0) - e_f(1)\rvert = \lvert\frac{q}{2} - \frac{q}{2}\rvert = 0$	$e_f(0) = \lceil\frac{q}{2}\rceil$ and $e_f(1) = \lfloor\frac{q}{2}\rfloor$ $\lvert e_f(0) - e_f(1)\rvert = \lvert\lceil\frac{q}{2}\rceil - \lfloor\frac{q}{2}\rfloor\rvert = 1$	$e_f(0) = \lceil\frac{q}{2}\rceil$ and $e_f(1) = \lfloor\frac{q}{2}\rfloor$ $\lvert e_f(0) - e_f(1)\rvert = \lvert\lceil\frac{q}{2}\rceil - \lfloor\frac{q}{2}\rfloor\rvert = 1$

Hence, we have proved that the cycle of n - antiprism graphs is cordial by satisfying thecordiality conditions. This is illustrated in Figure 4.

Remark: In this corollary, the cordiality condition $|e_f(0) - e_f(1)| \leq 1$ is not satisfied for $t \equiv 2(mod4)$.

Illustration:

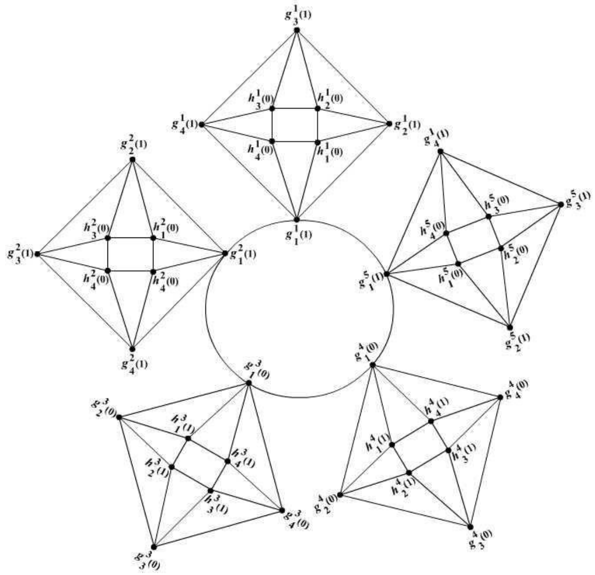

Figure 4: Cycle of 5 copies of 4-antiprism graph is cordial

IV. CONCLUSION

Hence, the path union and the cycle of n - antiprism graphs are proved to be cordial. The path union and cycle have various applications in network design, transportation planning, social network analysis and many more.

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