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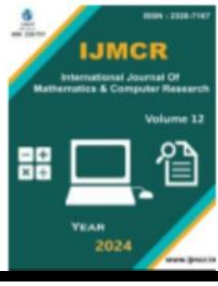

# Almost spgωα -Continuous Functions in Topological Spaces

**M. M. Holliyavar[1], T.D. Rayanagoudar[2] and Sarika M. Patil[3]**

[1]R and D Department, Bharathiar University, Coimbatore- 641046, Tamil Nadu, India

[2,3]Department of Mathematics, Government First Grade College, Rajnagar, Hubli-580 032, Karnataka, India

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Corresponding Author:
**T.D. Rayanagoudar**

**ABSTRACT**

The literature in 1968, almost continuous functions were introduced by Signal and Signal [14]. This paper introduced, a new class of functions called almost spgωα-continuous functions and faintly spgωα-continuous functions in topological spaces using the concept of spgωα-open sets. Authors investigated and introduced several basic properties of faintly spgωα-continuous functions and almost spgωα-continuous functions which are weaker than spgωα-continuous functions.

**KEYWORDS:** spgωα-open sets, spgωα-closed sets, spgωα-continuous functions, faintly spgωα-continuous functions and almost spgωα-continuous functions.

**AIMS Classification:** 54C08, 54C10

## 1. INTRODUCTION AND PRELIMINARIES

A crucial area of discussion in general topology is the concept of continuity. Signal and Signal [14] defined almost continuous functions as generalizations of continuity as weaker and stronger types of continuity. In 1978, Popa [7] generalized Signal's notion of virtually continuity by defining almost quasi continuous functions.

In this paper, a new class of weaker forms of spgωα-continuous functions, known as almost spgωα-continuous functions was introduced using spgωα-open sets. The examination of a new weaker class of functions known as faintly spgωα-continuous, along with various characterizations is covered in the next section. Finally, some essential characteristics of almost spgωα-functions are defined.

Throughout this paper, spaces R and S always means topological spaces (R, τ) and (S, σ) and $f$:(R, τ) → (S, σ) (simply $f$: R → S) denotes a function $f$ of a space (R, τ) into a space (S, σ).

**Definition1.1[10]:** A subset A of a topological space R is called spgωα-closed if spcl(A) ⊆ U whenever A ⊆ U and U is ωα-open in R.

The complement of a spgωα-closed set is called spgωα-open.

**Definition 1.2 [11]:** A function $f$ is said to be spgωα-continuous (spgωα- irresolute) if for every open (resp. spgωα-open) set V in S, $f^{-1}$(V) is spgωα-open in R.

**Definition 2.1.** A function $f$ is called almost continuous [14] (in the sense of Signal) at r ∈R if for every open set V in S containing $f$(r), there U ∈ O(R, r) with (U) ⊂ cl(int(V)).

If $f$ is almost continuous at every point of R, then it is called almost continuous.

## 2. ALMOST spgωα-CONTINUOUS FUNCTIONS

In this section we introduced almost spgωα-continuous functions in topological spaces and study some of their basic properties.

**Definition 2.1**: A function $f$: R → S is said to be almost spgωα-continuous (a.spgωα.C) if for each r ∈ R and V∈O(S, $f$(r)), there exists U ∈spgωα-O(R, r) such that $f$(U) ⊆ int(cl(V)).

**Theorem 2.2**: For a function $f$, the following statements are equivalent:

(i) f is a.spgωα.C.

(ii) for every V ∈ RO(S), $f^{-1}$(V) ∈spgωα-O(R).

(iii) for every F ∈ RC(S), $f^{-1}$(F) ∈spgωα-C(R).

(iv) If A ⊂ R, $f$(spgωα-cl(A)) ⊆ clδ($f$(A)).

(v) If B ⊂ S, spgωα-cl($f^{-1}$(B)) ⊂$f^{-1}$(clδ(B)).

(vi) for every F ∈δ-c(S), $f^{-1}$(F) ∈spgωα-C(R).

(vii) for every V ∈δ-O(S), $f^{-1}$(V) ∈spgωα-O(R).

**Proof.** (i) ⇒ (ii) Suppose V ∈ RO(S) and r ∈$f^{-1}$(V). Then

$f(r) \in V$. As $V \in O(R)$ and $f$ is a.spgωα.C, so $U \in$spgωα-O(R, r) with $f(U) \subset int(cl(V)) = V$. Thus, $r \in U \subset f^{-1}(f(U)) \subset f^{-1}(V)$ and so, $f^{-1}(V) \in$spgωα-O(R).

(ii) ⇒ (v) Let $B \subset S$. Then $f^{-1}(B) \subset S$. By (iv), $f$(spgωα-cl($f^{-1}(B)$)) ⊂cl-δ($f(f^{-1}(B))$) ⊂cl(δ(B)) and so, spgωα-cl($f^{-1}(B)$) ⊂ $f^{-1}$($f$(spgωα-cl($f^{-1}(B)$))) ⊂ $f^{-1}$(cl-δ(B)).

(v) ⇒ (vi) Let $F \in$ δ-C(S), then spgωα-cl($f^{-1}(F)$) ⊂$f^{-1}$(clδ(F)) = $f^{-1}(F)$.

So, spgωα-cl($f^{-1}(F)$) = $f^{-1}(F)$ and hence $f^{-1}(F) \in$spgωα-C(R).

(vi) ⇒ (vii) Let $V \in$ δ-O(S), then S - V ∈ δ-C(S). By hypothesis, $f^{-1}(S - V) \in$spgωα-C(R). Since $f^{-1}(S - V)$= R - $f^{-1}(V)$, we have R - $f^{-1}(V) \in$spgωα-C(R). Thus, $f^{-1}(V)$ ∈spgωα-O(R).

(vi) ⇒ (i) Let r ∈R and V ∈O(S) where $f(r) \in V$. Let us put W = int(cl(V)) and U = $f^{-1}(W)$. As cl(V) is a closed in S, so W = int(cl(V)) ∈ δ-O(S) and from (vii), U = $f^{-1}(W)$ ∈spgωα-O(R). Now, $f(r) \in V = int(V) \subset int(cl(V)) = W$, and so r ∈$f^{-1}(W)$= U, $f$ (U)= $f(f^{-1}(W)) \subset W = int(cl(V))$.

**Proposition 2.3:** Every a.spgωα.C is w.spgωα.C.

**Proof.** Let r ∈R and V ∈O(S) with $f(r) \in V$. As $f$ is a.spgωα.C, there exists U ∈spgωα-O(R) with r ∈ U and $f$ (U) ⊂ int(cl(V)) ⊂ cl(V). Hence, $f$ is w.spgωα.C.

**Theorem 2.4:** For a function $f$, the following statements are equivalent:

(i) $f$ is a.spgωα.C,

(ii) for each r ∈R and V ∈ O(S) containing $f(r)$, there exists U ∈spgωα-O(R, r) with $f(U)$ ⊂s-cl(V),

(iii) for each r ∈R and V ∈ RO(S) containing $f(r)$, there exists U ∈spgωα-O(R, r) with $f(U) \subset V$.

(iv) for each r ∈R and V ∈ δ-O(S) containing $f(r)$, there exists U ∈ spgωα-O(R, r) with $f(U) \subset V$.

**Theorem 2.5:** For a function $f$, the following statements are equivalent:

(i) $f$ is a.spgωα.C,

(ii) $f^{-1}$(int(cl(V))) ∈spgωα-O(R), for every V ∈O(S).

(iii) for every F ∈ C(S), $f^{-1}$(cl(int(F))) ∈spgωα-C(R).

**Proof.** (i)⇒(ii): Let V ∈ O(R). We need to show that $f^{-1}$(int(cl(V))) ∈spgωα-O(R).

Let r ∈$f^{-1}$(int(cl(V))). Then $f$ (r) ∈ int(cl(V)) and int(cl(V)) which is a regular open in S. As $f$ is a.spgωα.C, so U ∈spgωα-O(R, r) with $f(U)$ ⊂ int(cl(V)), that is r ∈ U ⊂ $f^{-1}$(int(cl(V))). In consequence, $f^{-1}$(int(cl(V))) ∈spgωα-O(R).

(ii)⇒(iii): Let F ∈ C(S). Then S - F ∈O(S). From (ii), $f^{-1}$(int(cl(S - F)))) ∈spgωα-O(R) and $f^{-1}$(int(cl(S - F))) = $f^{-1}$(int(S - int(F))) = $f^{-1}$(S - cl(int(F))) = R - $f^{-1}$(int(cl(F))). Hence $f^{-1}$(int(cl(F))) ∈spgωα-C(R).

(iii)⇒(i): Let F ∈RC(S). Then, F ∈C(S). From (iii), $f^{-1}$(cl(int(F))) ∈spgωα-C(R). As F ∈ RC(S), then $f^{-1}$(cl(int(F))) = $f^{-1}(F)$. Therefore, $f^{-1}(F)$ ∈spgωα-C(R). By Theorem 3.2, $f$ is a.spgωα.C.

**Theorem 2.6:** Let $f$ be a.spgωα.C and V ∈ O(S). If r ∈ spgωα-cl(($f^{-1}(V)$) - ($f^{-1}(V)$, then $f$ (r) ∈spgωα-cl(V).

**Proof.** Let r ∈R with r ∈ spgωα-cl(($f^{-1}(V)$) - ($f^{-1}(V)$. Suppose $f(r)$ ∉spgωα-cl(V). Then, H ∈ spgωα-O(S) containing $f(r)$ where H ∩ V = ϕ. So, cl(H) ∩ V = ϕ, and so int(cl(H)) ∩ V = ϕ and int(cl(H)) is a regular open in R. As $f$ is a.spgωα.C, U ∈spgωα-O(R, r) with $f(U)$ ⊂ int(cl(H)). Hence, $f(U)$ ∩ V = ϕ.

However, since r ∈spgωα-cl(($f^{-1}(V)$), U ∩ ($f^{-1}(V)$ = ϕ holds for every U ∈spgωα-O(R, r), so $f(U)$ ∩ V ≠ϕ, we have a contradiction. Then it follows that $f(r)$ ∈spgωα-cl(V).

**Definition 2.7:** Let R be a space. A filter base ∧* is said to be:

(i) spgωα-convergent to a point r in R, if for every U ∈spgωα-O(R, r), there exists B ∈ Λ* with B ⊂ U.

(ii) R-convergent [13] to a point r in R if for every U ∈ RO-(R, r), there exists B ∈ Λ* such that B ⊂ U.

**Theorem 2.8:** If $f$ is a.spgωα.C, then for each r ∈R and filter base Λ* in R is spgωα-converging to r, the filter base $f$(Λ*) is R-convergent to $f(r)$.

**Proof.** Let r ∈R and Λ* be any filter base in R, which is spgωα-converging to r. By Theorem 2.6, for any V ∈RO-(S) containing $f(r)$, there exists U ∈spgωα-O(R, r) with $f(U) \subset V$.

As Λ* is spgωα-converging to r, there exists B ∈ Λ* with B ⊂ U, that is $f(B)$ ⊂V. Hence the filter base $f$(Λ*) is R-convergent to $f(r)$.

**Definition 2.9**: A net ($r_\lambda$) is said to be spgωα-convergent to a point r, if for every V ∈spgωα-O(R, r), there exists an index $\lambda_0$ such that for $\lambda \geq \lambda_0$, $r_\lambda \in V$.

**Theorem 2.10**: If $f$ is a.spgωα.C, then for each point r ∈R and each net ($r_\lambda$) which is spgωα-convergent to r, then the net $f$(($r_\lambda$ )) is R-convergent to $f$ (r).

**Proof.** The proof is similar to that of Theorem 2.7.

**Theorem 2.11**: If $f$ is a.spgωα.C injective and S is r-$T_1$, then R is spgωα-$T_1$.

**Proof.** Suppose S is r-$T_1$. For any distinct points r and s in R, $f(r) \neq f(s)$. There exist V, W ∈O(R) with $f(r) \in V$, $f(s) \notin V$, $f(r) \notin W$ and $f(s) \in W$. As $f$ is a.spgωα.C, $f^{-1}(V)$, $f^{-1}(W)$ ∈spgωα-O(R) with r ∈$f^{-1}(V)$, s ∉$f^{-1}(V)$, r ∉$f^{-1}(W)$ and s ∈$f^{-1}(W)$, which shows that R is spgωα-$T_1$.

**Theorem 2.12:** If $f$ is a.spgωα.C injective and S is r-$T_2$, then R is spgωα-$T_2$.

**Proof.** For any pair of distinct points r and s in R. Then by the injectivity of $f$, $f(r) \neq f(s)$. There exist disjoint U, V ∈ RO-(R) such that $f$ (r) ∈ U and $f(s) \in V$. As $f$ is a.spgωα.C, $f^{-1}(U)$ ∈spgωα-O(R, r) and $f^{-1}(V)$ ∈spgωα-O(R, s). Thus, $f^{-1}(U) \cap f^{-1}(V)$=ϕ, as U ∩ V = ϕ. So R is spgωα-$T_2$.

**Definition 2.13.:** A function $f$ is said to be:
(i) spgωα-irresolute [11] if $f^{-1}(V)$ is spgωα-open in R for every spgωα-open set V of S.

**Definition 2.14:** A topological space R is said to be almost regular [10] if for any F ∈ RC(R) and any point r ∈R - F, there exist disjoint U, V ∈ O(R) such that r ∈ U and F ⊂ V.

**Theorem 2.15:** If $f$ is a w.spgωα.C and S is almost regular, then $f$ is a.spgωα.C.

**Proof.** Let r ∈ R and V ∈O(S, $f$(r)). By almost regularity of S, there exists spgωα ∈ RO(S) with $f$(r) ∈ spgωα ⊂ cl(spgωα) ⊂ int(cl(V)). As $f$ is w.spgωα.C, there exists U∈spgωα-O(R, r) with $f$(U) ⊂ cl(spgωα) ⊂ int(cl(V)). Thus, $f$ is a.spgωα.C.

**Definition 2.16 [10]:** A spgωα-frontier of a A is denoted by spgωα-Fr(A), is defined by spgωα-Fr(A) = spgωα-cl(A) ∩spgωα-cl(R - A).

**Theorem 2.17:** The set of all points r ∈R in which a function $f$ is not a.spgωα.C is identical with the union of spgωα-frontier of the inverse images of regular open sets containing $f$(r).

**Proof.** Suppose $f$ is not a.spgωα.C at r ∈ R. Then there exists V ∈ RO(S) containing $f$(r) such that U ∩ (R - $f^{-1}$(V)) ≠ϕ for every U ∈spgωα-O(R, r). Therefore, r ∈spgωα-cl(R - $f^{-1}$(V)) = R - spgωα-int($f^{-1}$(V)) and r ∈$f^{-1}$(V). Thus, r ∈spgωα-Fr($f^{-1}$(U)).
Conversely, suppose $f$ is a.spgωα.C at r ∈ R and V ∈ RO(S) containing $f$(r). Then there exists U ∈spgωα-O(R, r) such that U ⊂ $f^{-1}$(V), that is r ∈spgωα-int($f^{-1}$(V)). Thus, r ∈ R - spgωα-Fr($f^{-1}$(V)).

**Theorem 2.18:** If $f$ is a.spgωα.C, $f$* is w.spgωα.C with S is Hausdorff, then the set {r ∈ R: $f$(r)= $f$*(r)} is spgωα-closed in R.

**Proof.** Let A = {r ∈ R: $f$(r)= $f$*(r)} and r ∈ R - A. Then $f$(r) ≠ $f$*(r). As S is Hausdorff, there exist V, W ∈ O(S) with $f$(r) ∈ V, $f$*(r) ∈ W and V ∩ W = ϕ. Hence int(cl(V)) ∩ cl(W) = ϕ. Since $f$ is a.spgωα.C, there exists U ∈spgωα-O(R, r) with $f$(U) ⊂ int(cl(V)). As $f$* is w.spgωα.C, there exists H ∈spgωα-O(R) such that $f$*(H) ⊂ cl(W*). Now put U = spgωα ∩ H, then U ∈spgωα-O(R, r) and $f$(U) ∩ $f$*(U) ⊂ int(cl(V)) ∩ cl(W) = ϕ. Therefore, we obtain U ∩ A = ∅ and hence A is spgωα-C(R).

**Theorem 2.19**: Suppose the product of two spgωα-open sets is spgωα-open. If $f_1$: ($R_1$, τ) → (S, σ) is w.spgωα.C, $f_2$: ($R_2$, τ) → (S, σ) is a.spgωα.C and S is Hausdorff, then the set {($r_1$, $r_2$) ∈$R_1$ x $R_2$ : $f_1(r_1)$= $f_2(r_2)$} is spgωα-closed in $R_1$ x $R_2$.

**Proof.** Let A = {($r_1$, $r_2$) ∈$R_1$ x $R_2$ : $f_1(r_1)$ = $f_2(r_2)$}. If ($r_1$, $r_2$) ∈ ($R_1$ x $R_2$) – A, then $f_1(r_1)$ ≠ $f_2$ ($r_2$). As S is Hausdorff, there exist disjoint open sets $V_1$ and $V_2$ in S with $f_1(r_1)$ ∈ $V_1$ and $f_2$ ($r_2$) ∈ $V_2$ and cl($V_1$)∩ int(cl($V_2$)) = ϕ. As $f_1$ (resp. $f_2$) is w.spgωα.C (resp. a.spgωα.C), there exists $U_1$∈spgωα-O($R_1$, $r_1$) such that $f_1(U_1)$ ⊂ cl($V_1$) (resp. $U_2$∈spgωα-O($R_2$, $r_2$) with $f_2$(spgωα-cl($U_1$)) ⊂ int(cl($V_2$))). Hence, ($r_1$, $r_2$) ∈ $U_1$ x $U_2$ ⊂ $R_1$ x $R_2$ – A. Thus, ($R_1$ x $R_2$) – A is spgωα-open and so A is spgωα-closed in $R_1$ x $R_2$.

3. **Faintly spgωα-Continuous Functions**

**Definition 3.1:** A function $f$: R → S is called faintly spgωα-continuous (briefly f.spgωα.C) at a point r ∈ R if for each V ∈ θ-O(S, $f$(r)), there exists U ∈spgωα-O(R, r) such that $f$(U) ⊆ V.
If $f$ has the above property at each point of R, then $f$ is said to be f.spgωα.C.

Theorem 3.2: The following statements are equivalent for a function $f$:
(i) $f$ is f.spgωα.C
(ii) for each V ∈ θ-O(S), $f^{-1}$(V) ∈spgωα-O(R).
(iii) for each F ∈ θ-C(S), $f^{-1}$(F*) ∈spgωα-C(R).
(iv) $f$ is spgωα.C.
(v) for every B ⊆ S, spgωα-cl($f^{-1}$(B)) ⊆ $f^{-1}$(clθ(B)).
(vi) for every A ⊆ S, $f^{-1}$(intθ(A)) ⊆ spgωα-int($f^{-1}$(A)).

**Proof:** (i) → (ii) Let $f$ be f.spgωα.C and V ∈θ-O(S) such that r ∈ $f^{-1}$(V). Then there exists U ∈spgωα-O(R, r) with $f$(U) ⊆V, that is r ∈ U ⊆ $f^{-1}$(V). Thus $f^{-1}$(V) ∈spgωα-O(R).
(ii) → (i) Let r ∈ R and V ∈θ-O(S, $f$(r)). From (ii), $f^{-1}$(V) ∈spgωα-O(R, r). Let U = $f^{-1}$(V), then $f$(U) ⊆ V. Hence $f$ is f.spgωα.C.
(ii) → (iii) Let V ∈θ-C(S), then S-V ∈θ-O(R). From (ii), $f^{-1}$(S-V) = R - $f^{-1}$(V) ∈spgωα-O(R) and hence $f^{-1}$(V) ∈spgωα-C(R).
(iii) → (ii) Let V ∈θ-O(S), then S-V ∈θ-C(S). From (iii), $f^{-1}$(S-V) = R - $f^{-1}$(V) ∈spgωα-C(S) and hence $f^{-1}$(V) ∈spgωα-O(R).
From the definition 3.1, we can prove the other equivalent properties.

**Remark 3.3:** Every spgωα.C is f.spgωα.C.

**Example 3.4:** Let R = {$r_1$, $r_2$, $r_3$} and τ = {R, Φ, {$r_1$}, {$r_2$, $r_3$}} and σ = {S, φ, {$r_1$}, {$r_2$}, {$r_1$, $r_2$}, {$r_2$, $r_3$}}. Then the identity function $f$ is f.spgωα.C but not spgωα.C.

**Definition 3.5:** A function $f$ is said to be weakly spgωα-continuous (briefly w.spgωα.C) if for each point r ∈ R and for each V ∈O(S, $f$(r)), there exists U ∈spgωα-O(R, r) such that $f$ (U) ⊂ cl(V).

**Theorem 3.6:** Every weakly continuous function is f.spgωα.C.

Proof: Let r ∈ R and V ∈ θ-O(S, $f$(r)). Then there exists W ∈ O(S) such that $f$(r) ∈ W ⊂ V, that is $f$(r) ∈ W ⊂ cl(W) ⊂ V. By w.spgωα.C, there exists U ∈spgωα-O(R) such that $f$(U) ⊂ cl(W), that is $f$(U) ⊂ cl(W) ⊂ V. Thus, for each V ∈ θ-O(S, $f$(r)), there exists U ∈spgωα-O(R, r) such that $f$(U) ⊂ V. Hence $f$ is f.spgωα.C.

**Theorem 3.7:** Let $f$ be f.spgωα.C and S is regular space. Then $f$ is spgωα.C.

Proof: Let V ∈ O(S). As S is regular, V ∈ θ-O(S). Since $f$ is f.spgωα.C and from theorem 3.6, $f^{-1}(V)$ ∈spgωα-O(R). Therefore for every V ∈ O(S), $f^{-1}(V)$ ∈spgωα-O(R). Thus $f$ is spgωα.C.

**Theorem 3.8:** Every f.spgωα.C functions is s.spgωα.C.

Proof: Let r ∈ R and V be clopen set in S containing $f$(r). Then, V ∈ θ-O(S). Since $f$ is f.spgωα.C, there exists U ∈spgωα-O(R, r) such that $f$(U) ⊂ V. Thus, for every V ∈ θ-O(S), $f$(U) ⊂ V. Therefore $f$ is s.spgωα.C.

**Definition 3.9:** Let R be TS. Since the intersection of two clopen sets of R is clopen, the clopen sets of R may be use as a base for a topology for R. This topology is called the ultra-regularization of τ and is denoted by τu.

A topological space R is said to be ultra-regular if τ = τu.

**Theorem 3.10:** The following statements are equivalent for a function $f$: R → S, if S is ultra-regular space:

(i) $f$ is spgωα.C

(ii) $f$ is f.spgωα.C

(iii) $f$ is s.spgωα.C.

Proof: It follows from the theorem 3.2, 3.8 and definition 3.9.

**Definition 3.11:** A spgωα-frontier of a subset A of a space R is defined as

spgωα-Fr(A) = spgωα-cl(A) ∩spgωα-cl(R-A).

**Theorem 3.12:** The set of all points r ∈ R in which a function $f$ is not f.spgωα.C is the union of spgωα-frontier of the inverse images of θ-open set containing Ξ (r).

Proof: Suppose $f$ is not f.spgωα.C at each point r ∈ R. Then there exists V ∈θ-O(S, $f$(r)) such that $f$(U) is not contained in V and hence r ∈θ-cl(R − $f^{-1}(V)$).

On the other hand, let r ∈$f^{-1}(V)$ ⊂ spgωα-cl($f^{-1}(V)$) and hence r ∈spgωα-cl($f^{-1}(V)$). Therefore, we can observe that r ∈spgωα-fr($f^{-1}(V)$).

Conversely, assume that $f$ is f.spgωα.C at each point r ∈ R and V ∈θ-O(S, $f$(r)). Then, there exists U ∈spgωα-O(R, r) such that U ⊂ $f^{-1}(V)$. Hence r ∈spgωα-int($f^{-1}(V)$). Therefore r ∉ spgωα-fr($f^{-1}(V)$).

**Theorem 3.13:** Let $f$ be a function and $f$: (R, τ) → (R x S, τ x σ) the graph of $f$ defined by spgωα(x) = (r, $f$(r)) for every r ∈ R. If $f$ is f.spgωα.C then $f$ is f.spgωα.C.

Proof: Let U ∈θ-O(S), then R x U ∈θ-O(R x S). It follows that $f^{-1}(U) = (f)^{-1}(R \times U)$ ∈spgωα-O(R, r). Hence $f$ is f.spgωα.C.

**Theorem 3.14:** Faintly spgωα-continuous image of a spgωα-connected space is connected.

Proof: Assume that S is not connected. Then there exist two non-empty open sets $V_1$ and $V_2$ such that $V_1 \cap V_2 = \varphi$ and $V_1 \cup V_2 = S$. Hence $f^{-1}(V_1) \cap f^{-1}(V_2) = \varphi$ and $f^{-1}(V_1) \cup f^{-1}(V_2) = R$. As $f$ is surjective, $f^{-1}(V_1)$, $f^{-1}(V_2)$ are non-empty subsets of R. Then $V_1$, $V_2$∈θ-O(R), since $V_1$ and $V_2$ are both open and closed. As $f$ is f.spgωα.C, $f^{-1}(V_1)$, $f^{-1}(V_2)$ ∈spgωα-O(R) and hence R is not spgωα-connected which is contradiction to the assumption.

Hence S is connected.

**Theorem 3.15:** If $f$ is f.spgωα.C surjective and R is spgωα-compact then S is θ-compact.

Proof: Let $f$ be f.spgωα.C surjective. Let $\{G_\alpha : \alpha \in \lambda\}$ be any θ-open cover of S. Since $f$ is f.spgωα.C, $f^{-1}(G_\alpha)$ is spgωα-open cover of R. Then there exists a finite subcover $\{f^{-1}(G_i)$ :i = 1, 2, 3, ...} in R, that is $\{G_i$ : i = 1, 2, 3..} is a subfamily which covers the space S. Thus S is θ-compact.

**Theorem 3.16:** Let $f$ be f.spgωα.C, injective function. If

(i) S is θ-$T_1$ then R is spgωα-$T_1$

(ii) S is θ-$T_2$ then R is spgωα-$T_2$

Proof: (i) Let S be θ-$T_1$. Then for any $r_1$, $r_2$ ∈ R with $r_1 \cap r_2 = \varphi$, there exists $V_1$, $V_2$∈θ-O(S) such that $f(r_1) \in V_1$, $f(r_2) \notin V_1$ and $f(r_1) \notin V_1$, $f(r_2) \in V_2$. Then $f^{-1}(V_1)$, $f^{-1}(V_2)$ ∈spgωα-O(R) as $f$ is f.spgωα.C such that $r_1 \in f^{-1}(V_1)$, $r_1 \notin f^{-1}(V_2)$ and $r_2 \notin f^{-1}(V_1)$, $r_2 \in f^{-1}(V_2)$, implies that R is spgωα-$T_1$.

(ii) Let S be θ-$T_2$. Then for any $r_1$, $r_2$∈ R, there exist $V_1$, $V_2$ ∈ θ-O(S) such that $f(r_1) \in V_1$ and $f(r_2) \in V_2$. Then $f^{-1}(V_1)$, $f^{-1}(V_2)$ ∈spgωα-O(R) containing $r_1$ and $r_2$ respectively such that $f^{-1}(V_1) \cap f^{-1}(V_2) = \varphi$ as $V_1 \cap V_2 = \varphi$. Thus R is spgωα-$T_2$.

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