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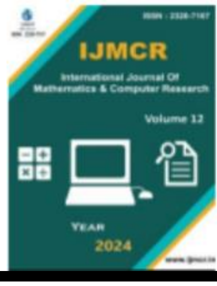

Unveiling Quantum Gates through Graph Theoretical Modelling: Exploring Single - Qubit Rotation Operators and Their Interplay with Linear Graph Theory

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ABSTRACT

This paper implicates the graph theoretic approach for quantum information systems modelling by employing linear graph theoretic modelling to analyse single qubit rotation operator gates. By representing qubit states as nodes and rotation operations at edges, a linear graph framework is developed to systematically examine the interactions and transformations involved in qubit rotation. This study establishes a link between linear graph theory and quantum computing. Through this perspective, the paper contributes to both the fields of linear graph theoretic modelling of general systems and quantum computing, paving the way for new insights and advancements in quantum algorithm design and optimization.

KEYWORDS: Quantum information systems modelling, Linear graph theory, Single-qubit rotation, rotation gates, Interaction and Transformation analysis.

I. INTRODUCTION

Graph theoretic modelling [1, 2] is a versatile tool applicable for mathematical modelling of enormous systems including the domains of computer science, biology, social networks [3] etc. It represents relationship between entities with nodes and edges [1, 2, 3]. In quantum computing [4, 5], graph theoretic modelling aids in analyzing and optimizing quantum circuits by visually representing quantum gates and their interactions [6, 7] which helps in sequencing gates efficiently and exploring error correction strategies. Quantum gates [5, 8] are categorized into Clifford [9, 10, 11, 12, 13] and non – Clifford gates [11, 12], each with distinct properties. Clifford gates are efficiently simulatable and crucial for error correction [11, 12, 13], while non-Clifford gates offer enhanced computational power [11, 12, 13]. Graph theoretic modelling, applied in understanding and modelling qubit rotation gates underneath non-Clifford category, provides a structural approach to analyze and optimize qubit manipulation by representing qubit states at nodes and rotation operation at edges.

An Introduction to Graph Theoretic Modelling of a Qubit:

A qubit encompasses a two-level quantum system that can exist in a coherent superposition of its orthogonal basis states, denoted as $|0\rangle$ and $|1\rangle$. The state of a qubit represented by equation (1) can be described by a complex vector in a two-dimensional Hilbert space [5].

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (1)$$

Where, **"α and β are complex numbers such that $|\alpha|^2$ is the probability of qubits of 'information/computation data' flow rate along unit directional (quantum across variable) vector $|0\rangle$; and $|\beta|^2$ is the probability of qubits of 'information/computation data' flow rate along unit directional (quantum across variable) vector $|1\rangle$."** [7]

The graph theoretic representation of a qubit [7] is given as:

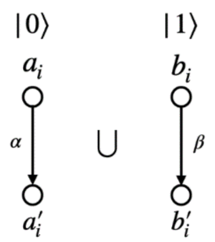

Figure 1: Graph Theoretic Model of a Qubit [7].

For the given graph, the qubit states are given at the nodes whereas the probability parameters α and β are given at the edges and for the given graph, the transfer function matrix is given as, $|\psi\rangle = Y^T\bar{X}$; where, $Y^T = [\alpha \quad \beta]$ and $\bar{X} = \begin{bmatrix}|0\rangle \\ |1\rangle\end{bmatrix}$ [6, 7].

II. GRAPH THEORETIC MODELLING OF OTHER QUBIT NOTATIONS

Hilbert space representation of a qubit [5, 14] is given by equation (2) as,

$$|\psi\rangle = \begin{pmatrix}\cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{\iota\varphi}\end{pmatrix} = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{\iota\varphi}|1\rangle \qquad (2)$$

Where, θ and φ are Euler angles [14].

The graph theoretic modelling for equation (2), illustrating the qubit in terms of Euler angles can be given as:

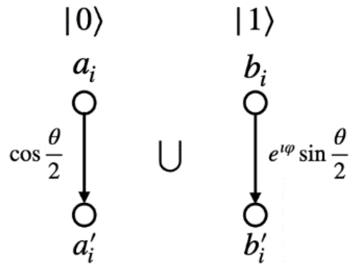

Figure 2- Alternate Graph Theoretic Model of a Qubit in Terms of Euler's Angles Visualizing the Qubit Rotation around the Axes of Bloch Sphere.

For the given graphs, nodes represent the qubit states and rotation operation (given in terms of θ and φ) is given at the edges.

The transfer function matrix for figure 2 can be given as, $|\psi\rangle = Y^T\bar{X}$; where, $Y^T = \left[\cos\frac{\theta}{2} \quad e^{i\varphi}\sin\frac{\theta}{2}\right]$ and $\bar{X} = \begin{bmatrix}|0\rangle \\ |1\rangle\end{bmatrix}$.

Modelling of a qubit in Two Orthogonal x - Basis States

Rotation of a qubit in the x-basis state is given as:

$$|+\rangle_x = \begin{pmatrix}\cos\frac{\theta}{2}e^{-\iota\frac{\varphi}{2}} \\ \sin\frac{\theta}{2}e^{\iota\frac{\varphi}{2}}\end{pmatrix} = e^{-\iota\frac{\varphi}{2}}\cos\frac{\theta}{2}|0\rangle + e^{\iota\frac{\varphi}{2}}\sin\frac{\theta}{2}|1\rangle \qquad (3)$$

$$|-\rangle_x = \begin{pmatrix}-\sin\frac{\theta}{2}e^{-\iota\frac{\varphi}{2}} \\ \cos\frac{\theta}{2}e^{\iota\frac{\varphi}{2}}\end{pmatrix} = -\sin\frac{\theta}{2}e^{-\iota\frac{\varphi}{2}}|0\rangle + e^{\iota\frac{\varphi}{2}}\cos\frac{\theta}{2}|1\rangle \; (4)$$

The graph theoretic modelling for these states is illustrated through figure 3.

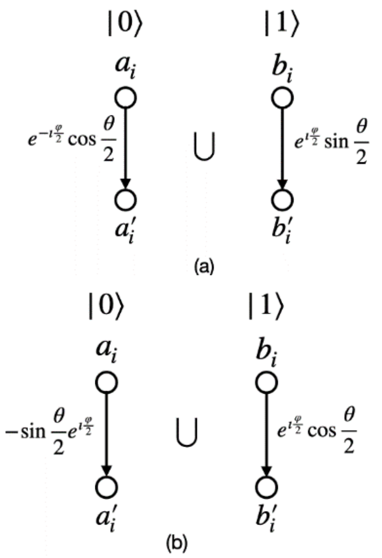

Figure 3 - Graph theoretical modelling of (a) Qubit state $|+\rangle_x$; (b) Qubit State $|-\rangle_x$.

For the given graphs, the transfer function matrices are given as, $|+\rangle_x = Y^T\bar{X}$;

where, $Y^T = \left[\cos\frac{\theta}{2}e^{-\iota\frac{\varphi}{2}} \quad \sin\frac{\theta}{2}e^{\iota\frac{\varphi}{2}}\right]$ and $\bar{X} = \begin{bmatrix}|0\rangle \\ |1\rangle\end{bmatrix}$

$$\text{Therefore, } |+\rangle_x = \left[\cos\frac{\theta}{2}e^{-\iota\frac{\varphi}{2}} \quad \sin\frac{\theta}{2}e^{\iota\frac{\varphi}{2}}\right]\begin{bmatrix}|0\rangle \\ |1\rangle\end{bmatrix} \qquad (5)$$

Similarly, $|-\rangle_x = Y^T\bar{X}$;

Where, $Y^T \left[-\sin\frac{\theta}{2}e^{-\iota\frac{\varphi}{2}} \quad \cos\frac{\theta}{2}e^{\iota\frac{\varphi}{2}}\right]$ and $\bar{X} = \begin{bmatrix}|0\rangle \\ |1\rangle\end{bmatrix}$

$$\text{Thus, } |-\rangle_x = \left[-\sin\frac{\theta}{2}e^{-\iota\frac{\varphi}{2}} \quad \cos\frac{\theta}{2}e^{\iota\frac{\varphi}{2}}\right]\begin{bmatrix}|0\rangle \\ |1\rangle\end{bmatrix} \qquad (6)$$

Modelling of a qubit in Two Spinor Basis States

Spinor basis states are essential quantum representations describing the intrinsic angular momentum, or spin of particles. In quantum mechanics, these states, such as $|\uparrow\rangle$ and $|\downarrow\rangle$ for spin-1/2 particles, encapsulate the possible orientations of spin along a given axis [14, 15]. Spinor basis states are foundational in quantum physics, underpinning phenomena like magnetic resonance and enabling advancements in quantum information processing [14,15].

Qubit representation in two spinor basis states is expressed through equations (7) and (8) as [14],

$$\xi(\uparrow) = e^{\iota\frac{\varphi}{2}}|+\rangle = \begin{pmatrix}cos\frac{\theta}{2} \\ e^{\iota\varphi}\,sin\frac{\theta}{2}\end{pmatrix} = cos\frac{\theta}{2}|0\rangle + sin\frac{\theta}{2}e^{\iota\varphi}|1\rangle$$

(7)

$$\xi(\downarrow) = e^{-\iota\frac{\varphi}{2}}|-\rangle = \begin{pmatrix}-e^{-\iota\varphi}\,sin\frac{\theta}{2} \\ cos\frac{\theta}{2}\end{pmatrix} = -sin\frac{\theta}{2}e^{-\iota\varphi}|0\rangle + cos\frac{\theta}{2}|1\rangle \qquad (8)$$

The Graph theoretic modelling of these Spinor basis states of a qubit are illustrated through figure 4.

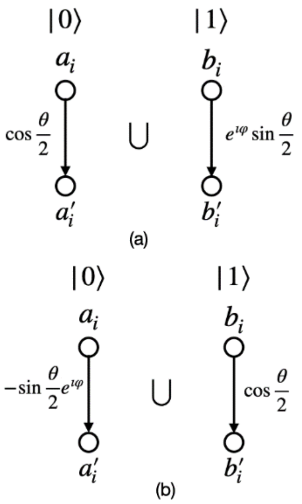

Figure 4 - Graph theoretical modelling of (a) Spinor basis state ξ(↑); (b) Spinor basis state ξ(↓).

For the modelled graphs, the transfer function matrices can be written as, $\xi(\uparrow) = e^{\iota\frac{\varphi}{2}}|+\rangle = Y^T\bar{X}$;

where, $Y^T = \begin{bmatrix}\cos\frac{\theta}{2} & e^{\iota\varphi}\sin\frac{\theta}{2}\end{bmatrix}$ and $\bar{X} = \begin{bmatrix}|0\rangle \\ |1\rangle\end{bmatrix}$

therefore, $\xi(\uparrow) = \begin{bmatrix}\cos\frac{\theta}{2} & e^{\iota\varphi}\sin\frac{\theta}{2}\end{bmatrix}\begin{bmatrix}|0\rangle \\ |1\rangle\end{bmatrix}$ (9)

and similarly, $\xi(\downarrow) = e^{-\iota\frac{\varphi}{2}}|-\rangle = Y^T\bar{X}$;

where, $Y^T = \begin{bmatrix}-e^{-\iota\varphi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{bmatrix}$ and $\bar{X} = \begin{bmatrix}|0\rangle \\ |1\rangle\end{bmatrix}$

and therefore, $\xi(\downarrow) = \begin{bmatrix}-e^{-\iota\varphi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{bmatrix}\begin{bmatrix}|0\rangle \\ |1\rangle\end{bmatrix}$ (10)

III. GRAPH THEORETIC MODELLING OF SINGLE QUBIT ROTATIONS

Single-qubit gates, the building blocks of quantum circuits, manipulate qubit information [5, 10, 14, 16]. General unitary transformations [5, 10, 14] provide the foundation for these manipulations. Three key gates, Rx, Ry, and Rz [5, 14], control rotations on the Bloch sphere (qubit state representation). Notably, any unitary transformation can be built from these [5]. The graph theoretic models of these gates viz., Rx, Ry, Rz gates have been illustrated in this section.

A. Graph Theoretic Modelling of R_x Gate

The R_x gate [5, 14] is a pivotal single-qubit quantum gate that rotates the qubit state around the x-axis of the Bloch sphere, allowing for precise control over the quantum state of a qubit. Represented by a parameterised angle, the R_x gate enables the creation of superposition states and facilitates the implementation of quantum algorithms [16]. Its significance extends to quantum error correction, quantum simulations, and quantum optimisation tasks [16].

The matrix representation of general single qubit rotation is given as [5, 14]:

$$R_x = \begin{pmatrix} cos\frac{\theta}{2} & -\iota\, sin\frac{\theta}{2} \\ -\iota\, sin\frac{\theta}{2} & cos\frac{\theta}{2}\end{pmatrix} \tag{11}$$

From equations (2) and (11), the net effect of R_x Gate on a qubit $|\psi\rangle$ can be calculated as,

$$R_x|\psi\rangle = \begin{pmatrix} cos\frac{\theta}{2} & -\iota\, sin\frac{\theta}{2} \\ -\iota\, sin\frac{\theta}{2} & cos\frac{\theta}{2}\end{pmatrix}\begin{pmatrix} cos\frac{\theta}{2} \\ sin\frac{\theta}{2}e^{\iota\varphi}\end{pmatrix}$$

$$= \begin{pmatrix} cos^2\frac{\theta}{2} - \iota e^{\iota\varphi}\, sin^2\frac{\theta}{2} \\ -\iota\, sin\frac{\theta}{2}cos\frac{\theta}{2} + e^{\iota\varphi}\, cos\frac{\theta}{2}sin\frac{\theta}{2}\end{pmatrix} = \begin{pmatrix} m \\ n\end{pmatrix} \tag{12}$$

$$R_x|\psi\rangle = (cos^2\frac{\theta}{2} - \iota e^{\iota\varphi}\, sin^2\frac{\theta}{2})|0\rangle + (-\iota\, sin\frac{\theta}{2}cos\frac{\theta}{2} + e^{\iota\varphi}\, cos\frac{\theta}{2}sin\frac{\theta}{2})|1\rangle \tag{13}$$

Or, $R_x|\psi\rangle = m|0\rangle + n|1\rangle$ (14)

Where, $\boldsymbol{m = cos^2\frac{\theta}{2} - \iota e^{\iota\varphi} sin^2\frac{\theta}{2}}$ (let)

and $\boldsymbol{n = -\iota sin\frac{\theta}{2}cos\frac{\theta}{2} + e^{\iota\varphi}cos\frac{\theta}{2}sin\frac{\theta}{2}}$ (let).

The inferences are summarised through the truth table given in table 1.

Table 1: Truth Table for R_x Gate

Input	Output
$|0\rangle$	$\cos\frac{\theta}{2}|0\rangle - \iota\sin\frac{\theta}{2}|1\rangle$
$|1\rangle$	$-\iota\sin\frac{\theta}{2}|0\rangle + \cos\frac{\theta}{2}|1\rangle$
$\cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{\iota\varphi}|1\rangle$ Or, $a|0\rangle + b|1\rangle$	$(\cos^2\frac{\theta}{2} - \iota e^{\iota\varphi}\sin\frac{\theta}{2})|0\rangle + (-\iota\sin\frac{\theta}{2}\cos\frac{\theta}{2} + e^{\iota\varphi}\cos\frac{\theta}{2}\sin\frac{\theta}{2})|1\rangle$ Or, $m|0\rangle + n|1\rangle$

These conclusions, mentioned in the table 1 can be graph theoretically modelled as,

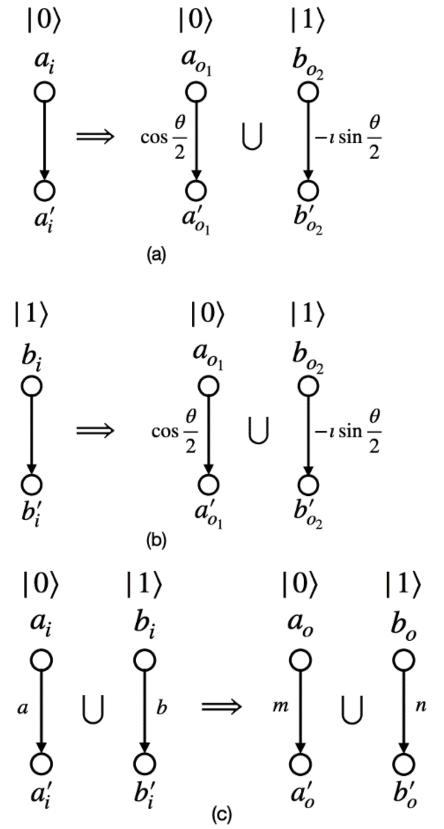

Figure 5 - Graph Theoretic Modelling for the Effect of Rx Gate on
(a) Qubit State $|0\rangle$, (b) Qubit State $|1\rangle$, (c) Qubit $|\psi\rangle$.

In the given graphs, nodes represent the qubit states (being through vector) and edges represent the net rotation effect on a qubit provided by the R_x gate.

For the graphs given in figure 5, the transfer function matrices are given as,

For the input end, $|\psi\rangle = Y_i^T \overline{X}_i = [a \quad b]\begin{bmatrix}|0\rangle \\ |1\rangle\end{bmatrix}$ (15)

Where, $Y_i = \begin{bmatrix}a \\ b\end{bmatrix}$; $\overline{X}_i = \begin{bmatrix}|0\rangle \\ |1\rangle\end{bmatrix}$; $a = \cos\frac{\theta}{2}$; $b = \sin\frac{\theta}{2} e^{\iota\varphi}$

For the output end, $R_x|\psi\rangle = Y_o^T \overline{X}_o = [m \quad n]\begin{bmatrix}|0\rangle \\ |1\rangle\end{bmatrix}$ (16)

Where, $Y_o = \begin{bmatrix}a \\ b\end{bmatrix}$; $\overline{X}_o = \begin{bmatrix}|0\rangle \\ |1\rangle\end{bmatrix}$;

And, $m = \cos^2\frac{\theta}{2} - \iota e^{\iota\varphi}\sin^2\frac{\theta}{2}$; $n = -\iota\sin\frac{\theta}{2}\cos\frac{\theta}{2} + e^{\iota\varphi}\cos\frac{\theta}{2}\sin\frac{\theta}{2}$

Thus, the input – output relationship equation can be given as, $Y_o = [R_x]Y_i$. (17)

B. Theoretic Modelling of R_y Gate

The R_y gate is a foundational single-qubit quantum gate which provides a rotation operation of a qubit state around the y - axis of the Bloch sphere [5, 14]. The matrix representation of general single qubit rotation is given as [5, 14]:

$$R_y = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \quad (18)$$

Employing equations (2) and (17), the net effect of R_y Gate on a qubit $|q\rangle$ can be calculated as,

$$R_y|\psi\rangle = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}\begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} e^{\iota\varphi} \end{pmatrix} = \begin{pmatrix} \cos^2\frac{\theta}{2} - e^{\iota\varphi}\sin^2\frac{\theta}{2} \\ \sin\frac{\theta}{2}\cos\frac{\theta}{2} + \cos\frac{\theta}{2} e^{\iota\varphi}\sin\frac{\theta}{2} \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix} \quad (19)$$

$$R_y|\psi\rangle = (\cos^2\frac{\theta}{2} - e^{\iota\varphi}\sin^2\frac{\theta}{2})|0\rangle + (\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \cos\frac{\theta}{2} e^{\iota\varphi}\sin\frac{\theta}{2})|1\rangle \quad (20)$$

Or, $R_y|\psi\rangle = p|0\rangle + q|1\rangle$ (21)

Where, $\boldsymbol{p = \cos^2\frac{\theta}{2} - e^{\iota\varphi}\sin^2\frac{\theta}{2}}$ (let) and $\boldsymbol{q = \sin\frac{\theta}{2}\cos\frac{\theta}{2} + \cos\frac{\theta}{2} e^{\iota\varphi}\sin\frac{\theta}{2}}$ (let).

These inferences can be shown through the truth table given as table 2.

Table 2: Truth Table for R_y Gate

Input	*Output*
$\|0\rangle$	$\cos\frac{\theta}{2}\|0\rangle + \sin\frac{\theta}{2}\|1\rangle$
$\|1\rangle$	$-\sin\frac{\theta}{2}\|0\rangle + \cos\frac{\theta}{2}\|1\rangle$
$\cos\frac{\theta}{2}\|0\rangle + \sin\frac{\theta}{2} e^{\iota\varphi}\|1\rangle$ Or, $a\|0\rangle + b\|1\rangle$	$(\cos^2\frac{\theta}{2} - e^{\iota\varphi}\sin^2\frac{\theta}{2})\|0\rangle + (\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \cos\frac{\theta}{2} e^{\iota\varphi}\sin\frac{\theta}{2})\|1\rangle$ Or, $p\|0\rangle + q\|1\rangle$

As mentioned in table 2, the conclusions can be graph theoretically modelled as,

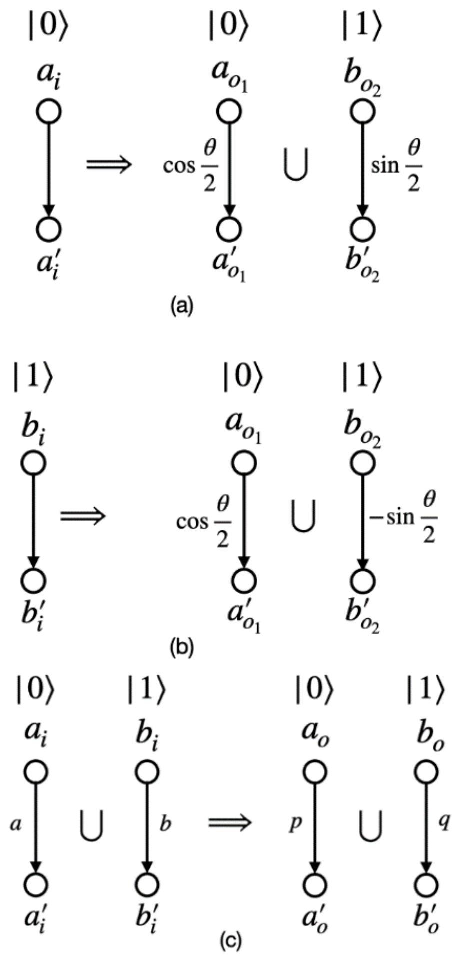

Figure 6 - Graph Theoretic Modelling for the Effect of Ry Gate on
(a) Qubit State $|0\rangle$, (b) Qubit State $|1\rangle$, (c) Qubit $|\psi\rangle$.

In the given graphs, nodes represent the qubit states (being through vector) and edges represent the net rotation effect on a qubit provided by the R_y gate.

For the graphs, shown in figure 6, the transfer function matrices are given as,

For the input end, $|\psi\rangle = Y_i^T \bar{X}_i = [p \quad q]\begin{bmatrix}|0\rangle\\ |1\rangle\end{bmatrix}$ (22)

Where, $Y_i = \begin{bmatrix}a\\ b\end{bmatrix}$; $\bar{X}_i = \begin{bmatrix}|0\rangle\\ |1\rangle\end{bmatrix}$; $a = \cos\frac{\theta}{2}$; $b = \sin\frac{\theta}{2}e^{\iota\varphi}$

For the output end, $R_y|\psi\rangle = Y_o^T \bar{X}_o = [p \quad q]\begin{bmatrix}|0\rangle\\ |1\rangle\end{bmatrix}$ (23)

Where, $Y_o = \begin{bmatrix}a\\ b\end{bmatrix}$; $\bar{X}_o = \begin{bmatrix}|0\rangle\\ |1\rangle\end{bmatrix}$;

And, $p = \cos^2\frac{\theta}{2} - \iota e^{\iota\varphi}\sin^2\frac{\theta}{2}$; $q = \sin\frac{\theta}{2}\cos\frac{\theta}{2} + e^{\iota\varphi}\cos\frac{\theta}{2}\sin\frac{\theta}{2}$.

Thus, the input – output relationship equation can be given as, $Y_o = [R_y]Y_i$. (24)

C. *Graph Theoretic Modelling of R_z Gate*

The R_z gate is a fundamental single-qubit quantum gate that holds a crucial role in quantum computing [5, 14]. By rotating qubit states around the z-axis of the Bloch sphere, the R_z gate enables precise control over qubit quantum states [5, 14]. Parameterised by an angle, this gate serves as a versatile tool for creating superposition states and executing diverse quantum algorithms [16].

The matrix representation of general single qubit rotation is given as [5, 14]:

$$R_z = \begin{pmatrix} e^{-\iota\frac{\theta}{2}} & 0 \\ 0 & e^{\iota\frac{\theta}{2}} \end{pmatrix} \quad (25)$$

By equations (2) and (25), the net effect of R_z Gate on a qubit $|q\rangle$ can be calculated as,

$$R_z|\psi\rangle = \begin{pmatrix} e^{-\iota\frac{\theta}{2}} & 0 \\ 0 & e^{\iota\frac{\theta}{2}} \end{pmatrix}\begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{\iota\varphi} \end{pmatrix} = \begin{pmatrix} e^{-\iota\frac{\theta}{2}}\cos\frac{\theta}{2} \\ e^{\iota\left(\frac{\theta+2\varphi}{2}\right)}\sin\frac{\theta}{2} \end{pmatrix}$$

$$= \begin{pmatrix} r \\ s \end{pmatrix} \quad (26)$$

$$R_z|\psi\rangle = \left(e^{-\iota\frac{\theta}{2}}\cos\frac{\theta}{2}\right)|0\rangle + \left(e^{\iota\left(\frac{\theta+2\varphi}{2}\right)}\sin\frac{\theta}{2}\right)|1\rangle \quad (27)$$

Or, $R_z|\psi\rangle = r|0\rangle + s|1\rangle$ (28)

Where, $\mathbf{r} = \mathbf{e}^{-\iota\frac{\theta}{2}}\mathbf{cos}\frac{\theta}{2}$ (let) and $\mathbf{s} = \mathbf{e}^{\iota\left(\frac{\theta+2\varphi}{2}\right)}\mathbf{sin}\frac{\theta}{2}$ (let).

These inferences are shown as a truth table, in table 3.

Table 3: Truth Table for R_z Gate

Input	*Output*
$\|0\rangle$	$e^{-\iota\frac{\theta}{2}}\|0\rangle$
$\|1\rangle$	$e^{\iota\frac{\theta}{2}}\|1\rangle$
$\cos\frac{\theta}{2}\|0\rangle + \sin\frac{\theta}{2}e^{\iota\varphi}\|1\rangle$ Or, $a\|0\rangle + b\|1\rangle$	$\left(e^{-\iota\frac{\theta}{2}}\cos\frac{\theta}{2}\right)\|0\rangle + \left(e^{\iota\left(\frac{\theta+2\varphi}{2}\right)}\sin\frac{\theta}{2}\right)\|1\rangle$ Or, $r\|0\rangle + s\|1\rangle$

Modelled graphs for the conclusions mentioned in the table 3 are given in figure 7 as,

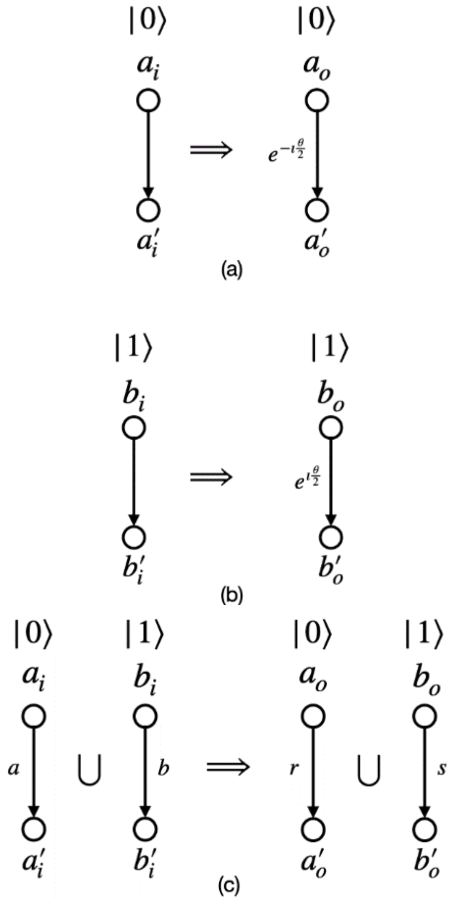

Figure 7 - Graph Theoretic Modelling for the Effect of Rz Gate on
(a) Qubit State |0⟩, (b) Qubit State |1⟩, (c) Qubit |ψ⟩.

In the given graphs, nodes represent the qubit states (being through vector) and edges represent the net rotation effect on a qubit provided by the R_z gate.

For the graphs given in figure 5, the transfer function matrices are given as,

For the input end, $|\psi\rangle = Y_i^T\bar{X}_i = [a \quad b]\begin{bmatrix}|0\rangle\\|1\rangle\end{bmatrix}$ (29)

Where, $Y_i = \begin{bmatrix}a\\b\end{bmatrix}$; $\bar{X}_i = \begin{bmatrix}|0\rangle\\|1\rangle\end{bmatrix}$; $a = \cos\frac{\theta}{2}$; $b = \sin\frac{\theta}{2}e^{\iota\varphi}$

For the output end, $R_z|\psi\rangle = Y_o^T\bar{X}_o = [r \quad s]\begin{bmatrix}|0\rangle\\|1\rangle\end{bmatrix}$ (30)

Where, $Y_o = \begin{bmatrix}a\\b\end{bmatrix}$; $\bar{X}_o = \begin{bmatrix}|0\rangle\\|1\rangle\end{bmatrix}$;

And, $r = e^{-\iota\frac{\theta}{2}}\cos\frac{\theta}{2}$; $s = e^{\iota\left(\frac{\theta+2\varphi}{2}\right)}\sin\frac{\theta}{2}$

Thus, the input – output relationship equation can be given as, $Y_o = [R_z]Y_i$. (31)

IV. EFFECT OF SINGLE QUBIT ROTATION GATES ON VON- NEUMANN ENTROPY

Von Neumann entropy [4, 17] serves as a fundamental concept in quantum information theory, quantifying the uncertainty or randomness associated with the quantum state of a system [4, 17, 18]. It provides a measure of the information content within a quantum state, highlighting the intricacies of quantum entanglement and the nature of quantum correlations [4, 17, 18]. This entropy, rooted in the density matrix formalism, plays a pivotal role in deciphering the properties of quantum systems and has far-reaching applications in quantum cryptography, quantum computing, and quantum thermodynamics [18].

The output states of single qubit rotation operator gates collapse to either |0⟩ or |1⟩ qubit states along with a phase provided by the respective rotation gate. Thus, the state of a qubit remains unchanged and due to which there is no change in the von Neumann entropy as a cause of single qubit rotation operation.

V. CONCLUSION

In conclusion, this paper demonstrates the interdisciplinary approach for systematic visualization of single qubit rotation gates by employing graph theoretic modelling where, for the modelled graphs, nodes represent qubit states and qubit rotations are given at the edges. By connecting the fields of graph theoretic approach for systems modelling and quantum computing, new possibilities in the theoretic modelling of the quantum information science can be explored.

VI. FUTURE SCOPE

Linear graph theory applied to qubit rotations opens exciting avenues for future research. Extending this approach to multi-qubit systems is crucial. This could unlock the understanding of entanglement dynamics and complex gate interactions in larger quantum circuits. Additionally, optimizing gate sequences and qubit connectivity using advanced graph algorithms could improve quantum algorithm performance. Furthermore, exploring the connection between graph theory and non-rotational gates, like entangling gates, could reveal new insights into broader quantum phenomena. Machine learning integration with graph theory might also lead to innovative circuit analysis and optimization strategies. As quantum technologies progress, the synergy between linear graph theory and quantum computing will likely shape a dynamic research landscape, driving advancements in both quantum algorithms and theoretical understanding.

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