[Volume 4 issue 10 October 2016]
PageNo.1643-1649 ISSN : 2320-7167

INTERNATIONAL JOURNAL OF MATHEMATICS AND COMPUTER RESEARCH

Gaps in Successive Primes till 1 Trillion

Neeraj Anant Pande
[1]Associate Professor, Department of Mathematics & Statistics,
Yeshwant Mahavidyalaya, Nanded – 431602,
Maharashtra, INDIA
napande@gmail.com

Abstract:
In this work, gaps between successive primes are analyzed. The range-wise distinct gap values, maximum gap, its occurrence frequency, the minimum gap, its occurrence frequency, gap occurring more and lesser times are analyzed for all primes in the range of 1 to 1 trillion.

Keywords: Prime numbers, successive primes, gaps.

Introduction:
Prime number p is a positive integer greater than 1 that is divisible only by ±1 and ±p. It is known from long that they are irregularly distributed amongst positive integers. There have been consistent efforts towards better understanding of occurrences of primes in natural numbers. The distribution of primes till 10^{12} in all arithmetic progressions $an + b$ with single digit a as well as for first double digit a have been exhaustively analyzed [9] to [22].

It is strongly conjectured that there are infinite number of twin primes, i.e., primes with gap of 2 only. The bounded gaps between successive primes is estimated recently in [23] as $\liminf_{n\to\infty}(p_{n+1} - p_n) < 7\times10^7$, where p_n denotes n^{th} prime.
It is proved fact that there are arbitrary gaps between successive primes. In fact, in a recent development [1], it is prove that there exist pairs of consecutive primes less than x whose difference is larger than $\frac{t(1+O(1))(\log x)(\log\log x)(\log\log\log\log x)}{(\log\log\log x)^2}$ for a fixed constant t. This again guarantees that there are as large (even) gaps between successive primes as needed within large ranges.

We continue this endeavor by analyzing gaps in successive primes in range as high as 1 trillion. The generation of primes till such high extent was possible by analyzing many algorithms and choosing the best amongst them possible due to their exhaustive comparison [2] to [8].

1. Distinct Gaps in Primes in Ranges of 1 – 10^n

As many as 255 different gaps are shown amongst themselves by successive primes in the range of 1 to 1 trillion. The gradual increase in the appearance of these different gaps is determined.

Sr. No.	Range of Numbers	Number of Distinct Gaps Between Successive Primes
1.	1 – 10	3
2.	1 – 100	5
3.	1 – 1,000	10
4.	1 – 10,000	19
5.	1 – 100,000	34
6.	1 – 1,000,000	52
7.	1 – 10,000,000	75
8.	1 – 100,000,000	97
9.	1 – 1,000,000,000	130
10.	1 – 10,000,000,000	167
11.	1 – 100,000,000,000	209
12.	1 – 1,000,000,000,000	255

Table 1: Distinct Gaps between Successive Primes in Increasing Ranges

The number of distinct gaps that appear in successive primes goes on increasing with increasing 10 power ranges and the extent of this rise is as depicted in the following figure.

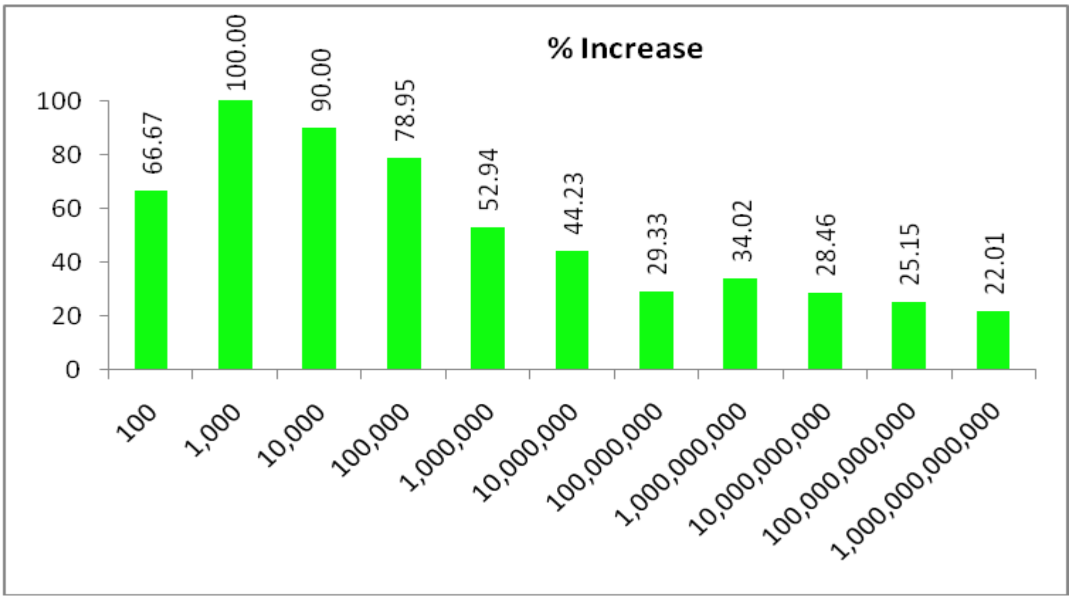

Figure 1: Percentage Increase in Number of Distinct Gaps between Successive Primes in Increasing Ranges

In this procedure, the first prime of the successive pair is considered even if the next one falls in higher range.

2. Early Appearance of Large Distinct Gaps than Small Ones

This section deals with distinct gap values between pairs of successive primes in range of $1 – 10^{12}$.
Out of 255 distinct gaps 191 gaps have appeared before first appearance of their smaller values till 1 trillion. The first gap to appear before smaller values is 14, which came before 2 gaps of 10 and 12 smaller than it. Gap 14 comes first between primes 113 & 127; while smaller gaps of 10 and 12 come first between 139 &

149 and 199 & 211, respectively.

The list of the gap values that have preceded the smaller gaps is as follows :

14, 18, 20, 22, 24, 34, 40, 42, 44, 48, 50, 52, 54, 58, 60, 62, 68, 72, 76, 78, 82, 84, 86, 90, 96, 98, 100, 104, 106, 110, 112, 114, 118, 120, 122, 126, 128, 130, 132, 136, 138, 146, 148, 152, 154, 160, 162, 164, 168, 170, 172, 174, 176, 178, 180, 182, 184, 188, 190, 192, 196, 198, 202, 204, 206, 208, 210, 212, 214, 216, 218, 220, 222, 230, 232, 234, 236, 238, 240, 242, 244, 246, 248, 250, 252, 258, 260, 262, 266, 268, 270, 272, 274, 276, 280, 282, 284, 286, 288, 290, 292, 296, 300, 302, 304, 306, 308, 310, 312, 318, 320, 322, 324, 326, 330, 332, 336, 338, 340, 342, 344, 346, 348, 350, 352, 354, 356, 358, 360, 364, 366, 372, 374, 376, 378, 380, 382, 384, 386, 390, 392, 394, 396, 398, 400, 402, 404, 406, 408, 410, 412, 414, 416, 418, 420, 424, 426, 428, 430, 432, 434, 438, 440, 444, 446, 448, 450, 454, 456, 458, 460, 462, 464, 468, 474, 476, 478, 480, 484, 486, 490, 494, 496, 498, 500, 504, 514, 516, 532, 534, 540

Sr. No.	Number of Precedences	Number of Distinct Lager Gaps Exhibiting These Precedences Over Distinct Smaller Gaps
1.	1	40
2.	2	25
3.	3	24
4.	4	11
5.	5	9
6.	6	13
7.	7	10
8.	8	13
9.	9	5
10.	10	5
11.	11	5
12.	12	4
13.	13	4
14.	14	4
15.	15	1
16.	16	4
17.	17	2
18.	18	1
19.	19	1
20.	20	2
21.	22	2
22.	24	2
23.	25	1
24.	30	1
25.	36	1
26.	41	1

Table 2: Lager Prime Distinct Gap Value Precedences Over Smaller Ones till 1 Trillion

Minimum & Maximum Gaps between Successive Primes

4.1 Minimum Gap Between Successive Primes

Out of these gaps, gap 1 is really unique universally. No other prime pair except the first one of 2 and 3 has gap of 1 and it is solo case. The gap of 1 cannot occur again between successive primes as gap of 1 means successive integer, out of successive integer, one is even and other is odd and even integer other than 2 cannot be prime.

4.2 Maximum Gap Between Successive Primes

The maximum gap between successive primes till one trillion is 540. For earlier ranges, the maximum gap values are found to be increasing with the increasing ranges.

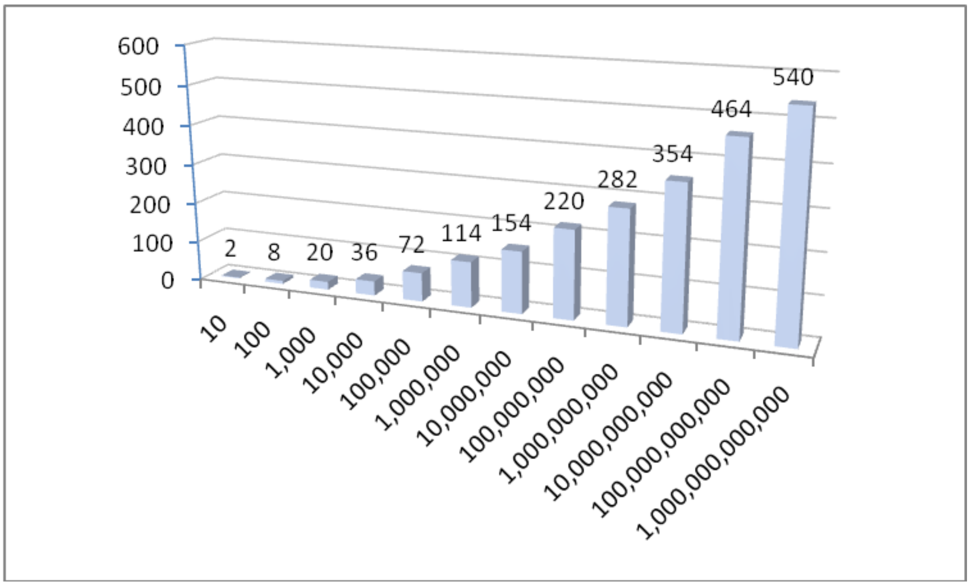

Figure 2: Maximum Gaps between Successive Primes in Increasing Ranges

3. Gap Counts between Successive Primes

Out of as many as 255 distinct gaps between successive prime pairs till one trillion, 8 gaps appear only once, within our range. They are 1, 490, 492, 496, 514, 532, 534, and 540. Out of these, gap 1 is really unique universally. No other prime pair except the first one of 2 and 3 have gap of 1 and it is solo case. For other values, outside our range, they do appear more than once.

As is well-known, barring first exception, there cannot be odd gaps between successive primes; so only even gaps appear between them. It is conjectured that there is always a successive prime pair with each even gap between its members. Till one trillion, all even gap from 2 to 540 appear except the following ones :

482, 488, 502, 506, 508, 510, 512, 518, 520, 522, 524, 526, 528, 530, 536, 538

Of course, these gaps do appear between successive primes, but for those greater than 1 trillion. In the graph below, on the horizontal axis, those gap values like odd ones greater than 1 and above mentioned even ones are omitted as there are no successive primes with those many gaps in between them.

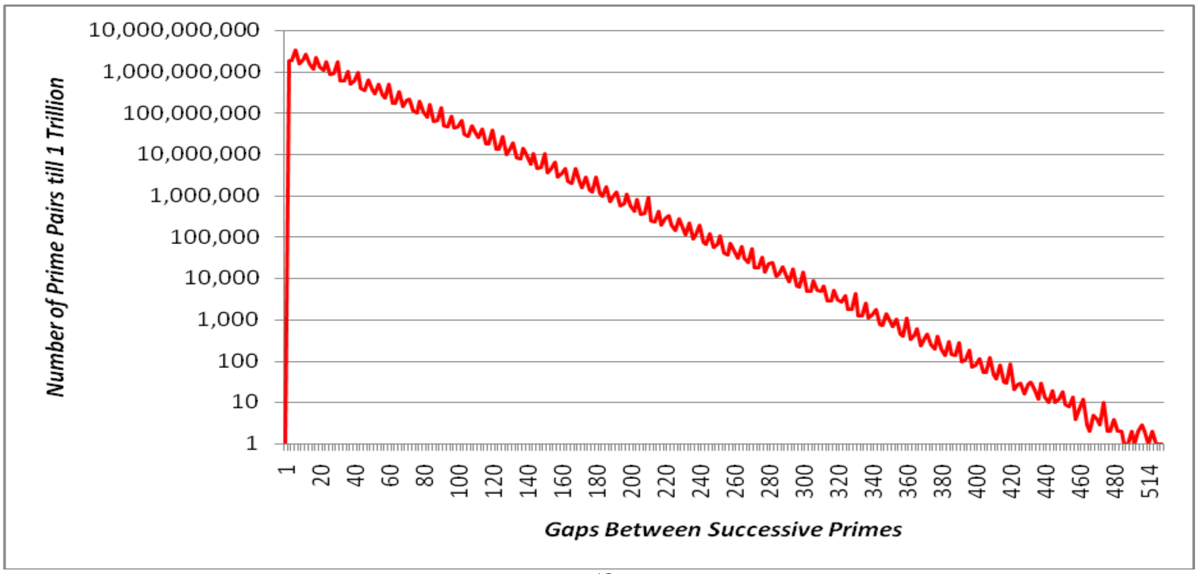

Figure 3: Number of Prime Pairs till 10^{12} with Different Gaps in between Them.

Till one trillion, gap 6 appears most frequently, as many as 3,435,528,229 times, followed by gap 12 which comes 2,753,597,777 times and then by gap 18 which is seen 2,246,576,317 times. Number of gaps with frequencies in different ranges are determined to be as follows :

	1 – 10	30
1.	1 – 100	56
2.	1 – 1,000	81
3.	1 – 10,000	108
4.	1 – 100,000	134
5.	1 – 1,000,000	159
6.	1 – 10,000,000	186
7.	1 – 100,000,000	213
8.	1 – 1,000,000,000	241
9.	1 – 10,000,000,000	255

4. First Primes with Different Gaps with Successive Primes

Barring those gap values which just don't appear till 1 trillion, the first prime in the first prime pair with different gaps give following pattern.

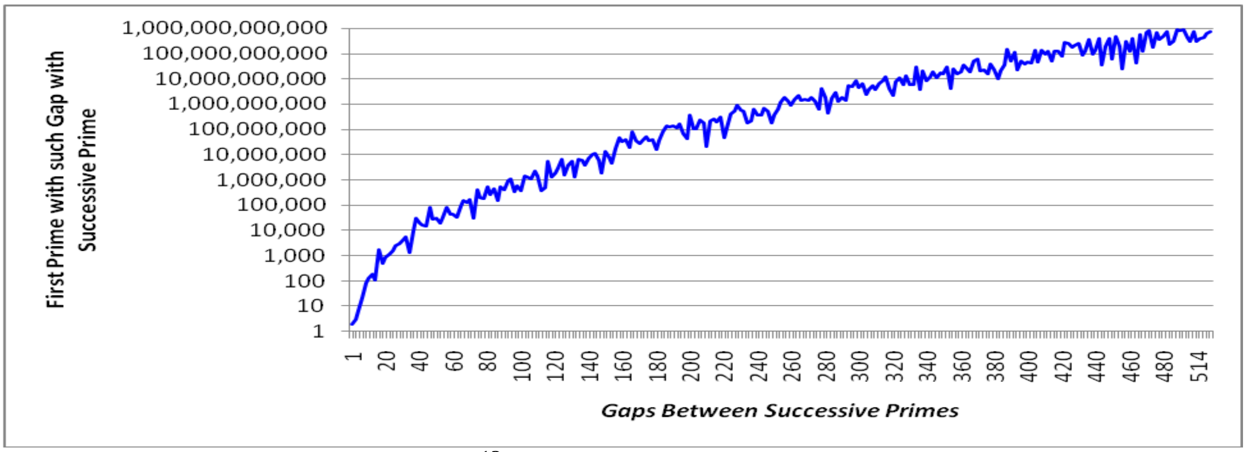

Figure 4: First Primes till 10^{12} with Different Gaps with their Successor Primes.

5. Last Primes with Different Gaps with Successive Primes

And again excepting gap values which don't appear till 1 trillion, the last prime in the first prime pair with different gaps have been determined.

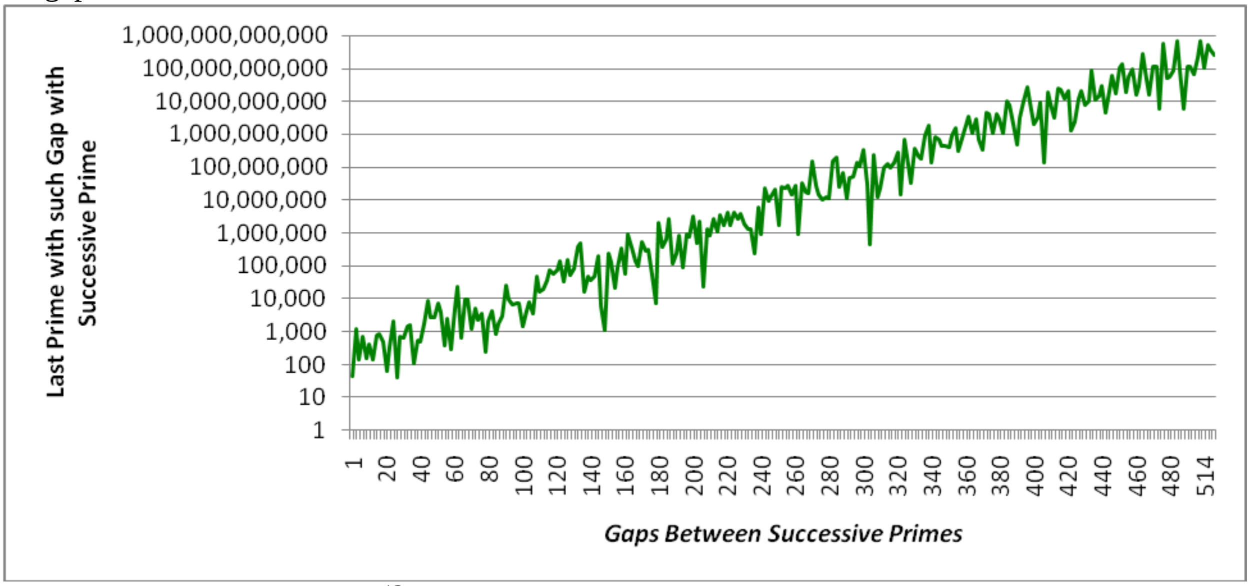

Figure 5: Distance from 10^{12} of Last Prime with Different Gaps with their Successor Primes.

Graphs in both Sections 7 & 8 are clear indicators of a supposed property that successive primes with higher gaps in between them are rare compared to those with lesser gaps in them. But these observations are within range of 1 to 1 trillion, which cannot be taken to be all-indicative as there are infinite primes and their distribution is not completely formulated yet to enough precision. But this study shows trends in their initial distribution through analysis of gaps between them.

Acknowledgements

The author is thankful to the anonymous referees of this paper.

References

[1] James Maynard, "Large Gaps between Primes", Ann. of Math, 183 (3), pp 915–933, 2016.

[2] Neeraj Anant Pande, "Evolution of Algorithms: A Case Study of Three Prime Generating Sieves", Journal of Science and Arts, Year 13, No.3(24), pp. 267-276, 2013.

[3] Neeraj Anant Pande, "Algorithms of Three Prime Generating Sieves Improvised Through Nonprimality of Even Numbers (Except 2)", International Journal of Emerging Technologies in Computational and Applied Sciences, Issue 6, Volume 4, pp. 274-279, 2013.

[4] Neeraj Anant Pande, "Algorithms of Three Prime Generating Sieves Improvised by Skipping Even Divisors (Except 2)", American International Journal of Research in Formal, Applied & Natural Sciences, Issue 4, Volume 1, pp. 22-27, 2013.

[5] Neeraj Anant Pande, "Prime Generating Algorithms through Nonprimality of Even Numbers (Except 2) and by Skipping Even Divisors (Except 2)", Journal of Natural Sciences, Vol. 2, No.1, pp. 107-116, 2014.

[6] Neeraj Anant Pande, "Prime Generating Algorithms by Skipping Composite Divisors", International Journal of Computer Science & Engineering Technology, Vol. 5, No. 09, pp. 935-940, 2014.

[7] Neeraj Anant Pande, "Improved Prime Generating Algorithms by Skipping Composite Divisors and Even Numbers (Other Than 2)", Journal of Science and Arts, Year 15, No.2(31), pp. 135-142, 2015.

[8] Neeraj Anant Pande, "Refinement of Prime Generating Algorithms", International Journal of Innovative Science, Engineering & Technology, Vol. 2 Issue 6, pp. 21-24, 2015.

[9] Neeraj Anant Pande, "Analysis of Primes Less than a Trillion", International Journal of Computer Science & Engineering Technology (ISSN: 2229-3345), Vol. 6, No. 06, pp. 332 – 341, 2015.

[10]Neeraj Anant Pande, “Analysis of Primes in Arithmetical Progressions $3n + k$ up to a Trillion”, IOSR Journal of Mathematics, Volume 11, Issue 3 Ver. IV, pp. 72-85, 2015.
[11]Neeraj Anant Pande, “Analysis of Primes in Arithmetical Progressions $4n + k$ up to a Trillion”, International Journal of Mathematics and Computer Applications Research, Vol. 5, Issue 4, pp. 1-18, 2015.
[12]Neeraj Anant Pande, “Analysis of Primes in Arithmetical Progressions $5n + k$ up to a Trillion”, Journal of Research in Applied Mathematics, Volume 2, Issue 5, pp. 14-29, 2015.
[13]Neeraj Anant Pande, “Analysis of Primes in Arithmetical Progressions $6n + k$ up to a Trillion”, International Journal of Mathematics and Computer Research, Volume 3, Issue 6, pp. 1037-1053, 2015.
[14]Neeraj Anant Pande, “Analysis of Primes in Arithmetical Progressions $7n + k$ up to a Trillion”, International Journal of Mathematics and Its Applications, Volume 4, 2-A, pp. 13-30, 2016.
[15]Neeraj Anant Pande, “Block-wise Distribution of Primes less than a Trillion in Arithmetical Progressions $8n + k$”, IOSR Journal of Mathematics, Volume 12, Issue 3, Ver. V, pp. 79-87, 2016.
[16]Neeraj Anant Pande, “Spacings Between and Units & Tens Place Digits in Primes till One Trillion in Arithmetical Progressions $8n + k$”, American International Journal of Research in Science, Technology, Engineering and Mathematics, 15 (1), pp. 1 - 7, 2016.
[17]Neeraj Anant Pande, “Block-wise Density Distribution of Primes less than a Trillion in Arithmetical Progressions $9n + k$”, International Journal of Advances in Mathematics and Statistics, 1 (2), pp. 13 – 23, 2016.
[18]Neeraj Anant Pande, “Spacings Between and Units & Tens Place Digits in Primes till One Trillion in Arithmetical Progressions $9n + k$”, International Journal of Mathematics and Statistics Invention, Volume 4, Issue 5, pp 13-20, 2016.
[19]Neeraj Anant Pande, “Block-wise Density Distribution of Primes less than a Trillion in Arithmetical Progressions $10n + k$”, Journal of Research in Applied Mathematics, Volume 2, Issue 9, pp. 10-18, 2016.
[20]Neeraj Anant Pande, “Spacings Between and Units & Tens Place Digits in Primes till One Trillion in Arithmetical Progressions $10n + k$”, International Journal of Engineering Maths and Computer Science, Vol. 4, No. 7, 2016.
[21]Neeraj Anant Pande, “Block-wise Density Distribution of Primes less than a Trillion in Arithmetical Progressions $11n + k$”, International Journal of Recent Research in Mathematics, Computer Science and Information Technology, Vol. 3, Issue 1, pp. 70-81, 2016.
[22]Neeraj Anant Pande, “Spacings Between and Units & Tens Place Digits in Primes till One Trillion in Arithmetical Progressions $11n + k$”, International Journal of Mathematics and Computer Research, Volume 4, Issue 6, pp. 1493 – 1501, 2016.
[23]Yitang Zhang, “Bounded Gaps between Primes”, Annals of Mathematics, 179 (3), pp. 1121–1174, 2014.

Author Profile

Dr. Neeraj Anant Pande received the B.Sc., M.Sc. and Ph.D. degrees in Mathematics from Swami Ramanand Teerth Marathwada University, Nanded. In 2000, he worked as lecturer in Mahatma Gandhi Mission’s College of Engineering, Nanded. During 2001-2004, he worked as lecturer in NES Science College, Nanded. From 2004 onwards, he is with Yeshwant Mahavidyalaya (College), Nanded and is now working there as Associate Professor in Mathematics.