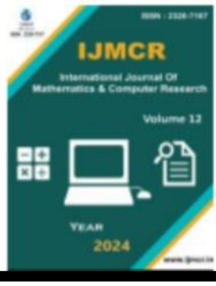
International Journal of Mathematics and Computer Research

ISSN: 2320-7167

Volume 12 Issue 05 May 2024, Page no. – 4226-4230

Index Copernicus ICV: 57.55, Impact Factor: 8.316

DOI: 10.47191/ijmcr/v12i5.06



Multiset Topological Space as Generalization of the Classical Topological Space via Support Set

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ARTICLE INFO	
Published Online:	
14 May 2024	
Corresponding Author:	
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ABSTRACT

Multiset as a genelization of the classical set has triggered the definition and introduction of some algebraic structures in classical set theory under multiset context such as multiset topological spaces. In this paper we defined a root (support) set of a multiset topological space and established that multiset topological spaces are a generalizations classical topological spaces via their root sets respectively.

KEYWORDS: classical set, multiset, topological space, multiset topological space, rootset

1.0 INTRODUCTION

The notion of multiset (mset, for short) is well established both in mathematics and in computer science. In mathematics, a mset is considered to be the generalization of a set. In classical set theory, a set is a well-defined collection of distinct objects. If repeated occurrences of any object are allowed in a set, then a mathematical structure, that is known as mset is obtained.

Research on the theory of msets has been gaining ground. The research carried out so far shows a strong analogy in the behaviour of classical sets and msets. It is possible to extend some of the main notion and results of sets to the setting of msets ([11] [12], [13], [14], [15], etc).

Girish and John [2], introduced the concept of topological spaces in the context of msets (called an M-topological space).

Mahanta and Das [16], studied the Semi Compactness in mset Topological space, by considering their properties. They introduced the concepts of semi open and semi closed msets in mset topological spaces.

Mahanta and Das [17], also introduced the concepts of exterior and boundary in mset topological space. They further established a few relationships between the concepts of boundary, closure, exterior and interior of a mset.

Sobhy. A. El-Sheikh et al [7] introduced the notion of Hausdorff topological space (T2 space) in mset context. Some related results were also studied. The notion of mset bitopological spaces also introduced and studied ([7]). The concepts of ij-pre-open msets, ij-α-open msets, ij-semi-open msets, and ij-β-open msets were further presented.

In this paper, we introduced the concept of Hausdorff bitopological space, root (support) set of mset topological space and established that a mset topological space and its submset space are generalizations of a classical topological space and its subspace via root set respectively. Thus, we presented some basic definitions, notations and some related results in section two, while in section three, we defined the submset space and root set of a mset topological space, Hausdorff mset topological space, mset bitopological space and Hausdorff mset bitopological space respectively and established that these submset spaces of mset topological spaces are generalizations of subspace of the classical topological spaces via root set. In section four, we present the summarized version of our findings.

2.0 BASIC DEFINITIONS AND NOTATIONS

Definition 2.1[1] **(mset):** A mset A drawn from the set X is represented by a count function m_A or C_A defined as $C_A: X \longrightarrow \mathbb{N}$, where $\mathbb{N}$ is the set of non-negative integers.

Here $C_A(x)$ is the number of occurrences of the element x in the mset A. The number $C_A(x)$ is assumed unique from known areas of application.

We present the mset A drawn from the se $X = \{x_1, x_2, x_3, \ldots, x_n\}$ as

$A = \{m_1/x_1, m_2/x_2, m_3/x_3, \ldots, m_n/x_n\}$ Where m_i is the number of occurrences of the

Element x_i, $i = 1,2,3, \ldots, n$ in the mset A.

One of the most natural and simplest examples is the mset of prime factors of a positive integer n. The number 504 has the factorization.
$504 = 2^3 3^2 7^1$ which gives the mset:
$X = [2,2,2,3,3,7] = \{3/2, 2/3, 1/7\}$
where $C_X(2) = 3, C_X(3) = 2, C_X(7) = 1$.
However, those elements which are not included in the mset A have zero count. $C_A(x) = 0 \leftrightarrow x \notin A$ and $C_A(x) > 0 \leftrightarrow x \in A$

Definition 2.2. Let A be a mset drawn from the set X. The root (support) set of A denoted A^* is defined:
$A^* = \{x \in X: C_A(x) > 0\}$.
Note that $x \in A^* \leftrightarrow x \in A$ for all x.

Definition 2.3[1] **(Cardinality of an mset):** The cardinality of an mset A denoted $|A|$ is the sum of the multiplicities of all the elements in A. i.e $|A| = \sum_{x \in X} C_A(x)$

Definition 2.4[10] **(Finite mset)**: A mset A is said to be finite if it has a finite number of distinct elements, and each element has a finite number of occurrences. i.e $|A| < \infty$

Definition 2.5[1] A domain X, is defined as a set of elements from which msets are drawn. We denote the mset space $\mathfrak{M}(X)$ as the set of all finite msets whose elements are in X. i.e If $X = \{x_1, x_2, \dots, x_n\}$, then
$\mathfrak{M}(X) = \{\{m_1/x_1, m_2/x_2, m_3/x_3, \dots, m_n/x_n\}\}$
$x_i \in X, i = 1,2,3, \dots, n$ and $m_i < \infty$.
Note that $M^* \in \mathfrak{M}(X)$ for which,
$C_{M^*}(x) = n \leftrightarrow n = 1$([3]).

Definition 2.6[1] **(mset relations)**.
Let $M, N \in \mathfrak{M}(X)$. Then
i. **(Equality)** $M = N$ if $C_M(x) = C_N(x)\ \forall x \in X$.
ii. (submset) $M \Subset N$
if $C_M(x) \leq C_N(x)\ \forall x \in X$

Definition 2.7[10] (mset operations):
i. **(mset union)** $P = M \uplus N$
if $C_P(x) = Max\{C_M(x), C_N(x)\}\ \forall x \in X$.
ii. (mset intersection) $P = M \Cap N$ if
$C_P(x) = Min\{C_M(x), C_N(x)\} \forall x \in X$
iii. **(mset addition)** $P = M \oplus N$ if
$C_P(x) = C_M(x) + C_N(x) \forall x \in X$
iv. **(mset Difference)** $P = M \ominus N$ if
$C_P(x) = Max\{C_M(x) - C_N(x), 0\}\ \forall x \in X$.
v. (mset arithmetic multiplication) $P = M \odot N$ iff $C_P(x) = C_M(x).C_N(x)\ \forall x \in X$.
vi. (mset raising to arithmetic power)
$P = M^n$ iff $C_P(x) = C_{M^n}(x) = (C_M(x))^n$.
vii. (mset scalar multiplication) $P = kM$ iff $C_P(x) = kC_M(x)$ all $x \in X$ and
$k \in \{1,2, \dots\}$.

Definition 2.8[1] **(empty mset):** Let $M \in \mathfrak{M}(X)$. If $C_M(x) = 0\ \forall x \in X$. Then, M is called empty mset and is denoted by ϕ. i.e, $C_\emptyset(x) = 0\ \forall x \in X$.

Definition 2.9[2] **(whole submset):** Let $M, N \in \mathfrak{M}(X)$. A submset N of M is a whole submset if
$C_N(x) = C_M(x)\ \forall x \in X$.

Definition 2.10[2] **(partial whole submset):**
Let $M, N \in \mathfrak{M}(X)$. A submset N of M is a partial whole submset if there exists $x \in X$ such that $C_N(x) < C_M(x)$.

Definition 2.11[2] **(full submset)**:
Let $M, N \in \mathfrak{M}(X)$. A submset N of M is a full submset of M if $M^* = N^*$

Definition2.12[3] **(power whole mset):**
Let $M \in \mathfrak{M}(X)$. The power whole mset of M denoted by $PW(M)$ is defined as the set of all whole submsets of M. The cardinality of $PW(M)$ is 2^n where n is the cardinality of the support set (root set) of M.

Definition 2.13[2]**(power full mset)**: Let $M \in \mathfrak{M}(X)$ be an mset. The power full mset of M denoted by $PF(M)$ is defined as the set of all full submsets of M. The cardinality of $PF(M)$ is the product of the counts of the elements in M i.e $|PF(M)| = \prod_{x \in X} C_M(x)$

Remark 2.14[2] $PW(M)$ and $PF(M)$ are ordinary sets whose elements are msets.

Definition 2.15[2] (power mset):
Let $M \in \mathfrak{M}(X)$ be an mset. The power mset $P(M)$ of M is the mset of all submsets of M.
The power set of an mset is the support set of the power mset and is denoted by $P^*(M)$.
Note that $P^*(M) \neq P(M^*)$.

Definition2.16[8] **(Topological space):** A topological space is an ordered pair (X, τ), where X is a nonempty set, τ a collection of subsets of X satisfying the following:

- $\phi, X \in \tau$
- Arbitrary union of elements of τ is in τ . that is $\{U_\alpha | \alpha \in I\}$ implies $\cup_{\alpha \in I} U_\alpha \in \tau$.
- Finite intersection of elements of τ is in τ. That is $U, V \in \tau$ implies $U \cap V \in \tau$.

Note that the elements of τ satisfying the above conditions are called open sets.

Definition 2.17[8] (**Subspace of a topological space)**: Let (X, τ) be a topological space, and $A \subseteq X$ be an arbitrary subset. Then the ordered pair (A, τ_A) such that
$\tau_A = \{U | U = A \cap V, V \in \tau\}$
is called subspace of the topological space (X, τ)

Definition1.18[6]
(Hausdorff topological space): Let (X, τ) be a topological space, then (X, τ) is said to be Hausdorff topological space if

for any $x, y \in X$ such that $x \neq y$, there exist $U, V \in \tau$ with $x \in U, y \in V$ such that
$U \cap V = \phi$.

Definition 2.19 [5] **(Bitopological Space)**:
Let τ_1 and τ_2 any two arbitrary topologies defined on a non-empty set X. Then the ordered triple (X, τ_1, τ_2) is called a bitopological space.

Definition2.20
(subspace of a bitopological space): Let (X, τ_1, τ_2) be bitopological space, and $Y \subseteq X$, then the ordered triple $(Y, \tau_{1Y}, \tau_{2Y})$ such that
$\tau_{1Y} = \{U: U = Y \cap V, V \in \tau_1\}$ and
$\tau_{2Y} = \{W: W = Y \cap Z, Z \in \tau_2\}$ is
called subspace of bitopological space (X, τ_1, τ_2).

Definition2.21[5]
(Hausdorff bitopological space): A bitopological space (X, τ_1, τ_2) is said to be Hausdorff if for each two points
$x, y \in X$ such that $x \neq y$, there exists.
τ_1-neighbourhood U of x and a τ_2-neighbourhood V of y such that $U \cap V = \phi$.

Definition2.22[2]
(mset Topological space): Let $M \in \mathfrak{M}(X)$ and $\tau \subseteq P^*(M)$. Then τ is called an mset topology on M if τ satisfies the following properties:
- The mset M and the empty mset ϕ are in τ.
- The mset union of the elements of any members of τ is in τ.
- The mset intersection of the elements of any finite subcollection of τ is in τ.

The ordered pair (M, τ) is called a
M-topological space. Each element in τ is called open mset.

Example2.23: Let X be a nonempty set and $M \in \mathfrak{M}(X)$. Then (M, τ) is a
M-topological space where $\tau = P^*(M)$

Example 2.24:
Let $M = \{3/a, 4/b, 2/c, 1/d\}$ be an mset,
$\tau_1 = \{M, \phi, \{3/a\}, \{2/b\}, \{3/a, 2/b\}\}$ and
$\tau_2 = \{M, \phi, \{2/a\}, \{2/c\}, \{2/a, 2/c\}\}$.

Definition 2.25[4]
(Submulti space of an M-topology space): Let (M, τ) be a M-topological space such that
$M \in \mathfrak{M}(X)$ and $N \sqsubseteq M$. Then the ordered pair (N, τ_N) such that
$\tau_N = \{U \in \mathfrak{M}(X): U = N \Cap V, V \in \tau\}$ is called the submulti space (submspace, for short) of the M- topological space (M, τ).

Definition 2.26[7]
(Hausdorff M-topological Space): Let (M, τ) be an M-topological space where $M \in \mathfrak{M}(X)$**.** If for every two simple msets
$\{k_1/x_1\}, \{k_2/x_2\} \sqsubseteq M$ such that $x_1 \neq x_2$, then there exist $G, H \in \tau$ such that $\{k_1/x_1\} \sqsubseteq G, \{k_2/x_2\} \sqsubseteq H$ and $G \Cap H \doteqdot \phi$. Then (M, τ) is said to be a Hausdorff M- topological space.

3.0 DEFINITIONS AND SOME RESULTS

Definition 3.1: The root (support) set of a
M- topological space (M, τ) where $M \in \mathfrak{M}(X)$ denoted by $(M, \tau)^*$ is defined by:
$(M, \tau)^* = (M^*, \tau^*)$ where $\tau^* = \{B^* | B \in \tau\}$..

Proposition 3.2: The root set of a
M-topological space (M, τ) is a topological space.

Proof:
Let (M, τ) be an M-topological space and let$(M, \tau)^*$ be its root set. We show that $(M, \tau)^*$ is a topological space.
By definition,
$(M, \tau)^* = (M^*, \tau^*)$ where $\tau^* = \{B^* | B \in \tau\}$ then;
- Since (M, τ) is an M-topological space

we have $\phi, M \in \tau$.(by definition)
In particular, $\phi^* = \phi, M^* \in \tau^*$.
- Note that for for any $B \in \tau$, we have $B \sqsubseteq M$ and $B \sqsubseteq M \rightarrow B^* \subseteq M^*$ [3].
Taking any collection of root sets in τ^* say $\{B_\alpha^*\}_{\alpha \in I}$. Then for each $\alpha \in I$

$\bigcup_{\alpha \in I} B_\alpha^* = (\biguplus_{\alpha \in I} B_\alpha)^*$ ([9]).
But $\biguplus_{\alpha \in I} B_\alpha \in \tau$ (by hypothesis)
Thus $(\biguplus_{\alpha \in I} B_\alpha)^* \in \tau^*$
In particular, $\bigcup_{\alpha \in I} B_\alpha^* \in \tau^*$
- For the finite intersection, supposed

$\{A_1^*, \dots, A_n^*\}$ be a finite collection of sets in τ^*. Then $\bigcap_{i=1}^n A_i^* = (\Cap_{i=1}^n A_i)^*$ [9]
But $\Cap_{i=1}^n A_i \in \tau$ (by hypothesis).
Thus $(\Cap_{i=1}^n A_i)^* \in \tau^*$
In particular, $\bigcap_{i=1}^n A_i^* \in \tau^*$
Hence, $(M, \tau)^*$ is a topological Space.

Proposition 3.3: The root set of a submspace of a M-topological space is a subspace of the root set of the M-topological space.

Proof:
Let (M, τ) be a M-topological space where $M \in \mathfrak{M}(X)$ and $N \sqsubseteq M$ such that (N, τ_N) a submspace of the M-topological space (M, τ).
We show that $(N, \tau_N)^*$ is subspace of $(M, \tau)^*$
i.e we show that (N^*, τ_N^*) is a subspace of (M^*, τ^*)
note that $N \sqsubseteq M \rightarrow N^* \subseteq M^*$ and (M^*, τ^*) is a topological space ([3]) and proposition 3.2 respectively).
Now $\tau_N^* = \{B^* | B \in \tau_N\}$ and $B = N \Cap V$ such that $V \in \tau$ (by definition)
But $B^* = (N \Cap V)^* = N^* \cap V^*$([9])
Note that $V \in \tau \rightarrow V^* \in \tau^*$(by definition) and $N \Cap V \sqsubseteq N \rightarrow (N \Cap V)^* = N^* \cap V^* \subseteq N^*$
Thus, $B^* = N^* \cap V^* \in \tau_N^*$ and (N^*, τ_N^*) is a subspace of (M^*, τ^*)(by definition)

Proposition 3.4: The root set of a Hausdorff M- topological space is a Hausdorff topological space.

Proof:

Let (M,τ) be a Hausdorff M-topological space, and let $(M,\tau)^*$ be it root set. Then we show that $(M,\tau)^*$ is Hausdorff topological space.

Now $(M,\tau)^* = (M^*,\tau^*)$ is a topological space

where $\tau^* = \{B^* | B \in \tau\}$(by definition) is a topological space (Proposition3.2).

We show that $(M,\tau)^*$ is Hausdorff topological space.

Let $\{k_1/x\}$ and $\{k_2/y\}$ be two simple msets such that $\{k_1/x\},\{k_2/y\} \Subset M$ and $x \neq y$.

We have $U,V \in \tau$ such that

$\{k_1/x\} \Subset U, \{k_2/y\} \Subset V$ and $U \Cap V = \phi$.

But $\{k_1/x\},\{k_2/y\} \Subset M \rightarrow x,y \in M^*$ (1)

$U,V \in \tau \rightarrow U^*,V^* \in \tau^*$ (2)

$\{k_1/x\} \Subset U \rightarrow x \in U^*$ (3)

$\{k_2/y\} \Subset V \rightarrow y \in V^*$ (4)

But $U \Cap V = \phi \rightarrow (U \Cap V)^* = \emptyset^* = \emptyset$ (5)

And $(U \Cap V)^* = U^* \cap V^*$ ([9]) (6)

From (5) & (6) we have

$U^* \cap V^* = \emptyset$ (7)

Hence, $(M,\tau)^*$ is a Hausdorff topological space. (from (2-7))

Definition 3.5[6] **(*M*-Bitopological Space):** A M-bitopological space is a triple (M,τ_1,τ_2) where $M \in \mathfrak{M}(X)$ and τ_1,τ_2 are arbitrary M-topologies on M.

Definition 3.6

(Root set of a M-bitopological space): The root (support) set of a M-bitopological space (M,τ_1,τ_2) denoted by $(M.\tau_1,\tau_2)^*$ is defined by:

$(M.\tau_1,\tau_2)^* = (M^*,\tau_1^*,\tau_2^*)$, where

$\tau_1^* = \{B^* | B \in \tau_1\}$ and $\tau_2^* = \{C^* | C \in \tau_2\}$.

Proposition 3.7: The root set of a M-bitopological space is a bitopological space.

Proof:

Let (M,τ_1,τ_2) be an M-bitopological space, and let$(M,\tau_1,\tau_2)^*$ be its root set. We show that $(M,\tau_1,\tau_2)^*$ is a bitopological space.

Now $(M,\tau_1,\tau_2)^* = (M^*,{\tau_1}^*,{\tau_2}^*)$ where

${\tau_1}^* = \{A^* | A \in \tau_1\}$ and ${\tau_2}^* = \{B^* | B \in \tau_2\}$

(by definition).

Since τ_1,τ_2 are M- topologies on M, then ${\tau_1}^*,{\tau_2}^*$ are topologies on M^*

(Proposition 3.2)

Hence, $(M,\tau_1,\tau_2)^* = (M^*,{\tau_1}^*,{\tau_2}^*)$

is a bitopological Space(definition 2.19).

Definition3.8: Let (M,τ_1,τ_2) be M-Bitopological Space where $M \in \mathfrak{M}(X)$ and $N \Subset M$. Then (N,τ_{1N},τ_{2N}) where $\tau_{1N} = \{A | A = N \Cap U, U \in \tau_1\}$ and $\tau_{2N} = \{B | B = N \Cap V, V \in \tau_2\}$ is called a submspace of the M-Bitopological Space.

Proposition 3.9. The root set of a submspace of a M-bitopological space is a subspace of the root set of the M-bitopological space

Proof:

Let (M,τ_1,τ_2) be M-Bitopological Space where $M \in \mathfrak{M}(X)$ and $N \Subset M$.

Then we show that $(N,\tau_{1N},\tau_{2N})^*$ is a subspace of $(M,\tau_1,\tau_2)^*$.

But $(M,\tau_1,\tau_2)^* = (M^*,{\tau_1}^*,{\tau_2}^*)$

where ${\tau_1}^* = \{U^* | U \in \tau_1\}$ and

${\tau_2}^* = \{V^* | V \in \tau_2\}$ (by definition)

Also, $(N,\tau_{1N},\tau_{2N})^* = (N^*,\tau_{1N}^*,\tau_{2N}^*)$ where

$\tau_{1N}^* = \{A^* | A \in \tau_{1N}\}$ and $\tau_{2N}^* = \{B^* | B \in \tau_{2N}\}$

Note that $N \Subset M \rightarrow N^* \subseteq M^*$ (1)

$A \in \tau_{1N} \rightarrow A = N \Cap U$ where $U \in \tau_1$ (2)

$B \in \tau_{2N} \rightarrow B = N \Cap V$ where $V \in \tau_2$ (3)

But $A = N \Cap U \rightarrow A^* = (N \Cap U)^*$ and

$(N \Cap U)^* = N^* \cap U^* \subseteq N^*$ (4)

$B = N \Cap V \rightarrow B^* = (N \Cap V)^*$ and

$(N \Cap V)^* = N^* \cap V^* \subseteq N^*$ (5)

The result is clear from (1)-(5)

i.e$(N,\tau_{1N},\tau_{2N})^*$ is a subspace of $(M,\tau_1,\tau_2)^*$.

Definition 3.10: A M-bitopological space (M,τ_1,τ_2) where $M \in \mathfrak{M}(X)$ is said to be a **Hausdorff M-bitopological space** if for any simple msets $\{k_1/x\},\{k_2/y\} \Subset M$ with $x \neq y$, there exist $U \in \tau_1$ and $V \in \tau_2$ such that $\{k_1/x\} \Subset U$, $\{k_2/y\} \Subset V$ and $U \cap V = \phi$,

Proposition 3,11: Let (M,τ_1,τ_2) be Hausdorff M-bitopological space. Then the root set of (M,τ_1,τ_2) is a Hausdorff bitopological space.

Proof:

Let (M,τ_1,τ_2) be Hausdorff M-bitopological space.

The root set of (M,τ_1,τ_2) denoted by $(M,\tau_1,\tau_2)^*$ is given by

$(M,\tau_1,\tau_2)^* = (M^*,\tau_1^*,\tau_2^*)$ where

${\tau_1}^* = \{A^* | A \in \tau_1\},{\tau_2}^* = \{B^* | B \in \tau_2\}$

(Definition 3.6).

Clearly $(M,\tau_1,\tau_2)^*$ is bitopological space (Proposition 3.7)

We show that $(M,\tau_1,\tau_2)^*$ is a Hausdorff bitopological space.

Let $x,y \in M^*$ such tha $x \neq y$

But $x,y \in M^* \rightarrow C_M(x),C_M(y) > 0$

Thus, for $C_M(x) = k_1$ and $C_M(y) = k_2$,

we have $\{k_1/x\},\{k_2/y\} \Subset M$ with $x \neq y$,

Thus there exists $U \in \tau_1$ and $V \in \tau_2$

such that $\{k_1/x\} \Subset U, \{k_2/y\} \Subset V$

and $U \cap V \doteqdot \phi$ (by hypothesis)

Note that $U \in \tau_1 \to U^* \in \tau_1^*$ and
$V \in \tau_2 \to V^* \in \tau_2^*$
$\{k_1/x\} \subseteq U \to x \in U^*$,
$\{k_2/y\} \subseteq V \to y \in V^*$
But $U^* \cap V^* = (U \cap V)^* = \emptyset^* = \emptyset$
Hence, $(M, \tau_1, \tau_2)^* = (M^*, \tau_1^*, \tau_2^*)$ is a Hausdorff bitopological space.

4.0 SUMMARY

Here it's been shown that the M-topological spaces and their submspaces are indeed the generalizations of the classical topological spaces and subspaces via their defined root sets respectively.

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