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Semiderivations on σ – Prime Rings

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ARTICLE INFO	ABSTRACT
Published Online: 20 February 2024 **Corresponding Author:** Dr. D. Bharathi	In this paper, we derive some results on semiderivation in σ – prime rings. If (R, σ) is a σ-prime ring with involution σ and char ≠ 2, let d be a nonzero semiderivation with g of R is centralizing, then R is commutative. Further we prove that if d commutes with σ and 0 ≠ I in a σ- Ideal of R such that either [d(x), d(y)] = 0 or d(xy) = d(yx), for all x, y ∈ I, then R is commutative. Finally, a σ-prime ring with char ≠ 2 possessing a nonzero semiderivation under surjective conditions must be commutative.

KEYWORDS: Derivation, Semiderivation, Char≠ 2, Commutator, Center, Commutative Ring, Prime ring, σ – prime ring, σ – prime ideal.

I. INTRODUCTION

The notion of derivation was introduced in rings and algebras long back. The theory of derivations and semiderivations play an important role not only in ring theory, but also in functional analysis and linear differential equations. In 1976, I.N.Herstein. [2] proved that an element ‘a’ of a semi-prime rings R centralizes all commutator sxy-yx ,x, y∈R then a ∈ Z (R).The study of such mappings was initiated by E.C.Posner in [1]. A famous result due to Herstein [4] states that if R is a prime ring of char ≠ 2 which admits a nonzero derivation d such that [d(x), a] = 0, for all x ∈ R, then a ∈ Z. Also Herstein showed that if d(R) ⊂ Z, then R must be commutative. Many authors have studied commutativity of prime and semiprime rings admitting derivations and generalized derivations which satisfy appropriate algebraic conditions on suitable subsets of the rings.

In [5] Bergen has introduced the notation of semiderivation of a ring R which extends the notation of derivation of a ring R.

In 1984, J.C.Chang [6] has given an extension of the I.N.Herstein [3] results in the following way. Let f ≠0 be a semiderivation of a prime ring R associated with an epimorphism of R such that [f(R), f(R)] = {0}. Then if Char R ≠ 2, R is commutative, and if char R = 2, then either R is a commutative or R is an order in a simple algebra which is 4-dimensional over its center.

In 1990, The structure of semiderivations of prime rings has been studied by C.L.Chuang [11] and he proved a structure theorem with the help of extended centroid of the classical associative rings. The same results have been obtained by M.Bresar[12].

II. PRELIMINARIES: We use these definitions in the main section.

Definition 2.1: An additive mapping d: R→R is called a **derivation** if

d(xy) = d(x) y+ x d(y), for allx, y ∈ R

Definition 2.2: An additive mapping d: R → R is called a **semiderivation** if there exists a function g: R → R such that (i) d(xy) = d(x)g(y) + x d(y) = d(x)y + g(x)d(y) and (ii) d(g(x)) = g(d(x)) (2.1)
holds for all x, y ∈ R.

Note: In case g is an identity map of R then all semiderivations associated with g are merely ordinary derivations. On the other hand, if g is a homomorphism of R such that g ≠ I then d = g – I is a semiderivation which is not a derivation.

Definition 2.3: Let S be a nonempty subset of R. A mapping d from R to R is called **centralizing** on S if [d(x), x] ∈ Z, for all x ∈ S and is called commuting on S if [d(x), x] = 0, for all x ∈ S.

Definition 2.4: A ring R is called **prime** if xRy = 0 implies either x=0 or y=0, for all x, y∈R.

Definition 2.5: If there exists a positive integer n such that nx = 0 for every element of the ring R, the smallest such positive integer is called the **characteristic of R**. We define a ring R to be characteristic $\neq$ n if nx = 0 implies x = 0, for all x in R.

Definition 2.6: The **commutator** [x,y] is defined by [x , y] = xy – yx, for all x, y$\in$R

Definition 2.7: The **anticommutator** is defined by xoy = xy + yx, for all x, y$\in$R

Definition 2.8: The **center** Z of R is defined as Z = { z $\in$R / [z, R] = 0 }

Definition 2.9: Let R be a semiprime ring and P be a nonzero prime ideal of R. If a, b $\in$ R such that b $\subseteq$ P, then either a$\in$ P or b$\in$ P.

Definition 2.10: An ideal L of R is said to be **σ- ideal** if σ(L) = L

Definition 2.11: A ring R equipped involution σ is said to be **σ-prime ring** if for any a,b $\in$R aRb=aRσ(b) = 0 $\Rightarrow$a=0 or b=0.

Definition 2.12: A ring R is said to be **commutative** if xy=yx, for all x,y$\in$R.

Also, we will make some extensive use of the basic commutator identities:

[x,yz]=y[x,z]+[x,y]z (2.2)

[xy,z]=[x,z]y+x[y,z] (2.3)

III.MAIN RESULTS:

In this paper, we derive some results on semi derivation in σ – prime rings

Throughout this paper R will be σ- prime ring and d be a nonzero semiderivation of R associated with surjective function g of R.

Now we prove the following lemmas:

Lemma 3.1: Let R be a σ- prime ring and let I be a nonzero σ- ideal of R. If a, b in R satisfy a I b = a I σ(b) = 0, then a = 0 or b = 0.

Proof: Suppose a $\neq$ 0, there exists some x $\in$ I such that ax $\neq$ 0 .

Otherwise a R x = 0 and a R σ(x) = 0, for all x $\in$ I and therefore a = 0.

Since a IR b = 0 and a IR σ(b) = 0, we then obtain

a x Rb = a x R σ(b) = 0.

In view of the σ- primeness of R this yields b = 0. ♦

Lemma 3.2 : Let I be a nonzero σ- ideal of R and 0 $\neq$ d be a semiderivation with surjective g on R which commutes with σ. If [x, R] I d(x) = 0, for all x $\in$ I, then R is commutative.

Proof : Let x $\in$ I.

Since k = x - σ(x)$\in$ I, then [k, r]Id(k) = 0, for all r $\in$ R.

As k $\in Sa_\sigma$(R), we get

[k, r] I d(k) = σ([k, r]) I d(k) = 0, for all r $\in$ R.

Which leads, in view of lemma 3.1, to

d(k) = 0 or [r, k] = 0, for all r $\in$ R.

If d(k) = 0, then

d(x - σ(x)) = 0, which implies d(x) = d(σ(x)) = σ(d(x)).

Therefore [x,r] I d(x) = [x, r] I σ(d(x)) = 0, thus

d(x) = 0 or [r, x] = 0, for all r $\in$ R, by lemma 3.1.

Consequently, either d(x) = 0 or x $\in$ Z(R).

If [r, k] = 0, for all r $\in$ R, then k $\in$ Z(R) and thus

[x, r] = [σ(x), r], for all r $\in$ R.

Hence [x, r] I d(x) = σ([x, r]) I d(x) = 0

Using lemma 2.3.1 we get d(x) = 0 or x $\in$ Z(R).

In conclusion, for each x $\in$ I either d(x) =0 or x $\in$ Z(R).

Let us consider A = { x $\in$ I / d(x) = 0} and B = { x $\in$ I / x in Z(R)}.

It is clear that A and B are additive subgroups of I such that I = AUB.

But a group cannot be a union of two its proper subgroups and hence I=A or I=B.

If I =A, then d(x) =0, for all x $\in$ I.

For any t $\in$ R, replace x by xt in d(x) =0, we get

d(xt) = 0 implies d(x)g(t) + xd(t) = 0 (by 2.1).

xd(t) = 0, for all x $\in$ I, t $\in$ R so that

Id(t) = 0, for all t $\in$ R.

In particular 1 I d(t) = σ(1) I d(t), for all t $\in$ R.

By lemma 3.1 gives d = 0, a contradiction.

Hence I = B so that I $\subseteq$Z(R).

Let r, t $\in$ R and x $\in$ I, from rtx = trx we conclude that [r, t] I = 0 and then

[r, t] I 1 = [r, t] I σ(1) = 0.

Therefore by lemma 3.1, [r, t]=0, for all r, t $\in$ R and consequently, R is commutative ring. ♦

Lemma 3.3: Let I be a nonzero σ- ideal of R. If R admits a semiderivation d with surjective function g such that $d^2(I)$ = 0 and d commutes with σ on R then d = 0.

Proof : If d $\neq$ 0 and let $t_0\in$ R such that $d(t_0) \neq 0$.

For any x $\in$ I we have $d^2(x) = 0$

Replacing x by xy, we obtain

d(d(x)g(y)+ xd(y)) = 0 (by 2.1).

$d^2(x)\, g^2(y) + d(x)g(d(y)) + d(x)d(g(y)) + x\, d^2(y) = 0$, for all x, y $\in$ I (by 2.1).

Since $d^2(I) = 0$ with char$\neq$ 2, we get

d(x)g(d(y)) = 0, for all x, y $\in$ I.

In particular $d(x)g(d(y\, t_0)) = 0$

$d(x)g(d(y))g(t_0) + d(x)g(y)g(d(t_0)) = 0$ (by 2.1).

Since g is on to we have

$d(x)yd(t_0) = 0$, for all y $\in$ I, $t_0\in$ R.

Thus $d(x)\, I\, d(t_0) = 0$.

As d commutes with σ, the fact that I is a σ –ideal gives $\sigma(d(x))\, I\, d(t_0) = 0$.

Consequently $d(x)\, I\, d(t_0) = \sigma(d(x))\, I\, d(t_0) = 0$.

By lemma 3.1, we have d(x) = 0, for all x $\in$ I

Replace x by x, t_0 in (3.1), then we get

$d(xt_0) = 0$,

$d(x)g(t_0) + x\, d(t_0) = 0$ which implies (by 2.1).

$xd(t_0) = 0$, for all x $\in$ I

i.e $I\, d(t_0) = 0$.

In particular, $1 I d(t_0) = \sigma(1) I d(t_0) = 0$.
So that $d(t_0) = 0$, a contradiction.
Consequently, $d = 0$. ♦

Theorem 3.1: Let R be a σ- prime ring with char ≠ 2 and let d be a nonzero semiderivation associated with surjective g of R . If $[d(x), x] \in Z(R)$, for all $x \in R$, then R is commutative.

Proof : Linearizing $[d(x), x] \in Z(R)$ gives
$[d(x), y] + [d(y), x] \in Z(R)$, for all $x, y \in R$. (3.2)
Replacing y by x^2 in (3.2)
$x[d(x), x] +[d(x), x]x + d(x)[g(x), x]+ [d(x), x]g(x) + x[d(x), x] +[x, x]d(x) \in Z(R)$
(by 2.2, 2.1 & 2.3).
Since char ≠ 2 and g is on to, we have
$x[d(x), x] \in Z(R)$ which implies
$[r, x[d(x), x]] = 0$.
Henceforth $[r, x] [d(x), x] = 0$, for all $x \in R$.
Replacing r by d(x), we obtain
$[d(x), x]^2 = 0$.
Since $[d(x), x]$ in $Z(R)$, then
$[d(x), x] R [d(x), x] \sigma([d(x), x]) = 0$.
As $[d(x), x] \sigma([d(x), x]) \in Sa_\sigma(R)$, and R is σ-prime, then
$[d(x), x] = 0$ or $[d(x), x] \sigma([d(x), x]) = 0$.
Assume $[d(x), x] \sigma([d(x), x]) = 0$, the fact that $[d(x), x] \in Z(R)$ gives
$[d(x), x] R \sigma([d(x), x]) = [d(x), x] R [d(x), x] = 0$.
Since R is a σ-prime, we have $[d(x), x] = 0$, for all $x \in R$. (3.3)
$[d(x), y] + [d(y), x] = 0$, for all $x, y \in R$. (3.4)
Replace y by xy in (3.4) and using (3.4), we get
$x[d(x), y] + [d(x), x]y + d(x)[g(y), x] +[d(x),x]g(y) + x[d(x), x] + [x, x]d(y) = 0$
(by 2.2, 2.1 & 2..3).
$d(x)[g(y), x] = 0$, g is on to gives
$d(x)[y, x] = 0$, for all $x, y \in R$ (3.5)
Replace y by yz in (3.5), we get
$d(x) y [z, x] = 0$, for all $x, y, z \in R$ (by 2.3).
And hence $d(x) R [z, x] = 0$, for all $x, z \in R$.
In particular,
$d(\sigma(x) R [\sigma(z), \sigma(x)] = \sigma(d(x)) R \sigma([z, x])$.
Because d commutes with σ. Applying σ to this last equality, we obtain
$[z, x] R d(x) = 0$, for all $x, z \in R$.
Hence by lemma 3.2, R is commutative. ♦

Theorem 3.2 : Let R be a σ- prime ring with char ≠ 2 and let I be a nonzero σ-ideal of R. If R admits a nonzero semiderivation d associated with surjective function g such that $[d(x), d(y)]=0$, for all $x, y \in I$ and d commutes with σ, then R is commutative.

Proof : By hypothesis we have
$[d(x), d(y)] = 0$, for all $x, y \in I$ (3.6)
Replace y by xy in (3.6), we get
$[d(x), d(x)g(y)+xd(y)] = 0$, for all $x, y \in I$ (by 2.1).
$d(x)[d(x), g(y)] + [d(x), x]d(y) = 0$ (by 2.2).
Since g is on to we have
$d(x)[d(x), y] + [d(x), x]d(y) = 0$, for all $x, y \in I$.
Replace y by yr, $r \in R$ in the above equation
$d(x)[d(x), yr] + [d(x), x]d(yr) = 0$, for all $x, y \in I, r \in R$.
$d(x)[d(x), y]r + d(x)y[d(x), r] + [d(x), x]d(y)g(r) + [d(x), x] y d(r) = 0$, for all $x, y \in I, r \in R$. (by 2.2 & 2.1)
Using (3.7) and g is on to we obtain
$d(x)y[d(x), r] + [d(x), x] y d(r) = 0$, for all $x, y \in I, r \in R$.
Substitute r by d(z) in the above equation
$d(x)y[d(x), d(z)] + [d(x), x] y d(d(z)) = 0$, for all $x, y, z \in I$.
Using the hypothesis, we get
$[d(x), x] y d^2(z) = 0$, for all $x, y, z \in I$.
So that $[d(x), x] I d^2(z) = 0$, for all $x, z \in I$
As d commutes with σ and I is a σ-ideal, then
$[d(x), x] I d^2(z) = \sigma([d(x), x]) I d^2(z) = 0$.
By lemma 3.1, we say that either
$d^2(z) = 0$, for all $z \in I$ or $[d(x), x] = 0$, for all $x \in I$
If $d^2(z) = 0$, for all $z \in I$, then by lemma 3.3 assures $d= 0$ which is impossible.
Now suppose that
$[d(x), x] = 0$, for all $x \in I$
Linearizing (3.8), we get
$[d(x), y] + [d(y), x] = 0$, for all $x, y \in I$
Replace y by yx in(3.9) and using (3.9), we get
$[y, x]d(x) = 0$.
Hence $[x, y]d(x) = 0$, for all $x, y \in I$
Again replace y by ry, $r \in R$, we get
$[x, r] y d(x) = 0$, for all $x, y \in I, r \in R$ and thus (by 2.2).
$[x, r] I d(x) = 0$, for all $x \in I, r \in R$.
Then by lemma 3.2, R is commutative. ♦

Theorem 3.3: Let R be a σ- prime ring with char ≠ 2 and let I be a nonzero σ-ideal of R. If R admits a nonzero semiderivation d associated with surjective function g such that $d(xy) =d(yx)$, for all $x, y \in I$ and d commutes with σ, then R is commutative.

Proof : Since $d[x, y] = 0$, for all $x, y \in I$, the condition that $d([x, y]z) = d(z[x,y])$, for all $x, y, z \in I$ yields
$d[x,y]g(z) + [x, y] d(z) = d(z)g([x, y]) + z d([x, y])$ (by 2.1).
Using the hypothesis and g is on to we have
$[x, y] d(z) = d(z)[x, y]$, for all $x, y, z \in I$.
By hypothesis $d(xy) =d(yx)$, for all $x, y \in I$
$d(x)g(y) + xd(y) = d(y)g(x) + yd(x)$, for all $x, y \in I$ (by 2.1.1).
Which implies
$[d(x), y] = [d(y), x]$, for all $x, y \in I$.
In particular $[d(x^2), y] = [d(y), x^2]$, for all $x, y \in I$
$[d(x)g(x), y] + [xd(x), y] = x[d(y), x] + [d(y), x]x$ (by 2.1).
Since g is onto
$d(x)[x, y] + [d(x), y]x + x[d(x), y] + [x, y]d(x) = x[d(y), x] + [d(y), x]x$
Using (3.11), we obtain

$d(x)[x, y] + [x, y]d(x) = 0$, for all $x, y \in I$.
Using (3.10) in the above equation
$[x, y]d(x) = 0$, for all $x, y \in I$. (3.12)
For any r in R, replace y by ry in (3.12), we obtain
$[x, r]\, y\, d(x) = 0$, for all $x, y \in I$.
Hence $[x, R]\, I\, d(x) = 0$, for all $x \in I$.
By lemma 3.2 we conclude that R is commutative. ♦

Theorem 3.4 : Let R be a σ- prime ring with char $\neq 2$ and let I be a nonzero σ-ideal of R. If R admits a nonzero semiderivation d associated with surjective function g. If $r \in Sa_{\sigma}(R)$ satisfies $[d(x), r]=0$, for all $x \in I$, then $r \in Z(R)$. Furthermore, if $d(I) \subset Z(R)$, then R is commutative.

Proof :
By hypothesis $[d(x), r] = 0$, for all $x \in I$
Replace x by xy implies $[d(xy), r] = 0$, for all $x, y \in I$
$[d(x)g(y) + xd(y), r] = 0$, for all $x, y \in I$ (by 2.1).
$d(x)g(y)r + xd(y)r – rd(x)g(y) – rxd(y) = 0$,
$d(x)[g(y), r] + [x, r]d(y) = 0$ (by 2.3).
Since g is on to we have
$d(x)[y, r] + [x, r]d(y) = 0$, for all $x, y \in I$ (3.13)
Taking y by yr, $r \in Sa_{\sigma}(R)$ in (3.13)
$d(x)[yr, r] + [x, r]d(yr) = 0$,
$d(x)[y, r]r + [x, r]d(y)g(r) + [x, r]yd(r) = 0$ (by 2.3 & 2.1).
Since g is on to and using (3.13), we get
$[x, r]yd(r) = 0$, we conclude that $[x, r]\, Id(r) = 0$.
The fact that I is a σ-ideal together with $r \in Sa_{\sigma}(R)$, give
$\sigma([x, r])\, Id(r) = [x, r]\, Id(r) = 0$.
Using lemma 3.1, either $d(r) = 0$ or $[x, r] = 0$.
If $d(r) \neq 0$, then $[x, r] = 0$, for all $x \in I$
Replace x by tx, $t \in R$, we get $[t, r]x= 0$ (by 2.1.3).
Let $0 \neq x_0 \in I$, as $[t, r]\, R\, x_0 = [t, r]\, R\, \sigma(x_0)$, then $[t, r] = 0$.
Since R is a σ- prime, which proves $r \in Z(R)$.
Now $d(r) = 0$, then $d([x, y]) = [d(x), r] = 0$ and consequently
$d([I, r]) = 0$, (3.14)
Replace y by yw, $w \in I$ in(3.13), we get
$d(x)[yw, r] + [x, r]d(yw) = 0$, for all $x, y, w \in I$,
$d(x)y[w, r] + d(x)[y, r]w + [x, r]d(y)g(w) + [x, r]yd(w) = 0$ (by 2.3 & 2.1).
Since g is on to and using (3.13), we get
$d(x)y[w, r] + [x, r]yd(w) = 0$. (3.15)
Now taking [w, r] instead of w in (3.15) and using (3.14), we get
$d(x)y[[w, r], r] = 0$, for all $x, y, w \in I$, so that
$d(x)\, I\, [[w, r], r] = 0 = d(x)I\, \sigma([[w, r], r])$, for all $x, y, w \in I$, $r \in Sa_{\sigma}(R)$.
By lemma 3.1, we have either $d(I) = 0$ or $[[w, r], r] = 0$.
If $d(I) = 0$, then for any $t \in R$, we get
$d(tu) = 0$, for all $t \in R, u \in I$,
$d(t)u + g(t)d(u) = 0$, which gives $d(t)u = 0$, for all $t \in R$, $u \in I$.
Therefore, $d(t)\, RI = d(t)R\, \sigma(I) = 0$,
And as $I \neq 0$, then $d(t) = 0$ in such way that $d = 0$.
Consequently, $[[w, r], r] = 0$.
Replace w by wu in (3.16), we obtain
$[[wu, r], r] =[w[u, r] +[w, r]u, r]$ (by 2.3)
$=w[[u, r], r] + [w, r][u, r] + [w, r][u, r] + u[[w, r], r]$.
Since R is a char$\neq 2$ and using (3.16), we get
$[w, r][u, r] = 0$.
Hence $[tw, r][u, r] = [t, r]\, w\, [u, r] = 0$ and consequently
$[t, r]\, I\, [u, r] = 0$, for all $u \in I$.
Therefore $[t, r]\, I\, [u, r] = [t, r]\, I\, \sigma([u, r]) = 0$.
Again using lemma 3.1, we see that $[t, r] = 0$ or $[u, r] = 0$.
If $[t, r] = 0$ then $r \in Z(R)$.
If $[u, r] = 0$, for all $u \in I$, then for any $t \in R$
$[tu, r] = t[u, r] + [t, r]u = 0$, which implies $[t, r]u = 0$.
Hence $[t, r]I =[t, r]\, I\, 1 = [t, r]I\, \sigma(1) = 0$.
Again using lemma .3.1, we conclude that $[t, r] = 0$, which proves that $r\in Z(R)$.
Now suppose that $d(I) \subset Z(R)$and let$r \in R$.
From the first part of the theorem we conclude $Sa_{\sigma}(R) \subset Z(R)$.
Using the fact that $r + \sigma(r)$ and $r – \sigma(r)$ are elements of $Sa_{\sigma}(R)$.
We then obtain $r + \sigma(r) \in Z(R)$ and $r – \sigma(r) \in Z(R)$ and hence $2r \in Z(R)$.
Since R is char$\neq 2$, then $r \in Z(R)$.
Hence we conclude that R is commutative. ♦

Theorem 2.3.5 : Let R be a σ- prime ring with char $\neq 2$ and let $a \in Sa_{\sigma}(R)$. If R admits a nonzero semiderivation d associated with surjective function g such that $d([R, a]) = 0$, then $a \in Z(R)$. In particular, if $d(xy) – d(yx) = 0$, for all $x, y \in R$, then R is commutative ring.

Proof :
If $d(a) = 0$, from our hypothesis, we have for any $r \in R$.
$d([r, a]) = 0$,
$d(ra) – d(ar) = 0$,
$d(r)a + g(r)d(a) – d(a)r –g(a)d(r) = 0$ (by 2.1)
Since g is on to, which gives
$[d(r), a] = 0$, for all $r \in R$
Now using Theorem 3.4 we say $a \in Z(R)$ and the proof is complete.
Next assume that $d(a) \neq 0$.
Replace r by ar, $r \in R$ in (.3.16), we get $d[ar, a] = 0$.
$d(a[r, a]) = d(a)[r, a] + g(a)d([r, a]) = 0$ (by 2.3 & 2.1).
Using (3.17) we get $d(a)[r, a] = 0$.
Replace r by rt, $t \in R$ in (3.18), we get
$d(a)r[t, a] + d(a)[r, a]t = 0$ (by 2.3).
Using (3.18) implies $d(a)\, r\, [t, a] = 0$, so that
$d(a)\, R\, [t, a] = 0$, for all $t \in R$.
Since $a \in Sa_{\sigma}(R)$, then
$d(a)\, R\, [t, a] = d(a)\, R\, \sigma([t, a]) = 0$.
Therefore R is a σ-primeness yields that $[t, a] =0$, which proves $a \in Z(R)$.

Now assume that $d([x, y]) = 0$, for all $x, y \in R$.
Using the first part of our theorem, then we get $Sa_\sigma(R) \subset Z(R)$.
By the fact that $r + \sigma(r)$ and $r - \sigma(r)$ are elements of $Sa_\sigma(R)$
We then obtain $r + \sigma(r) \in Z(R)$ and $r - \sigma(r) \in Z(R)$ and hence $2r \in Z(R)$.
Since R is char$\neq 2$, then $r \in Z(R)$.
Hence we conclude that R is commutative. ♦

IV COCLUSION

The study of commutative rings has innovative results not only in algebra but also other branches of Mathematics. A σ -prime ring R of Char $\neq 2$ possessing a nonzero semiderivation under some conditions is shown as Commutative.

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