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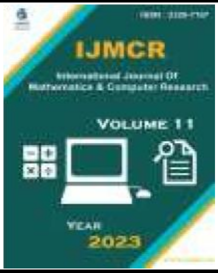

Qualitative Study of Controlling Pest of Jatropha curcas for Linear Functional Response

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ABSTRACT
In today's world, Jatropha curcas plants are a renewable energy feedstock for producing biodiesel to overcome the limitations of natural fuel. The alternative fuel biodiesel can be produced from the Jatropha plant as its seeds contain oil which is one of the important resources for biodiesel. Despite being toxic to many insects and animals, the Jatropha curcas plant is not pest and disease-free. In view of this, our research article presents the formulation and analysis of a mathematical model that describes the Jatropha plantation along with a prediction to control its natural pests by biological pesticides. We assume linear functional response of predators for susceptible pests, and infected pests as infected pests are weaker than susceptible pests and easy to catch. The dynamics of the system around each of the ecologically feasible equilibria are studied. The reduction of disease eradication and coexistence of predator and pest are observed around the predator-free and disease-free equilibrium respectively.

KEYWORDS: Jatropha curcas plant, Biodiesel, Pest control, Stability, Mathematical modeling, Functional response, Biological pesticide

I. INTRODUCTION

Conventional energy resources are very limited and due to excessive use of them, these conventional energy resources will last for maximum next few decades. To fulfill our dependency on fuel resources and maintain daily requirements, the demand for alternative energy sources is thriving. To meet these demands, the production of renewable and sustainable energy sources is greatly required. Biodiesel is one of the most useful alternative fuels which is renewable, clean-burning, and cost-effective [1]. The higher oil content and non-edible nature of the seed have made Jatropha curcus one of the most effective resources for biodiesel [2].

Among many biodiesel-producing resources like Soybean oil, Mustard oil, Palm oil, etc. Jatropha oil is the most promising resource because it produces the purest quality of biodiesel. Jatropha plants are generally affected by pests. This affects the growth of the plant as well as the oil production. How to control pests for this plant is a global problem in agricultural ecosystem management [3]. Hence controlling these pests for the healthy growth of the plant and for the improvement of oil productivity is urgently required. Many researchers have formulated mathematical models for controlling pests and they have studied the different perspectives of pest management tools with probable results by analyzing the system within the mathematical illustration. Chemical pesticides affect our health and plant growth and they also cause environmental pollution, and health problems and affect economic crop production [4]. This leads us to find out biological control methods for plant pests. Besides that, the most effective measures in pest management are determined by the ecology of a pest. Thus, the concept of Integrated Pest Management (IPM) [5] is being generated. Its application has been increased in the field by the farmers recently.

IPM reduces reliance on pesticides by emphasizing biological control methods. Bio-pesticides, components of an integrated approach, can play an effective role in pest control [6]. Potentially, the use of viruses is one of the most significant biological methods for pest control. In American and European countries, practical evidences of where the virus is used against insect pests are being noticed [7]. The experimental and field use of pathogenic viruses in Europe is listed by Falcon et al. [8].

As the importance of fuel crops is economically very high, our aim is to provide protection to the crop from exigency and increase its oil production. Pests are the main obstacles for Jatropha curcus plant to grow naturally. Jatropha plants are attacked by many pests but less than ten types of pests occur quite often, e.g., the leaf miner Stomphastis thraustica, the leaf and stem miner Pempelia morosalis, and the shield-backed bug Calidea panaethiopica etc. [9]. The natural enemy (predator) in the system survives consuming the susceptible and infected pests. Since the viral infection makes some behavioral changes and sub-lethal effects on the host, the predator consumes the infected pest in linear mass action.

In this research article to get a healthy production of the Jatropha plant, we want to show the change in nature of the various biomass of the system due to the consumption of susceptible pests by predators with the Holling type I functional response by the means of mathematical and numerical analysis. In the next section, we will formulate the mathematical model to represent the above-described biological phenomenon. After that local stability analysis around different equilibria will be performed. Then numerical simulations will be shown and discussed. Finally, we will conclude our findings.

II. MODEL FORMULATION

In this section, a five-dimensional mathematical model has been formulated, which consists of the biomass of Jatropha curcas plant J(t), susceptible pest S(t), infected pest I(t), predator P(t), and virus population V(t). Plant growth normally follows a logistic fashion where r_J is the maximum growth rate and k_J is the carrying capacity of the said plant. Here we have described two classes of pest populations, i.e., Susceptible pests and Infected pests. The plant resource is consumed by pests at a rate α which is again converted into the susceptible pest with r_S as the maximum growth rate. k_S is considered as the carrying capacity of the susceptible pest. The virus population attacks the susceptible pests and converts them into infected pests at a rate λ. Here we have considered the linear functional response of the predator population on the susceptible pest population which helps the predators in their growth at a rate θ_1. The predators consume the infected pests at a rate l. Infected pests have a natural death rate ξ. d_P is the natural mortality rate of predators and ε_P denotes the intra-specific competition coefficient among predators present in the predatory guild of infected pests. The predators grow at a rate θ_2,due to predation of the infected pests. π_V is assumed as the constant recruitment rate of the virus to the system and κ is the virus replication rate. The reduction rate constant of the virus population is γ. The mortality rate of the virus population is assumed as μ_V.

With the above assumptions, the following mathematical model has been formulated.

$$
\begin{aligned}
\frac{dJ}{dt} &= r_J J\left(1-\frac{J}{k_J}\right) - \alpha JS \\
\frac{dS}{dt} &= r_S JS\left(1-\frac{S+I}{k_S}\right) - \lambda SV - \beta SP \\
\frac{dI}{dt} &= \lambda SV - \xi I - lIP \\
\frac{dP}{dt} &= P(-d_P - \epsilon_P P) + \theta_1 \beta SP + \theta_2 IP \\
\frac{dV}{dt} &= \pi_V + \kappa \xi I - \mu_V V - \gamma SV
\end{aligned}
\tag{1}
$$

Where

$J(0) \geq 0, S(0) \geq 0, I(0) \geq 0, P(0) \geq 0, V(0) \geq 0$

and all the parameters are assumed to be non-negative.

III. EXISTENCE OF EQUILIBRIA AND STABILITY

Theorem 1: The axial equilibrium point $E = (0,0,0,0,\pi_V)$ exists and the system (1) is unstable around E for all the parametric values.

Theorem 2: The pest-free equilibrium point $E_0 = (k_J, 0, 0, 0, \frac{\pi_V}{\mu_V})$ exists and the system (1) is stable around E_0 if $k_J < \frac{\lambda \pi_V}{r_S \mu_V}$.

Pest-free equilibrium: $E_0 = (k_J, 0, 0, 0, \frac{\pi_V}{\mu_V})$.

The Jacobian matrix for pest-free equilibrium point is given by,

$$
J = \begin{pmatrix}
-r_j & -\alpha k_j & 0 & 0 & 0 \\
0 & r_s k_j - \lambda \frac{\pi_v}{\mu_v} & 0 & 0 & 0 \\
0 & \lambda \frac{\pi_v}{\mu_v} & -\xi & 0 & 0 \\
0 & 0 & 0 & -d_p & 0 \\
0 & -\gamma \frac{\pi_v}{\mu_v} & \kappa \xi & 0 & -\mu_v
\end{pmatrix}
$$

At E_0 the above system is stable if $k_J < \frac{\lambda \pi_V}{r_S \mu_V}$.

Theorem 3: The predator-free equilibrium point E_1 exists if $\kappa > \frac{\gamma}{\lambda}$ and $\frac{r_J}{\alpha} < \bar{S} < \frac{\mu_V}{\kappa\lambda - \gamma}$ and the system (1) is stable around E_1 for condition (4).

The predator-free equilibrium point $E_1 = (\bar{J}, \bar{S}, \bar{I}, 0, \bar{V})$, where $\bar{S}$ is the positive root of the cubic equation $A\bar{S}^3 + B\bar{S}^2 + C\bar{S} + D = 0$.

Where $A = \frac{A'}{r_J k_S \xi [\mu_V - (\kappa\lambda - \gamma)\bar{S}]}$, $B = \frac{B'}{r_J k_S \xi [\mu_V - (\kappa\lambda - \gamma)\bar{S}]}$,

$$
C = \frac{C'}{r_J k_S \xi [\mu_V - (\kappa\lambda - \gamma)\bar{S}]}, D = \frac{D' - \lambda \pi_V r_J k_S \xi}{r_J k_S \xi [\mu_V - (\kappa\lambda - \gamma)\bar{S}]}
$$

And

$A' = r_S k_J \alpha\xi(\gamma - \kappa\lambda)$,

$B' = r_S k_J(\alpha\xi k_S \kappa\lambda - \alpha\xi\lambda k_S + \alpha\xi\mu_V + \xi r_J \kappa\lambda - r_J\xi\gamma + \alpha\mu\lambda)$,

$C' = r_S k_J(\xi\lambda r_J k_S - \alpha\xi\mu_V k_S - \xi k_S \kappa\lambda r_J - \xi\mu_V r_J - \lambda\mu_V r_J)$

$$D' = \xi\mu_V r_S r_J k_J k_S$$

The Jacobian matrix for predator-free equilibrium point for Holling type I functional response is given by

$$J = \begin{pmatrix} a^{11} & a^{12} & 0 & 0 & 0 \\ a^{21} & a^{22} & a^{23} & a^{24} & a^{25} \\ 0 & a^{32} & a^{33} & a^{34} & a^{35} \\ 0 & 0 & 0 & a^{44} & 0 \\ 0 & a^{52} & a^{53} & 0 & a^{55} \end{pmatrix}$$

Where,

$a^{11} = r_J - \alpha\bar{S} - 2\bar{J}\frac{r_J}{k_J}, a^{12} = -\alpha\bar{J}, a^{21} = r_S\bar{S}(1 - \frac{\bar{S}+\bar{I}}{k_S})$,

$a^{22} = r_S\bar{J}\left(1 - \frac{\bar{S}+\bar{I}}{k_S}\right) - \frac{r_S}{k_S}\bar{J}\bar{S} - \lambda\bar{V}, a^{23} = -\frac{r_S}{k_S}\bar{J}\bar{S}$,

$a^{24} = -\beta\bar{S}, a^{25} = -\lambda\bar{S}, a^{32} = \lambda\bar{V}, a^{33} = -\xi$,
$a^{34} = -l\bar{I}, a^{35} = \lambda\bar{S}, a^{44} = -d_P + \theta_1\beta\bar{S} + \theta_2\bar{I}$,
$a^{52} = -\gamma\bar{V}, a^{53} = k\xi, a^{55} = -(\mu_V + \gamma\bar{S})$.

$\bar{V} = \frac{\pi_V}{\mu_V - (\kappa\lambda - \gamma)\bar{S}}, \bar{J} = k_J\left(1 - \frac{\alpha\bar{S}}{r_J}\right), \bar{I} = \frac{\pi_V\lambda\bar{S}}{\xi[\mu_V - (\kappa\lambda - \gamma)\bar{S}]}$.

The predator-free equilibrium exists when $\kappa > \frac{\gamma}{\lambda}$ and $\frac{r_J}{\alpha} < \bar{S} < \frac{\mu_V}{\kappa\lambda - \gamma}$. The characteristic equation corresponding to the variational matrix at predator-free equilibrium point given before is,

$\lambda^5 + b_1\lambda^4 + b_2\lambda^3 + b_3\lambda^2 + b_4\lambda + b_5 = 0.$ (2)

Where $b_i s(i = 1,2,3,4)$ are given as follows.

$b_1 = -\sum a^{ii}$,

$b_2 = \sum a^{ii}a^{jj} - \sum a^{ij}a^{ji}$,

$b_3 = -\sum a^{ii}a^{jj}a^{kk} + \sum a^{ij}a^{ji}a^{kk} - \sum a^{ij}a^{jk}a^{ki}$,

$b_4 = \sum a^{ii}a^{jj}a^{kk}a^{ll} - \sum a^{ij}a^{ji}a^{kk}a^{ll} + \sum a^{ij}a^{jk}a^{ki}a^{ll} - \sum a^{ij}a^{ji}a^{kl}a^{lk}$

,

$b_5 = -\sum a^{ii}a^{jj}a^{kk}a^{ll}a^{mm} + \sum a^{ij}a^{ji}a^{kk}a^{ll}a^{mm} - \sum a^{ij}a^{jk}a^{ki}a^{ll}a^{nn} + \sum a^{ij}a^{ji}a^{kl}a^{lk}a^{mm}$.

$(i, j, k, l, m = \{1,2,3,4,5\}\ and\ i \neq j \neq k \neq l \neq m)$

Then by Routh-Hurwitz criterion, it follows that the predator-free equilibrium point $E_1(\bar{J}, \bar{S}, \bar{I}, 0, \bar{V})$ is locally asymptotic*ally stable if*

- $b_i(i = 1,2,3,4,5) > 0$
- $b_1b_2b_3 > b_3^2 + b_1^2b_4$
- $(b_1b_4 - b_5)(b_1b_2b_3 - b_3^2 - b_1^2b_4) > b_5(b_1b_2 - b_3)^2 + b_1b_5^2$

. (3)

Theorem 4: The interior equilibrium point E^* exists if $S^* < \frac{r_J}{\alpha}$ and the system (1) is stable around E^* for condition (7).

The interior equilibrium: $E^*(J^*, S^*, I^*, P^*, V^*)$.

Here $J^* = k_J(1 - \frac{\alpha S^*}{r_J})$,

I^* is the positive root of the equation $c_1 I^{*2} + c_2 I^* + c_3 = 0.$ (4)

Where $c_i s$ are given by

$c_1 = l\mu_V\theta_2\varepsilon_P S^* + \gamma l\theta_2$,

$c_2 = k\lambda\xi - \mu_V\xi - \frac{(l\mu_V\theta_1\beta)}{\varepsilon_P} - \frac{l\mu_V d_P}{\varepsilon_P S^*} - \gamma\xi - \frac{\gamma l}{\varepsilon_P}(\theta_1\beta S^* - d_P)$

,

$c_3 = \lambda\pi_V$.

$P^* = \frac{\theta_1\beta S^* + \theta_2 I^* - d_P}{\varepsilon_P}, V^* = \frac{I^*[\xi\varepsilon_P + l(\theta_1\beta S^* + \theta_2 I^* - d_P)]}{\varepsilon_P\lambda S^*}$.

In Eq. (3) we have a single variation of sign. Then by Descartes' Rule of Sign there should be unique positive root. Therefore$c_2 < 0$.

$i.e. \kappa\lambda\xi - \mu_V\xi - \frac{l\mu_V\theta_1\beta}{\varepsilon_P} - \frac{l\mu_V d_P}{\varepsilon_P S^*} - \gamma\xi - \frac{\gamma l}{\varepsilon_P}(\theta_1\beta S^* - d_P) < 0$

.

$l\gamma\beta S^{*2} + (\varepsilon_P\mu_V\xi + l\mu_V\theta_1\beta + \varepsilon_P\gamma\xi - \kappa\varepsilon_P\xi - l\gamma d_P)S^* + l\mu_V d_P > 0.$

This can be written as $(S - a)(S - b) > 0 \cdot (a < b)$

a, b are given as follows:

$a = \frac{-(\varepsilon_P\mu_V\xi + l\mu_V\theta_1\beta + \varepsilon_P\gamma\xi - \kappa\varepsilon_P\xi - l\gamma d_P) - \sqrt{(\varepsilon_P\mu_V\xi + l\mu_V\theta_1\beta + \varepsilon_P\gamma\xi - \kappa\varepsilon_P\xi - l\gamma d_P)^2 - 4l^2\gamma\theta_1\beta\mu_V d_P}}{2l\gamma\theta_1\beta}$

and

$b = \frac{-(\varepsilon_P\mu_V\xi + l\mu_V\theta_1\beta + \varepsilon_P\gamma\xi - \kappa\varepsilon_P\xi - l\gamma d_P) + \sqrt{(\varepsilon_P\mu_V\xi + l\mu_V\theta_1\beta + \varepsilon_P\gamma\xi - \kappa\varepsilon_P\xi - l\gamma d_P)^2 - 4l^2\gamma\theta_1\beta\mu_V d_P}}{2l\gamma\theta_1\beta}$

The Jacobian matrix for the interior equilibrium point for Holling type I functional response is given by,

$$J = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ 0 & a_{32} & a_{33} & a_{34} & a_{35} \\ 0 & a_{42} & a_{43} & a_{44} & 0 \\ 0 & a_{52} & a_{53} & 0 & a_{55} \end{pmatrix}$$

Where,

$a_{11} = r_J - \alpha S^* - 2J^* \frac{r_J}{k_J}, a_{12} = -\alpha J^*, a_{21} = r_S S^*(1 - \frac{S^*+I^*}{k_S}),$

$a_{22} = r_S J^*\left(1 - \frac{S^*+I^*}{k_S}\right) - \frac{r_S}{k_S} J^* S^* - \lambda V^*, a_{23} = -\frac{r_S}{k_S} J^* S^*,$

$a_{24} = -\beta S^*, a_{25} = -\lambda S^*, a_{32} = \lambda V^*, a_{33} = -\xi,$
$a_{34} = -lI^*, a_{35} = \lambda S^*, a_{44} = -d_P + \theta_1 \beta S^* + \theta_2 I^*,$
$a_{52} = -\gamma V^*, a_{53} = k\xi, a_{55} = -(\mu_V + \gamma S^*).$

The interior equilibrium point E^* exists when $S^* < \frac{r_J}{\alpha}$.

The characteristic equation corresponding to the variational matrix at the interior equilibrium point given before is,

$$\lambda_1^5 + d_1\lambda_1^4 + d_2\lambda_1^3 + d_3\lambda_1^2 + d_4\lambda_1 + d_5 = 0. \quad (5)$$

Where $d_i s(i = 1, 2, 3, 4)$ are as follows:

$$d_1 = -\sum a_{ii},$$

$$d_2 = \sum a_{ii}a_{jj} - \sum a_{ij}a_{ji},$$

$$d_3 = -\sum a_{ii}a_{jj}a_{kk} + \sum a_{ij}a_{ji}a_{kk} - \sum a_{ij}a_{jk}a_{ki},$$

$$d_4 = \sum a_{ii}a_{jj}a_{kk}a_{ll} - \sum a_{ij}a_{ji}a_{kk}a_{ll} + \sum a_{ij}a_{jk}a_{ki}a_{ll} - \sum a_{ij}a_{ji}a_{kl}a_{lk},$$

$$d_5 = -\sum a_{ii}a_{jj}a_{kk}a_{ll}a_{mm} + \sum a_{ij}a_{ji}a_{kk}a_{ll}a_{mm} - \sum a_{ij}a_{jk}a_{ki}a_{ll}a_{mm} + \sum a_{ij}a_{ji}a_{kl}a_{lk}a_{mm} - \sum a_{ii}a_{jk}a_{kl}a_{lm}a_{mj}.$$

$(i, j, k, l, m = \{1, 2, 3, 4, 5\}$ and $i \neq j \neq k \neq l \neq m)$

Then by Routh-Hurwitz criterion, it follows that the interior equilibrium point $E^* = (J^*, S^*, I^*, P^*, V^*)$ is locally asymptotically stable if

- $d_i(i = 1,2,3,4,5) > 0$
- $d_1d_2d_3 > d_3^2 + d_1^2 d_4$ (6)
- $(d_1d_4 - d_5)(d_1d_2d_3 - d_3^2 - d_1^2 d_4) > d_5(d_1d_2 - d_3)^2 + d_1d_5^2.$

IV. NUMERICAL SIMULATION AND DISCUSSION

In this section, we have shown the numerical simulation results using Matlab to validate our analytical findings. We have introduced virus spraying to reduce the pest population. Here our main goal is to control the pest population with the help of virus for healthy Jatropha plantation. This numerical simulation has been performed under parameters given in Table I.

Table I: Parameters value used for numerical simulation [3, 10, 11]

Parameters	Definition	Values (Unit)
r_J	The growth rate of plant biomass	$0.03kg\ day^{-1}$
k_J	The carrying capacity of plant biomass	$50\ kg\ plant^{-1}$
r_S	Maximum growth rate of susceptible pest	$0.05\ day^{-1}$
k_S	The carrying capacity of susceptible pest	$300\ plant^{-1}$
α	Interaction rate between pest and plant	$0.0001\ plant^{-1}day^{-1}$
γ	Reduction rate constant of virus	$0.008\ day^{-1}$
λ	The infection rate of pest by virus	$0.003\ pest^{-1}day^{-1}$
ξ	The mortality rate of infected pest	$0.01\ day^{-1}$
d_P	Death rate of predator	$0.006\ day^{-1}$
ϵ_P	Intra specific competition coefficient	$0.002\ day^{-1}$
θ_1	Conversion factor for predator	0.05
θ_2	Conversion factor for predator	0.01
μ_V	Decay rate of virus	$0.1\ g\ day^{-1}$
β	Consumption rate of susceptible pest by predator	$0.015\ pest^{-1}day^{-1}$
l	Consumption rate of infected pest by predator	$0.7\ pest^{-1}day^{-1}$

In Fig. I, we have plotted the model variables as function of time. It is clear that the system moves towards its stable region as time increases. The trajectories of plant biomass, susceptible pest, infected pest, predator and virus population for Holling type I functional response have been shown. Here we can see that the system moves towards stability after a certain time It has been also observed that susceptible pest is transformed into infected pest for and exterminated by the virus interference. The predator population is initially slightly decreased and then increased gradually, finally reaches its steady state.

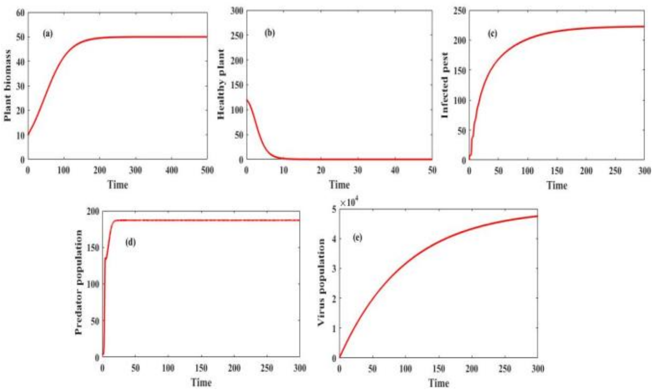

Fig. I Trajectories for different biomass: Plant biomass, Healthy pest, Infected pest, Predator population and Virus population for Holling type I Functional response at κ = 500, other parameter values are given in Table I.

Fig. II depicts the region of existence for different equilibria. Here, R_1 is the existence region of predator-free equilibrium point for the system with linear functional response, whereas R_2 is the existence region of interior equilibrium points, which is represented by Fig. III.

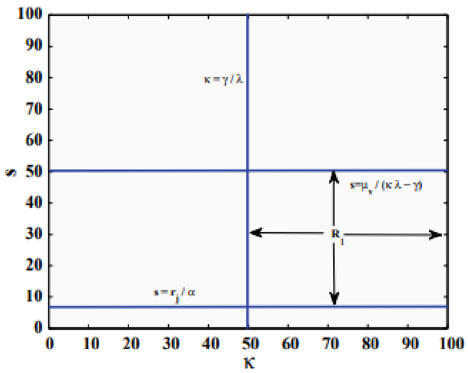

Fig. II The existence region of predator-free equilibrium point for the system with linear functional response

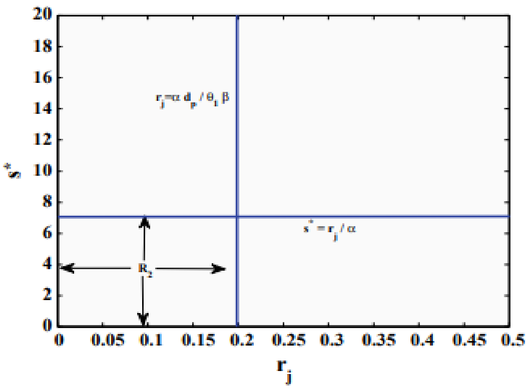

Fig. III The existence region of interior equilibrium point for the system with linear functional response

In Figure IV, we have shown a mesh plotting in κ − λ-susceptible pest plane. This figure shows that for Holling type I functional response with the increasing value of κ and λ, though the pest population does not get eradicated totally from the system, it has been decreased significantly.

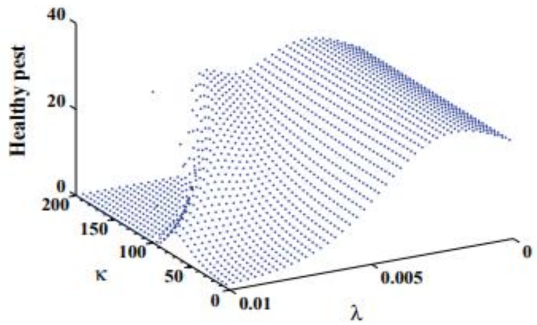

Fig. IV Mesh plotting for linear functional response in κ − λ-susceptible pest plane

V. CONCLUSION

In our research article, we aim to control pest of Jatropha curcas plant. To do so, here we have used virus as controlling agent. The stability and existence of the system have been inspected analytically. We have checked the local stability at pest-free equilibrium, predator-free equilibrium point and interior equilibrium point for Holling type I or linear functional response on predator. Numerically, we have also examined the effect of virus replication. The dynamical behavior of all the biomass, considered in this study, have been studied and depicted with respect to different time intervals. If the pest becomes dominant in the system, then Jatropha plant will get affected severely which will lead to economic loss and consequently production of biodiesel will not be maximum. On the other hand, if the prey density becomes very less or they become extinct, the natural predator will be evanished which may also affect the biological balance of the ecosystem. Thus, it is very important to maintain the biological balance of the ecosystem in such a way so that on one hand crop yield will be maximized and predators also survive. We can easily see that linear functional response is effective in controlling and reducing the pest population within 150 days in order to get healthy plant production.

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