[Volume 4 issue 07 July 2016]
PageNo.1541-1542 ISSN : 2320-7167

INTERNATIONAL JOURNAL OF MATHEMATICS AND COMPUTER RESEARCH

Note on uniform domination number and Non-domination Parameter in Graphs

Dr.A.Vijayalekshmi

Associate Professor

Department of Mathematics S.T.Hindu College,Nagercoil

Tamil Nadu-629 002,India

Email:vijimath.a@gmail.com

Abstract

Let G = (V, E) be a graph. A non-dominating set A $\subseteq$ V is said to be a maximal non-dominating set (mn-d-set) if every superset of A is a dominating set of G. The non-domination number λ of G is the minimum cardinality taken over all mn-d-sets of G. The upper non-domination number Λ of G is the maximum cardinality of a non-dominating set of G. The uniform domination number γ_u (G), is the least positive integer k such that any k-element subset of V is a dominating set of G. In this paper, we obtain a relation between non- domination number and uniform domination number

Key Words: Non- domination number, uniform domination number and upper domination number.

1. Introduction.

All graphs considered in this paper are finite, undirected graphs and we follow standard definitions of graph theory as found in [2].

Let G = (V, E) be a graph of order n. The open neighborhood N(v) of a vertex $v \in V(G)$ consists of the set of all vertices adjacent to v. The closed neighborhood of v is N[v] = N (v) $\cup\{v\}$.For a set S $\subseteq$ V, the open neighborhood N(S) is defined to be $\bigcup_{v \in S} N(v)$, and the closed neighborhood of S is N[S] = N(S) $\cup$ S. The uniform domination number γ_u (G), is the least positive integer k such that any k-element subset of V is a dominating set of G. The concept of uniform domination in graphs was introduced by S.Arumugam and Paul raj Joseph[1].We need the following theorem from [1].

Theorem A[1]. For any graph G, $\gamma_u(G) = p - \delta(G)$.

2.Non-Domination.

A subset A of V is called a non-dominating set (or, in short, n-d set) if there exists a vertex u in V – A which is not adjacent to any vertex in A. A maximal n-d set (mn-d set) is defined in an obvious way. Clearly n-d set A is a mn-d set if and only if A $\cup$ {x} is a dominating set, for every x $\in$ V – A. The n-d number λ of G is minimum cardinality taken over all mn-d-sets of G. The upper n-d number Λ of G is the maximum cardinality of n-d set of G.

The next theorem gives a necessary and sufficient condition for a subset of V(G) to be a mn-d set.

Theorem1. A set $A \subseteq V$ is a mn-d set of G if and only if there exists a vertex $x \in V - A$ such that $A = V - N[x]$, where $N[x]$ is a minimal element of the family of closed neighborhoods.

Proof. Let $A \subseteq V$ be a mn-d set of G and f be the family of closed neighborhood sets of G. Since A is a n-d set, there exists $x \in V - A$ such that $N[x] \cap A = \phi$.Then $A \subseteq V - N[x]$.

As A is a mn-d set, $A = V - N[x]$.Suppose there exists $y \in V$ such that $N[y]$ is a proper subset of $N[x]$. Then $A = V - N[x]$, which is a proper subset of $V - N[y]$, which is not true. Hence $N[x]$ is minimal among the family of closed neighborhoods, f.

Conversely, let $A = V - N[x]$, where $N[x]$ is minimal element in f. Then clearly A is a n-d set. Let $y \in N[x]$, $y \neq x$.Suppose that $A \cup \{y\}$ is also a n-d set. Then there exists a vertex z in V such that $N[z] \subseteq N[x] - \{y\}$ which contradicts the minimality of $N[x]$.Thus A is a mn-d set. ■

Proposition2.A graph G is totally disconnected if and only if $\lambda(G) = p - 1$.

Corollary 3. For any graph G, $\lambda = p - \delta - 1$ and $\Lambda \geq p - \Delta - 1$.

Theorem 4. For any graph G, $\lambda = \gamma_u - 1$.

Proof. By definition of λ, there exists a n-d set of cardinality λ. Therefore $\gamma_u \geq \lambda + 1$.

Also every set of cardinality $\lambda + 1$ is a dominating set. Therefore $\gamma_u \leq \lambda + 1$.

Hence $\gamma_u = \lambda + 1$. ■

Remark 5. Corollary (2) and theorem (3) provide another proof for the fact $\gamma_u = p - \delta$. [1].

Acknowledgement

I am very grateful to the University Grants Commissions for grants-in-aid to Minor Research Projects No.F MRP-5344/14(SERO/UGC).

Referrences

[1] Arumugam,S. and Joseph,P.,The uniform domination numbers of a graph, Int.J.Management.Syst.,**11**,(1995)111 – 116.

[2] F.Harary, Graph Theory,Addison-Wesley Reading Mass,1969.

[3] Terasa W.Haynes, Stephen T.Hedetniemi, Peter J.Slater, Fundamentals in Domination on Graphs,Marcel Dekker,New York,1998.

[4] Terasa W.Haynes, Stephen T.Hedetniemi, Peter J.Slater,Domination in Graphs-Advanced Topics,Marcel Dekker,New York,1998.