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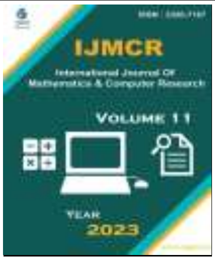

An Advanced Method to Find Optimal Solution for Fully Interval Integer Transportation Problem

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ABSTRACT
In this paper, we develop an efficient algorithm is proposed for finding an optimal solution to fully interval integer transportation problem (TP), in which shipping cost, supply and demand are real intervals. A numerical example is established and the optimality is checked by this method.

KEYWORDS: Transportation Problem (TP), Transportation Table (TT), Real Intervals, Optimal solution

INTRODUCTION

Transportation model defines a vital role to derive the efficient role of availability of materials and goods from sources to destinations. Transportation problem is a linear programming problem deals from a network structure consisting of a number of nodes and arcs [1, 2]. The objective of the TP is to determine the transportation schedule that minimizes the total transportation cost. To find optimal solutions to TP different methods are discussed in many papers [3].

To find the initial basic feasible solution of a TP, several researchers have developed alternative methods. This problem has been studied, since long and is well known by Pandian P, Natarajan G[4] deals a new method for finding an optimal solution for TPs, and a new method for finding an optimal solution of fully interval integer TPs[5], Pandian P, Natarajan G, Akilbasha A[6,8], deals fully rough integer interval TP and another one paper an innovative exact method for solving fully interval integer TPs, Kumar BR, Murugesan S[7], New optimal solution to fuzzy interval TP and Padma Karthiyayini G, Dr.S.Ananthalakshmi and Dr.R.Usha Parameswari deals an inventive method for solving fully interval TP.

In this paper a proposed algorithmic approach is to find an optimal solution for the TPs.

Preliminaries:

Let $\mathfrak{D}$ denote the set of all closed bounded intervals on the real line $\mathfrak{R}$.

$\mathfrak{D}$= {[a, b]: a≤ b, a and b are in $\mathfrak{R}$}

Definition:

Let $\mathcal{A}$= [a, b] and $\mathfrak{B}$=[c, d] be in $\mathfrak{D}$. Then,

(i) $\mathcal{A} \oplus \mathfrak{B}$ =[a+c , b+d] and

(ii) $\mathcal{A} \otimes \mathfrak{B}$ =[$\wp$, q] , where $\wp = \min\{ac, ad, bc, bd\}$ and $q = \max\{ac, ad, bc, bd\}$.

Definition:

Let $\mathcal{A}$= [a, b] and $\mathfrak{B}$=[c,d] be in $\mathfrak{D}$. Then,

(i) $\mathcal{A} \leq \mathfrak{B}$ if and only if a≤c and b≤ d

(ii) $\mathcal{A} = \mathfrak{B}$ if and only if a=c and b=d.

Definition:

Let $\mathcal{A}$= [a,b] in $\mathfrak{D}$. Then, $\mathcal{A}$ is said to be positive denoted by $\mathcal{A} \geq 0$ if a≥ 0.

Definition:

Let $\mathcal{A}$= [a, b] in $\mathfrak{D}$. $Then, \mathcal{A}$ is said to be integer if a and b are integers.

The half width-value of an interval $\mathcal{A}$= [a, b],

$\mathfrak{W}(\mathcal{A})$ *is* Defined as $\mathfrak{W}(\mathcal{A})=\frac{b-a}{2}$

Fully Interval Integer Transportation Problem:

Consider the fully interval integer transportation problem (*) is

Minimize $\mathcal{Z}$= [$\mathcal{Z}_1, \mathcal{Z}_2$]=$\sum_{i=1}^{m}\sum_{j=1}^{n}[\mathcal{C}_{ij},\mathcal{D}_{ij}] \otimes [\mathcal{X}_{ij}, \mathcal{Y}_{ij}]$

$\sum_{j=1}^{n}[\mathcal{X}_{ij}, \mathcal{Y}_{ij}]$= [$a_i$, $\wp_i$], i=1,2,…m -------(I)

$\sum_{j=1}^{n}[\mathcal{X}_{ij}, \mathcal{Y}_{ij}]$= [$b_j$, q_j], j=1,2,…n -------(II)

$\mathcal{X}_{ij} \geq 0, \mathcal{Y}_{ij} \geq 0$, i=1,2,…m and j=1,2,…n are integers. -----(III)

Where $\mathcal{C}_{ij}, \mathcal{D}_{ij}$, a_i, p_i, b_j, $and\ q_j$ are all positive real numbers for all i and j.

Definition:

The set $\{[\mathcal{X}_{ij}, \mathcal{Y}_{ij}]$, for all i=1, 2 …m and j=1, 2…n} is said to be a feasible solution, if they satisfy equations (I) – (III).

Definition:

A feasible solution $\{[\mathcal{X}_{ij}, \mathcal{Y}_{ij}]$, for all i=1, 2 …m and j=1, 2…n} of the problem (*) is said to be an optimal solution of (*) if,

$\sum_{i=1}^{m}\sum_{j=1}^{n}[\mathcal{C}_{ij,}\mathcal{D}_{ij}] \otimes [\mathcal{X}_{ij}, \mathcal{Y}_{ij}] = \sum_{i=1}^{m}\sum_{j=1}^{n}[\mathcal{C}_{ij,}\mathcal{D}_{ij}] \otimes [\mathcal{U}_{ij}, \mathcal{V}_{ij}]$, for all feasible solution $\{[\mathcal{U}_{ij}, \mathcal{V}_{ij}]$ for all i=1, 2 …m and j=1, 2…n}.

LOWER CASE METHOD

Step 1:

First construct the independent transportation problems from the given problem.

Step 2:

In the interval value, choose the lower case value to form the transportation table.

Step 3:

Choose the smallest value for each row of the transportation table and subtract it from the corresponding row wise.

Step 4:

In the same way, choose the smallest value for each column of the transportation table and subtract it from the corresponding column wise.

Step 5:

In the reduced transportation table, there will be at least one zero value in each row and column.

Step 6:

Mark all the zeros in the respective row and column.

Step 7:

Choose a zero with minimum supply or demand and then delete the row or column.

Step 8:

If all the supply and demands are satisfied then we get the optimal solution of the given problem. $\{l_{ij}^{*}$, for all i and j} be an optimal solution of the problem.

Half width value method:

Step 1:

In the interval value, find the half width value to form the transportation table.

Step 2:

Choose the smallest value for each row of the transportation table and subtract it from the corresponding row wise.

Step 3:

In the same way, choose the smallest value for each column of the transportation table and subtract it from the corresponding column wise.

Step 4:

In the reduced transportation table, there will be at least one zero value in each row and column.

Step 5:

Mark all the zeros in the respective row and column.

Step 6:

Choose a zero with minimum supply or demand and then delete the row or column.

Step 7:

If all the supply and demands are satisfied then we get the optimal solution of the given problem. $\{\mathfrak{W}_{ij}^{*}$, for all i and j} be an optimal solution of the problem.

Step 8:

The optimal solution of the given problem (*) is $\{[l_{ij}^{*} - \mathfrak{W}_{ij}^{*}, l_{ij}^{*} + \mathfrak{W}_{ij}^{*}], for\ all\ i\ and\ j\}$

Example:

A Company produces a product in its three factories $\mathcal{F}1, \mathcal{F}2\ and\ \mathcal{F}3$. the product will be sent to four destinations $\mathcal{D}1, \mathcal{D}2, \mathcal{D}3\ and\ \mathcal{D}4$ from the three factories. Determine a shipping cost for the company from three factories to four destinations such that the total shipping cost should be minimum using the following data obtained from the company, the minimum supply from $\mathcal{F}1, \mathcal{F}2\ and\ \mathcal{F}3$ are 7,17 and 16 respectively and maximum supply from $\mathcal{F}1, \mathcal{F}2\ and\ \mathcal{F}3$ are 9,21 and 18 respectively. The minimum demand for $\mathcal{D}1, \mathcal{D}2, \mathcal{D}3\ and\ \mathcal{D}4$ are 10,2,13 and 15 respectively and the maximum demand for $\mathcal{D}1, \mathcal{D}2, \mathcal{D}3\ and\ \mathcal{D}4$ are 12,4,15 and 17 respectively.

Unit shipping cost range from supply points to demand points

Table 1. Interval Transportation Table

	$\mathcal{D}1$	$\mathcal{D}2$	$\mathcal{D}3$	$\mathcal{D}4$	Supply
$\mathcal{F}1$	[1,2]	[1,3]	[5,9]	[4,8]	[7,9]
$\mathcal{F}2$	[1,2]	[7,10]	[2,6]	[3,5]	[17,21]
$\mathcal{F}3$	[7,9]	[7,11]	[3,5]	[5,7]	[16,18]
Demand	[10,12]	[2,4]	[13,15]	[15,17]	[40,48]

Table 2. Lower case Transportation Table

Lower case value transportation problem of the problem is

	$\mathcal{D}1$	$\mathcal{D}2$	$\mathcal{D}3$	$\mathcal{D}4$	Supply
$\mathcal{F}1$	1	1	5	4	7
$\mathcal{F}2$	1	7	2	3	17
$\mathcal{F}3$	7	7	3	5	16
Demand	10	2	13	15	40

Table 3. Zero Cost Value Method

0	0	4	1
0	6	1	0
4	4	0	0

Table 4. Zero Cost Value Method

[0]	[0]	4	1
[0]	6	1	[0]
4	4	[0]	[0]

Table 5. Zero Cost Value Method

0	0 [2]		1
0	6	1	0
4	4	0	0

Table 6. Zero Cost Value Method

0 [5]	4	1
0 [5]	1	0 [12]
4	0 [13]	0 [3]

Table 7. Lower Case Method Allotment Table

1 [5]	1 [2]	5	4
1 [5]	7	2	3 [12]
7	7	3 [13]	5 [3]

The given problem can be modeled as an interval integer transportation problem

The feasible value of lower case value method is

l_{11}^{*}=5, l_{12}^{*}=2, l_{21}^{*}=5, l_{24}^{*}=12, l_{33}^{*}=13, l_{34}^{*}=3

Half –width TP (W) of the problem (P)

Table 8. Half-width Transportation Table

	D_1	D_2	D_3	D_4	Supply
F_1	0.5	1	2	2	1
F_2	0.5	1.5	2	1	2
F_3	1	2	1	1	1
Demand	1	1	1	1	4

By using zero cost value method.

Choose the smallest entry for each row and column of the transportation table and subtract it from the corresponding row wise and column wise.

Table 9. Zero Cost Value Method

0	0	1.5	1.5
0	0.5	1.5	0.5
0	0.5	0	0

First mark all the zeros in respective row and column

Table 10. Zero Cost Value Method

0	0	1.5	1.5
0	0.5	1.5	0.5
0	0.5	0	0

The first allocation is

Table 11. Zero Cost Value Method

0 1	0	1.5	1.5
0	0.5	1.5	0.5
0	0.5	0	0

Table 12. Zero Cost Value Method

0	1.5	1.5
0 1	1	0 1
0.5	0 1	0

The allocated cell values are in the following transportation table is

Table 13. Half Width Allotment Table

0.5 [1]	1	2	2
0.5	1.5 [1]	2	1 [1]
1	2	1 [1]	1

The feasible value of half width method is

$\mathfrak{W}^*_{11}$=1, $\mathfrak{W}^*_{22}$=1, $\mathfrak{W}^*_{24}$=1, $\mathfrak{W}^*_{33}$=1

An optimal solution to the given transportation problem is,

$[\mathcal{X}_{11}, \mathcal{Y}_{11}]$=[4,6];$[\mathcal{X}_{12}, \mathcal{Y}_{12}]$=[2,2];

$[\mathcal{X}_{21}, \mathcal{Y}_{21}]$=[5,5]; $[\mathcal{X}_{22}, \mathcal{Y}_{22}]$=[-1,1];$[\mathcal{X}_{24}, \mathcal{Y}_{24}]$=[11,13];

$[\mathcal{X}_{33}, \mathcal{Y}_{33}]$=[13,13]; $[\mathcal{X}_{34}, \mathcal{Y}_{34}]$=[2,4] .

The minimum interval transportation cost

= [4 6][1 2]+[2 2][1 3]+[5 5][1 2]+[-1 1][7 10]+[11 13][3 5]+[13 13][3 5]+[2 4][5 7]

= [4 12]+[2 6]+[5 10]+[-7 10]+[33 65]+[39 65]+[10 28]

= [86 196]

CONCLUSION

In this paper, a new method along with an algorithm for finding an optimal solution of a transportation problem is introduced. The proposed method is based on two independent transportation problems that are obtained from the interval transportation problem. As this method is very easy to understand and apply, so it will be very helpful for dealing with chain problems.

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