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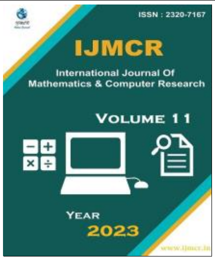

On Finding Integer Solutions to the Homogeneous Cone $x^2+(k^2+2k)y^2=(k+1)^4z^2$

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ARTICLE INFO	ABSTRACT
Published online: **20 July 2023** **Corresponding Name** J. Shanthi	This paper aims at determining non-zero distinct integer solutions to the homogeneous cone given by $x^2+(k+2k)y^2=(k+1)^4z^2$. A few interesting properties between the solutions are presented.

KEYWORDS: Homogeneous cone ,Ternary quadratic ,Integer solutions

INTRODUCTION

The subject of quadratic Diophantine equations has a rich variety of fascinating problems. The homogeneous or non-homogeneous quadratic equations with three unknowns are rich in variety. For an extensive review of sizable literature and various problems, one may refer [1-14].This paper concerns with yet another interesting homogeneous ternary quadratic Diophantine equation given by $x^2+(k+2k)y^2=(k+1)^4z^2$ for determining its distinct integer solutions.

METHOD OF ANALYSIS

The ternary quadratic equation representing homogeneous cone under consideration is $x^2+(k^2+2k)y^2=(k+1)^4z^2$ (1)The introduction of the linear transformations $x=X+(k^2+2k)T, y=X-T$ (2)in (1) leads to $X^2+(k^2+2k)T^2=(k+1)^2z^2$ (3)Assume $z=a^2+(k+2k)b^2$ (4) Write $(k+1)^2$ on the R.H.S. of (3) as $(k+1)^2=(1+i\sqrt{k^2+2k})(1-i\sqrt{k^2+2k})$ (5)Substituting (4) and (5) in (3) and employing the method of factorization ,the corresponding values of X, T obtained through equating the rational and irrational parts are as follows:

$$X=a^2-(k^2+2k)b^2-2(k^2+2k)ab,$$
$$T=a^2-(k^2+2k)b^2+2ab$$

In view of (2), it is seen that

$$\left.\begin{aligned} x&=(k+1)^2a^2-(k^2+2k)(k+1)^2b^2, \\ y&=-2ab(k+1)^2 \end{aligned}\right\} \quad (6)$$

Thus, (4) and (6) represent the integer solutions to (1).

PROPERTIES

i) $x+y$ is expressed as difference of two squares

ii) x is a perfect square when $a=(k^2+2k)p^2+q^2, b=2pq$

iii) z is a perfect square when $a=(k^2+2k)p^2-q^2, b=2pq$

iv) The members of the triple $(x, (k+1)^2 a^2, (k+1)^2 z)$ form an Arithmetic progression

v) z is a cubical integer when $a = A(A^2 + (k+2k)B^2), b = B(A^2 + (k+2k)B^2)$

vi) Each of the following expressions is a nasty number $3(x + (k+1)^2 z), 3(k+1)^2 (z + 2b^2) - 3x$ Albeit tacitly ,there are other sets of integer solutions to (1) that are illustrated below:

Set 1: Write (3) as $(k+1)^2 z^2 - (k^2 + 2k)T^2 = X^2$ (7) Assume $X = (k+1)^2 a^2 - (k^2 + 2k)b^2$ (8)

Using (8) in (7) and applying the method of factorization, define $(k+1)z + \sqrt{k^2 + 2k}\,T = ((k+1)a + \sqrt{k^2 + 2k}\,b)^2$ (9)

Equating the rational and irrational parts in (9) and replacing b by $(k+1)B$,the corresponding integer solutions to (1) after some algebra are given by

$$x = (k+1)^4 a^2 - (k^2 + 2k)(k+1)^2 (B - a)^2,$$
$$y = (k+1)^2 \left[(a - B)^2 - (k+1)^2 B^2\right],$$
$$z = (k+1)a^2 + (k^2 + 2k)(k+1)B^2$$

Set 2:

Rewrite (7) as $(k+1)^2 z^2 - (k^2 + 2k)T^2 = X^2 * 1$ (10) Write 1 on the R.H.S. of (10) as

$1 = (k + 1 + \sqrt{k^2 + 2k})(k + 1 - \sqrt{k^2 + 2k})$ (11)

Substituting (8) & (11) in (10) and following the procedure as above ,the corresponding integer solutions to (1) are given by

$$x = \left[(k^2 + 2k)b + (k+1)^2 a\right]^2 - (k^2 + 2k)b^2,$$
$$y = -2(k+1)^2 ab - 2(k^2 + 2k)b^2,$$
$$z = (k^2 + 2k)(a + b)^2 + a^2$$

Set 3:

Write (3) in the form of ratios as

$$\frac{(k+1)z + X}{kT} = \frac{(k+2)T}{(k+1)z - X} = \frac{P}{Q}, Q \neq 0 \quad (12)$$

Which is equivalent to the system of double equations

$$QX - kPT + (k+1)Qz = 0,$$
$$PX + (k+2)QT - (k+1)Pz = 0$$

Employing the method of cross-multiplication and using (2) , the corresponding integer solutions to (1) are found to be

$$x = k(k+1)^3 P^2 - (k+1)(k+2)(Q - kP)^2,$$
$$y = (k+1)(kP^2 - (k+2)Q^2 - 2PQ),$$
$$z = (k+2)Q^2 + kP^2$$

Note: One may write (3) in the form of ratios as $\frac{(k+1)z + X}{(k+2)T} = \frac{kT}{(k+1)z - X} = \frac{P}{Q}, Q \neq 0$

The repetition of the above process leads to a different set of integer solutions Remark: In addition to (2), on introducting the following linear transformations $x = X - (k^2 + 2k)T, y = X + T$, different sets of integer solutions to (1) are obtained .

CONCLUSION

In this paper,an attempt has been made to obtain non-zero distinct integer solutions to the considered homogeneous cone. To conclude, one may search for the integral solutions to the other choices of homogeneous or non-homogeneous cones.

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