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D^{dM}- Distance in Cycle Related Graphs

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ABSTRACT

For two vertices u and v of a graph G, the usual distance d (u, v), is the length of the shortest path between u and v. In this paper we introduced the concept of D^{dM}- distance by considering the degrees of various vertices presented in the path, in addition to the length of the path. We study some properties with this new distance. We define the eccentricities of vertices, radius and diameter of G with respect to the D^{dM}- distance. First we prove that the new distance is a metric on the set of vertices of G. We compare the usual, detour and D^{dM}-distances of two vertices u,v of V.

KEYWORDS: Detour distance, D^{dM}- Distance, D^{dM}- Eccentricity, D^{dM}- Radius and D^{dM}- Diameter.

I. INTRODUCTION

By a graph G, we mean a non-trivial finite undirected connected graph without multiple edges and loops. Following standard notations (for any unexplained notation and terminology we refer [2]) V(G) or V is the vertex set of G and E(G) or E is the edge set of G = G(V, E). Let u, v be two vertices of G. The standard or usual distance d(u, v) between u and v is the length of the shortest u - v path in G. Chartrandel al [3] introduced the concept of detour distance in graphs as follows: For two vertices u, v in a graph G,the detour distance D(u, v) is defined as the length of the longest u - v path in G. In this article we introduce a new distance, which we call as D^{dM}- distance between any two vertices of a graph G, and study some of its properties. This distance is significantly different from other distances. In some of the earlier distances, only path length was considered. Here we, in addition, consider the degree of u and v vertices present in a u - v path while defining its length. Using this length we define the D^{dM}- distance. Chartand et al introduced the concept ofdetour distance by considering the length of the longest path between u andv. Kathiresan et al [4] introduced the concept of superior distance and signal distance.Goldern Ebenezer et al [13] introduced the concept of d^d - distance. In some of these distances only the lengths of various paths were considered. We introduce the concept of detour mean D^{dM}- distance in some graphs *G*.

II. D^{dpM}- DISTANCE IN GRAPHS

DEFINITION 2.1

Let *u*, *v* be two vertices of a connected graph *G*. Then the detour meanD^{dM}- *length* of a *u*-*v* path defined as $D^{dM}(u,v) = D(u, v) + \deg(u) + \deg(v) + \left\lceil \frac{\deg(u)+\deg(v)}{2} \right\rceil$ or $\left\lfloor \frac{\deg(u)+\deg(v)}{2} \right\rfloor$, where *D*(*u*, *v*) is the longest distance between the vertices *u* and *v*. The D^{dM}*-distance* between two vertices *u* and *v* is defined as the D^{dM}- length of a *u*-*v* path.

DEFINITION 2.2

The D^{dM}- eccentricity of any vertex v, $e^{D^{dM}}(v)$, is defined as the maximum distance from v to any other vertex, i.e., $e^{D^{dM}}(v) = \max\{D^{dM}(u,v): u, v □□V(G)\}$

DEFINITION 2.3

Any vertex u for which $D^{dM}(u, v) = e^{D^{dM}}(v)$ is called D^{dM}- eccentric vertex of v. Further, a vertex u is said to be D^{dM}- eccentric vertex of G if it is the D^{dM}- eccentric vertex of some vertex.

DEFINITION 2.4

The D^{dM}-radius, denoted by $r^{D^{dM}}(G)$, is the minimum D^{dM}- eccentricity among all vertices of G i.e.,

$r^{D^{dM}}(G) = \min\{e^{D^{dM}}(v) : v□V(G)\}$. Similarly the D^{dM}-diameter, $D^{dM}(G)$, is the maximum D^{dM}- eccentricity among all vertices of G.

DEFINITION 2.5

A graph is called D^{dM} self centered if radius is same as diameter, i.e., $r^{D^{dM}} = d^{D^{dM}}$.

REMARK 2.6

If *G* is any connected graph, then the D^{dM}- distance is a metric on the set of vertices of *G*.

THEOREM 2.7

Consider the cyclic graph C_n, with n vertices. Then the product mean $\boldsymbol{D^{dM}}$ - eccentricity of any vertex $\boldsymbol{v}$ in C_n is $\boldsymbol{n+5}$.

PROOF:

Let $\boldsymbol{D(u,v) = n-1, \deg(u) = 2, \deg(v) = 2}$

$$\boldsymbol{D^{dpM}(u,v) = D(u,v) + \deg(u) + \deg(v) + \left\lceil \frac{\deg(u)+\deg(v)}{2} \right\rceil}$$

$$= \boldsymbol{(n-1) + 2 + 2 + \left\lceil \frac{2+2}{2} \right\rceil}$$

$$= \boldsymbol{n - 1 + 2 + 2 + 2}$$

$$= \boldsymbol{n + 5}.$$

COROLLARY 2.8

The cyclic graph, C_n is product mean $\boldsymbol{D^{dM}}$ self centered.

THEOREM 2.9

In wheel graph $W_{1,n}$, with n + 1 vertices the product mean $\boldsymbol{D^{dpM}}$ of any vertex is given below

If n is odd, $\boldsymbol{D^{dpM} = 5\left(\frac{n+1}{2}\right) + 2}$

If n is even, $\boldsymbol{D^{dpM} = 5\left(\frac{n}{2}\right) + 5}$.

PROOF:

i) For Odd n,

$\boldsymbol{D(u,v) = n, \deg(u) = 3, \deg(v) = n}$.

$$\boldsymbol{D^{dpM}(u,v) = D(u,v) + \deg(u) + \deg(v) + \left\lceil \frac{\deg(u)+\deg(v)}{2} \right\rceil}.$$

$$= \boldsymbol{n + 3 + n + \left\lceil \frac{3+n}{2} \right\rceil}.$$

$$= \boldsymbol{2n + 3 + \left\lceil \frac{3+n}{2} \right\rceil}.$$

$$= \boldsymbol{\frac{5(n+1)}{2} + 2}.$$

ii) For Even n,

$$\boldsymbol{D^{dpM}(u,v) = \frac{5(n+1)}{2} + 2}.$$

$$= \boldsymbol{\frac{5n+5}{2} + 2}.$$

$$= \boldsymbol{\frac{5n+5+4}{2}}.$$

$$= \boldsymbol{\frac{5n+9}{2}}.$$

$$= \boldsymbol{\frac{5n+9+1-1}{2}}.$$

$$= \boldsymbol{5\left(\frac{n}{2}\right) + 5}.$$

COROLLARY 2.1

The wheel graph $W_{1,n}$, is product mean $\boldsymbol{D^{dM}}$ self centered.

THEOREM 2.11

In friendship graph F_n, the product mean $\boldsymbol{D^{dM}(u,v) = 3n + 5}$.

PROOF

Let $\boldsymbol{D(u,v) = 2, \deg(u) = 2, \deg(v) = 2n}$

$$\boldsymbol{D^{dM}(u,v) = D(u,v) + \deg(u) + \deg(v) + \left\lceil \frac{\deg(u)+\deg(v)}{2} \right\rceil}.$$

$= 2 + 2 + 2n + \left\lceil \frac{2+2n}{2} \right\rceil$

= 4 + 2n + (n + 1)

= 4 + 2n + n + 1

= 5 + 3n

= 3n + 5.

COROLLARY 2.12

The friendship graph $F_{n,}$ is product mean $\boldsymbol{D^{dM}}$ self-centered.

THEOREM 2.13

In fan graph f_n, with n + 1 vertices the product mean $\boldsymbol{D^{dM}}$ of any vertex is given below

The radius $\boldsymbol{r^{D^{dM}}}$ If n is odd then $\boldsymbol{r^{D^{dM}}(u,v) = 4\left(\frac{n+1}{2}\right) + 3}$.

If n is even then $\boldsymbol{r^{D^{dM}}(u,v) = 4\left(\frac{n}{2}\right) + 6}$.

The diameter $\boldsymbol{d^{D^{dM}}}$ If n is odd then $\boldsymbol{d^{D^{dM}}(u,v) = \frac{5(n+1)}{2} + 1}$

If n is even then $\boldsymbol{d^{D^{dM}}(u,v) = \frac{5n}{2} + 4}$

THEOREM 2.14

In flower graph (C_m, K_n), the product mean $\boldsymbol{D^{dM}}$ is given below

Diameter $\boldsymbol{d^{D^{dpM}} = 4n + 7}$.

Radius $\boldsymbol{r^{D^{dpM}} = 4n + 5}$.

CONCLUSION

Many researchers are concentrating various distance concepts in graphs. In this paper we have studied about D^{dM}-distance in graphs. Then we have defined D^{dM}- eccentricity, D^{dM}- radius and D^{dM} – diameter. Many results have been found.

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