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Range Labeling for Some Graphs

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ABSTRACT

In this paper, we focus on one type of labeling is called range labeling, we have introduced range labeling for certain graphs as for P_z, C_z, $S_{y,z}$ (Double star), sun graph and for some trees.

INTRODUCTION

In this paper consider for all graphs are finite and simple. The graph G is the vertex set S(G) and edge set T(G). A graph labeling G is a function that carries graph elements to integers. The labeling was first introduced by Rosa in 1967.

In this paper range labeling apply for some Graphs. Neelam kumari and Seema mehra was first developed one-edge magic labeling and n-edge magic labeling in 2013(5). Prime edge magic labeling origin in Dr. R. Jahir Hussain and J. Senthamizh Selvan in 2021(2). Range labeling was first developed in J.Senthamizh Selvan and Dr.R.Jahir Hussain in 2023(7).

1. PRELIMINARY

Definition1.1. n-edge magic labeling:

Let G= (S, T) be a graph S= $\{S_k, 1\le k\le z\}$ and T = $\{S_kS_{k+1}, 1\le k\le z\text{-}1\}$. Let $\propto$: S $\rightarrow$ [-1, n+1] and $\propto^*$: T $\rightarrow$ [n] such that for every $S_kS_{k+1}\in T$, $\propto^*[S_kS_{k+1}] = \propto[S_k] + \propto[S_{k+1}] = n$. This result is called n-edge magic labeling(5).

Definition1.2. Prime edge magic labelling

Let G= (S, T) be a graph, S = $\{S_k, 1\le k\le z\}$ and T = $\{S_kS_{k+1}, 1\le k\le z\text{-}1\}$. Let$\propto$: S $\rightarrow$[-p,2p] and$\propto^*$: T$\rightarrow$[p] such that if p is a Prime number. For every $S_kS_{k+1}\in T$, $\propto^*$ $[S_kS_{k+1}]$ $=\propto[S_k]+\propto[S_{k+1}]$ =P. This result is called Prime edge magic labeling (2).

Definition1.3. Graceful labelling

A graceful labeling of a graph G is a vertex labeling $\propto$: S$\rightarrow$ [0, m] such that $\propto$ is injective and the edge labeling $\propto^*$: T$\rightarrow$ [1, m] is defined by$\propto^*[S_kS_{k+1}]$ $=|\propto(S_k)-\propto(S_{k+1})|$ is also injective. If a graph G admits a graceful labeling. We say G is a graceful graph (4).

Definition1.4. Symmetrical tree

A rooted tree in which every level contains vertices of the same degree is called Symmetrical tree (4).

2.MAIN RESULTS

The idea of graceful labeling and prime edge magic labeling stimulate us to decide coming new result of Range labeling.

2.1Range labeling:

Let G= (S, T) be a graph with n vertices. An injective function on $\propto$: S$\rightarrow$ {1,3,6,10,15,21,28..., $(n^2+n)/2$} is called a range labeling if the edge labels are $\propto^*$: T$\rightarrow$ {1,2, 3..., $((n^2+n)/2)-1)$} and T is defined by $\propto^*$(T)= Maximum value (S_k,S_{k+1}) – Minimum value (S_k,S_{k+1}). If a graph G admits range labeling,we say G is a range graph(7).

2.2Range Value;

Let G= (S, T) be a range graph, then the range value is
Range value of graph G=Maximum edge value of G-Minimum edge value of G.
That is, RV(G)=MAEV(G)-MIEV(G).

Theorem 1. The Path P_z is a Range graph.

Proof:

Let G= (S, T) be a graph. Let S(G)= $\{S_k, 1\le k\le z\}$, T (G)= $\{S_kS_{k+1}, 1\le k\le z\text{-}1\}$ if $\propto$:S$\rightarrow$ {1,3,6,10,15,21,28..., $(n^2+n)/2$}, $\propto^*$(T)=Maximum value of (S_k,S_{k+1}) − Minimum value of (S_k,S_{k+1}).

If S_k is a minimum value and S_{k+1} is a maximum value.

$\propto^*(T) = S_{k+1} - S_k = Z$ is an integer.
Suppose, S_{k+1} is a minimum value and S_k is a maximum value.
$\propto^*(T) = S_k - S_{k+1} = Z$
∴ Z is an integer. Hence every path graph is accepting Range labeling.
∴ Any path graph is range graph.

Example 1: Range labelling of P_7 , P_{10} shown in the figure Remark

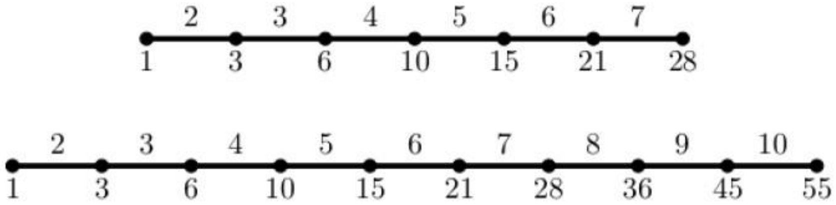


Figure 1

Remark 1:

Range value of P_7 = Maximum edge value of (P_7)-Minimum edge value of (P_7)

RV(P_7) =7-2
=5

Range value of P_{10} = Maximum edge value of (P_{10})-Minimum edge value of (P_{10})

RV ($_{10}$) =10-2
=8

Theorem 2. The cycle C_z is a Range graph.

Proof:

Let G= (S, T) be a graph. Let S(G)= {S_k, $1 \leq k \leq z$}, T(G)= {S_kS_{k+1}, $1 \leq k \leq z-1$}. If $\propto$:S→ {1,3,6,10,15,21,28..., $(n^2+n)/2$},

$\propto^*(T)$= Maximum value of (S_k,S_{k+1}) – Minimum value of (S_k,S_{k+1}).

If S_k is a minimum value and S_{k+1} is a maximum value.
$\propto^*(T) = S_{k+1} - S_k = Z$
∴ Z is a integer.
Suppose, S_{k+1} is a minimum value and S_k is a maximum value.
$\propto^*(T) = S_k - S_{k+1} = Z$
∴ Z is a integer. Hence, every cycle graph is accepting Range labeling.
∴ Any cycle graph is Range graph.

Example 2: Range labeling of C_5, C_6 shown in the figure 2.

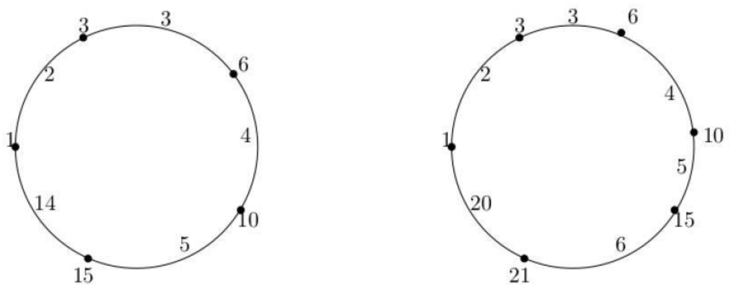


Figure 2

Remark 2:

Range value of C_5 = Maximum edge value of (C_5)-Minimum edge value of (C_5)

RV(C_5) =14-2
=12.

Range value of C_6 = Maximum edge value of (C_6)-Minimum edge value of (C_6)

RV(C_6) =20-2
=18.

Theorem 3.A sun graph S_z is a Range graph.

Proof:

Let G = (S, T) be a graph. Let S_1, S_2, S_3,.,S_z is a vertices of cycle S_z and r_1,r_2,...,r_z is a end vertices of every edge fixed to S_1, S_2, S_3,....,S_z. If $\propto$:S→ {1,3,6,10,15,21,28..., $(n^2+n)/2$} &$\propto_1$: R→ {1,3,6,10,15,21,28..., $(n^2+n)/2$}.

$\propto^*(T)$= Maximum value of (S_k,S_{k+1}) – Minimum value of (S_k,S_{k+1}).

If S_k is a minimum value and S_{k+1} is a maximum value.
$\propto^*(T) = S_{k+1} - S_k = Z$
∴Z is an integer.
Suppose S_{k+1} is minimum value and S_k is a maximum value.
= S_k- S_{k+1}= Z
∴Z is an integer.

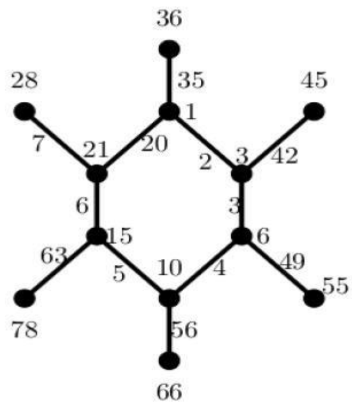
$\propto^*(T)$= Maximum value of (S_k,R_k) – Minimum value of (S_k,R_k).

If S_k is a minimum value, R_k is a maximum value.
$\propto^*(T) = R_k - S_k = Z$
∴Z is a integer.
Suppose R_k is a minimum value and S_k is a maximum value.
$\propto^*(T) = S_k - R_k = Z$

∴Z is an integer.
∴Any sun graph is a Range graph.

Hence, Every Sun graph is accepting Range labelling.

Example 3:



Remark 3:
Range value of S_6 = Maximum edge value of (S_6)-Minimum edge value of (S_6)

RV(S_6) =63-2
=61.

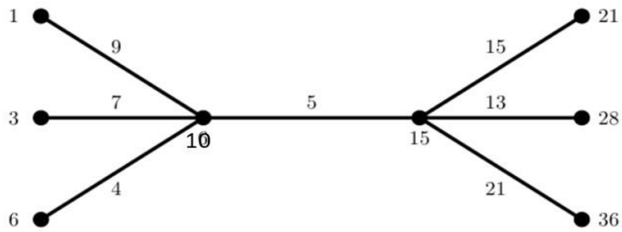
Theorem 4. The double star graph $S_{m,n}$ is a Range graph.
Proof:

Let G= (S, T) be a double star graph. It is fixed by $S_{m,n}$ and S_1 , S_2 are two vertices in $S_{m,n}$ which are not pendent. Consider r_i's are m pendent vertices to S_1 and r_j's are n pendent vertices to S_2.

Let ∝:S→ {1,3,6,10,15,21,28..., (n^2+n)/2} and $\propto_1$: R→ {1,3,6,10,15,21,28..., (n^2+n)/2}
$\propto^*$(T)= Maximum value of (S_1, S_2)– Minimum value of (S_1, S_2).
If $S_2$is a minimum value and $S_1$is a maximum value.
=S_1 – S_2= Z is an integer.
Suppose $S_1$is a minimum value and $S_2$is a maximum value.
=S_2 – S_1= Z is a integer.
$\propto^*$(T)= Maximum value of (S_1, r_i)– Minimum value of ($S_{1, ri}$).
If S_1 is a minimum value and r_i is a maximum value.
=r_i – S_1 = Z is an integer.
Suppose r_i is a minimum value and S_1 is a maximum value.
= S_1−r_i = Z is an integer.
$\propto^*$(T)= Maximum value of (S_2, r_j) – Minimum value of (S_2, rj).
If rj is a minimum value and S_2 is a maximum value.
=S_2 – r_j = Z is an integer.
Suppose $S_2$is a minimum value and r_j is a maximum value.
= r_j – S_2= Z is an integer.

Hence, Every Double Star graph $S_{m,n}$ is accept range labeling. ∴ Any Double Star graph is a range graph.

Example 4. Range labeling for star graph shown in the figure4



Remark 4:
Range value of $S_{3,3}$ = Maximum edge value of ($S_{3,3}$)-Minimum edge value of ($S_{3,3}$) RV($S_{3,3}$) =21-4
=17

Theorem 5. The symmetrical tree is a Range graph.
Proof:

Let G= (S, T) be a graph.

Let ∝: S_1→ {1,3,6,10,15,21,28..., (n^2+n)/2}, $\propto_1$: S_{1i}→ {1,3,6,10,15,21,28..., (n^2+n)/2}, for i=1,2 and $\propto_2$: S_{1ij}→ {1,3,6,10,15,21,28..., (n^2+n)/2}, for i,j=1,2.
$\propto^*$(T)= Maximum value of (S_1, S_{1i})– Minimum value of (S_1, S_{1i}).
If $S_{1i}$is a minimum value and S_1 is a maximum value.
=S_1 – S_{1i} = Z is an integer.
Suppose $S_1$is a minimum value and S_{1i} is a maximum value.
= S_{1i} – S_1 = Z is an integer.

$\propto^{*}(T)$= Maximum value of (S_{1i}, S_{1ij})– Minimum value of (S_{1i}, S_{1ij}).

If $S_{1ij}$is a minimum value and S_{1i} is a maximum value.

$=S_{1i} - S_{1ij} = Z$ is an integer.

Suppose $S_{1i}$is a minimum value and S_{1ij} is a maximum value.

$= S_{1ij} - S_{1i}= Z$ is an integer.

Hence, every symmetrical tree is accepting range labeling.

∴ Any symmetrical tree is a range graph.

Example 5: Range labeling of symmetrical tree shown in the

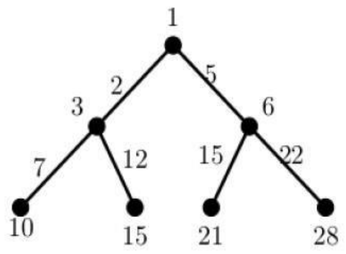


Figure 5

Remark 5: Range value of symmetrical tree= Maximum edge value of symmetrical tree-Minimum edge value of symmetrical tree $RV(S_{1,2})$=22-2

=20.

CONCLUSION

In this paper we have discussed some graphs which accept Range labeling.Further investigation can be done to obtain the condition at which some special graph accept range 'labeling.

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