International Journal of Mathematics and Computer Research

ISSN: 2320-7167

Volume 11 Issue 02 February 2023, Page no. – 3250-3254

Index Copernicus ICV: 57.55, Impact Factor: 7.362

DOI: 10.47191/ijmcr/v11i2.05

Neutrosophic Fuzzy Bi-Ideal of BS-Algebras

P. Ayesha Parveen[1], M. Himaya Jaleela Begum[2]

[1,2] Department of Mathematics, Sadakathullah Appa College, Tirunelveli-11, Tamil Nadu, India.

ARTICLE INFO	ABSTRACT
Published Online: 17 February 2023 Corresponding Author: **P. Ayesha Parveen**	The main aim of this paper is to introduce the new concept of neutrosophic fuzzy bi-ideal of BS-algebras. Some algebraic nature are investigated. Neutrosophic fuzzy bi-ideal of BS-algebras is also applied in Cartesian product. Finally, we also provide the homomorphic behaviour of neutrosophic fuzzy bi-ideal of BS-algebras.

KEYWORDS: BS-algebras, Neutrosophic fuzzy bi-ideal, Cartesian product, Homomorphism.

1. INTRODUCTION

After the introduction of fuzzy subsets by L.A.Zadeh[7], several researches explored on the generalization of the notion of fuzzy subset. In 1966, Imai and Iseki introduced two classes of abstract algebras viz. BCK-algebras and BCI-algebras[2]. J.Neggers and H.S. Kim introduced the notion of B-algebras[3] which is a generalisation of BCK-algebras. We introduce the notion of BS-algebras which is a generalisation of B-algebras and established the notion of Doubt fuzzy bi-ideal of BS-algebras[1]. F. Smarandache[4] extented the concept of fuzzy logic to neutrosophic logic which includes indeterminancy. Neutrosophic set theory played a major role in decision making problem, medical diagnosis, robotics, image processing, etc.,

In this paper, we introduced the notion of Neutrosophic fuzzy bi-ideal of BS-algebras and studied their algebraic properties. We obtained the Cartesian product of neutrosophic fuzzy bi-ideal for BS-algebras. Finally, we studied how to deal with homomorphism in neutrosophic fuzzy bi-ideal for BS-algebras.

2. PRELIMINARIES

In this Section, We recall some basic definitions which are needed for our study.

Definition 2.1 [1]. A BS-Algebra $\mathfrak{B}$ is a non empty set with a constant 1 and a binary operation * satisfying the following axioms

(i) a*a=1

(ii) a*1=a

(iii) (a*b)*c = a*(c*(1*b)) ∀ a, b, c ∈ $\mathfrak{B}$

Definition 2.2 [1]. A fuzzy subset Ƒ in a BS-Algebra $\mathfrak{B}$ is called Fuzzy Bi-Ideal if

(i) Ƒ(1) ≥ Ƒ(a)

(ii) Ƒ(b*c) ≥ min{Ƒ(a), Ƒ(a*(b*c))} ∀ a, b, c ∈ $\mathfrak{B}$

Example 2.3 [1]. Let $\mathfrak{B}$ = {1,x,y,z} be a set with the following Cayley table

*	1	x	y	z
1	1	x	y	z
x	x	1	z	y
y	y	z	1	x
z	z	y	x	1

Then ($\mathfrak{B}$, *, 1) is a BS-algebra. Define a fuzzy set Ƒ:$\mathfrak{B}$→[0,1] by Ƒ(1)= Ƒ(b)=0.9 and Ƒ(a)= Ƒ(c)=0.7. Then Ƒ is a fuzzy bi-ideal of $\mathfrak{B}$.

Definition 2.4 [5]. A Neutrosophic fuzzy set $\mathcal{N}$ on the Universe of discourse X characterised by a truth membership function $Ʈ_{\mathcal{N}}$(a), an indeterminacy function $Ɩ_{\mathcal{N}}$(a) and a falsity membership function $Ƒ_{\mathcal{N}}$(a) is defined as $\mathcal{N}$={<a, $Ʈ_{\mathcal{N}}$(a), $Ɩ_{\mathcal{N}}$(a), $Ƒ_{\mathcal{N}}$(a)>: a ∈ X } where $Ʈ_{\mathcal{N}}$, $Ɩ_{\mathcal{N}}$, $Ƒ_{\mathcal{N}}$: X⟶[0,1] and $0 \leq Ʈ_{\mathcal{N}} + Ɩ_{\mathcal{N}} + Ƒ_{\mathcal{N}} \leq 3$

Definition 2.5 [5]. Let $\mathcal{M}$ and $\mathcal{N}$ be two neutrosophic fuzzy bi-ideal of BS-algebra $\mathfrak{B}$. Then a ∈ $\mathfrak{B}$

i) $\mathcal{M} \cup \mathcal{N}$={<a, $Ʈ_{\mathcal{M}\cup\mathcal{N}}$(a), $Ɩ_{\mathcal{M}\cup\mathcal{N}}$(a), $Ƒ_{\mathcal{M}\cup\mathcal{N}}$(a)>}, where $Ʈ_{\mathcal{M}\cup\mathcal{N}}$(a) = max($Ʈ_{\mathcal{M}}$(a), $Ʈ_{\mathcal{N}}$(a)); $Ɩ_{\mathcal{M}\cup\mathcal{N}}$(a) = min($Ɩ_{\mathcal{M}}$(a), $Ɩ_{\mathcal{N}}$(a)); $Ƒ_{\mathcal{M}\cup\mathcal{N}}$(a) = min($Ƒ_{\mathcal{M}}$(a), $Ƒ_{\mathcal{N}}$(a))

ii) $\mathcal{M}\cap\mathcal{N}=\{<a, T_{\mathcal{M}\cap\mathcal{N}}(a), I_{\mathcal{M}\cap\mathcal{N}}(a), F_{\mathcal{M}\cap\mathcal{N}}(a)>\}$, where $T_{\mathcal{M}\cap\mathcal{N}}(a) = \min(T_{\mathcal{M}}(a), T_{\mathcal{N}}(a))$; $I_{\mathcal{M}\cap\mathcal{N}}(a) = \max(I_{\mathcal{M}}(a), I_{\mathcal{N}}(a))$; $F_{\mathcal{M}\cap\mathcal{N}}(a) = \max(F_{\mathcal{M}}(a), F_{\mathcal{N}}(a))$

3. NEUTROSOPHIC FUZZY BI IDEAL OF BS-ALGEBRAS

In this section, we give the definition for Neutrosophic Fuzzy Bi-Ideal of BS-Algebras and studied some of their algebraic properties.

Definition 3.1. A Neutrosophic fuzzy set $\mathcal{N}$ of BS-Algebras $\mathfrak{B}$ is called a Neutrosophic Fuzzy Bi-Ideal of $\mathfrak{B}$ if $\forall\, a, b, c \in \mathfrak{B}$

($\mathcal{N}_1$) $T_{\mathcal{N}}(1) \geq T_{\mathcal{N}}(a)$; $I_{\mathcal{N}}(1) \leq I_{\mathcal{N}}(a)$; $F_{\mathcal{N}}(1) \leq F_{\mathcal{N}}(a)$;

($\mathcal{N}_2$) $T_{\mathcal{N}}(b*c) \geq \min\{T_{\mathcal{N}}(a), T_{\mathcal{N}}(a*(b*c))\}$;

$I_{\mathcal{N}}(b*c) \leq \max\{I_{\mathcal{N}}(a), I_{\mathcal{N}}(a*(b*c))\}$;

$F_{\mathcal{N}}(b*c) \leq \max\{F_{\mathcal{N}}(a), F_{\mathcal{N}}(a*(b*c))\}$

Theorem 3.2. Let $\mathcal{M}$ and $\mathcal{N}$ be two neutrosophic fuzzy bi-ideal of BS-algebra $\mathfrak{B}$. Then $\mathcal{M}\cup\mathcal{N}$ is a neutrosophic fuzzy bi ideal of $\mathfrak{B}$.

Proof

Let $\mathcal{M}$ and $\mathcal{N}$ be two neutrosophic fuzzy bi-ideal of BS-algebra $\mathfrak{B}$. For any $a, b, c \in \mathfrak{B}$

i) $T_{\mathcal{M}\cup\mathcal{N}}(1) = \max\{T_{\mathcal{M}}(1), T_{\mathcal{N}}(1)\}$

$\geq \max\{T_{\mathcal{M}}(a), T_{\mathcal{N}}(a)\}$

$= T_{\mathcal{M}\cup\mathcal{N}}(a)$

Therefore, $T_{\mathcal{M}\cup\mathcal{N}}(1) \geq T_{\mathcal{M}\cup\mathcal{N}}(a)$

And $I_{\mathcal{M}\cup\mathcal{N}}(1) = \min\{I_{\mathcal{M}}(1), I_{\mathcal{N}}(1)\}$

$\leq \min\{I_{\mathcal{M}}(a), I_{\mathcal{N}}(a)\}$

$= I_{\mathcal{M}\cup\mathcal{N}}(a)$

Therefore, $I_{\mathcal{M}\cup\mathcal{N}}(1) \leq I_{\mathcal{M}\cup\mathcal{N}}(a)$

And $F_{\mathcal{M}\cup\mathcal{N}}(1) = \min\{F_{\mathcal{M}}(1), F_{\mathcal{N}}(1)\}$

$\leq \min\{F_{\mathcal{M}}(a), F_{\mathcal{N}}(a)\}$

$= F_{\mathcal{M}\cup\mathcal{N}}(a)$

Therefore, $F_{\mathcal{M}\cup\mathcal{N}}(1) \leq F_{\mathcal{M}\cup\mathcal{N}}(a)$

ii) $T_{\mathcal{M}\cup\mathcal{N}}(b*c) = \max\{T_{\mathcal{M}}(b*c), T_{\mathcal{N}}(b*c)\}$

$\geq \max\{\min\{T_{\mathcal{M}}(a), T_{\mathcal{M}}(a*(b*c))\}, \min\{T_{\mathcal{N}}(a), T_{\mathcal{N}}(a*(b*c))\}\}$

$= \min\{\max\{T_{\mathcal{M}}(a), T_{\mathcal{N}}(a)\}, \max\{T_{\mathcal{M}}(a*(b*c)), T_{\mathcal{N}}(a*(b*c))\}\}$

$= \min\{T_{\mathcal{M}\cup\mathcal{N}}(a), T_{\mathcal{M}\cup\mathcal{N}}(a*(b*c))\}$

Therefore, $T_{\mathcal{M}\cup\mathcal{N}}(b*c) \geq \min\{T_{\mathcal{M}\cup\mathcal{N}}(a), T_{\mathcal{M}\cup\mathcal{N}}(a*(b*c))\}$

And $I_{\mathcal{M}\cup\mathcal{N}}(b*c) = \min\{I_{\mathcal{M}}(b*c), I_{\mathcal{N}}(b*c)\}$

$\leq \min\{\max\{I_{\mathcal{M}}(a), I_{\mathcal{M}}(a*(b*c))\}, \max\{I_{\mathcal{N}}(a), I_{\mathcal{N}}(a*(b*c))\}\}$

$= \max\{\min\{I_{\mathcal{M}}(a), I_{\mathcal{N}}(a)\}, \min\{I_{\mathcal{M}}(a*(b*c)), I_{\mathcal{N}}(a*(b*c))\}\}$

$= \max\{I_{\mathcal{M}\cup\mathcal{N}}(a), I_{\mathcal{M}\cup\mathcal{N}}(a*(b*c))\}$

Therefore, $I_{\mathcal{M}\cup\mathcal{N}}(b*c) \leq \max\{I_{\mathcal{M}\cup\mathcal{N}}(a), I_{\mathcal{M}\cup\mathcal{N}}(a*(b*c))\}$

And $F_{\mathcal{M}\cup\mathcal{N}}(b*c) = \min\{F_{\mathcal{M}}(b*c), F_{\mathcal{N}}(b*c)\}$

$\leq \min\{\max\{F_{\mathcal{M}}(a), F_{\mathcal{M}}(a*(b*c))\}, \max\{F_{\mathcal{N}}(a), F_{\mathcal{N}}(a*(b*c))\}\}$

$= \max\{\min\{F_{\mathcal{M}}(a), F_{\mathcal{N}}(a)\}, \min\{F_{\mathcal{M}}(a*(b*c)), F_{\mathcal{N}}(a*(b*c))\}\}$

$= \max\{F_{\mathcal{M}\cup\mathcal{N}}(a), F_{\mathcal{M}\cup\mathcal{N}}(a*(b*c))\}$

Therefore, $F_{\mathcal{M}\cup\mathcal{N}}(b*c) \leq \max\{F_{\mathcal{M}\cup\mathcal{N}}(a), F_{\mathcal{M}\cup\mathcal{N}}(a*(b*c))\}$

Hence, $\mathcal{M}\cup\mathcal{N}$ is a neutrosophic fuzzy bi-ideal of $\mathfrak{B}$

Theorem 3.3. Let $\mathcal{M}$ and $\mathcal{N}$ be two neutrosophic fuzzy bi-ideal of BS-algebra $\mathfrak{B}$. Then $\mathcal{M}\cap\mathcal{N}$ is a neutrosophic fuzzy bi ideal of $\mathfrak{B}$.

Proof

Let $\mathcal{M}$ and $\mathcal{N}$ be two neutrosophic fuzzy bi-ideal of BS-algebra $\mathfrak{B}$. For any $a, b, c \in \mathfrak{B}$

i) $T_{\mathcal{M}\cap\mathcal{N}}(1) = \min\{T_{\mathcal{M}}(1), T_{\mathcal{N}}(1)\}$

$\geq \min\{T_{\mathcal{M}}(a), T_{\mathcal{N}}(a)\}$

$= T_{\mathcal{M}\cap\mathcal{N}}(a)$

Therefore, $T_{\mathcal{M}\cap\mathcal{N}}(1) \geq T_{\mathcal{M}\cap\mathcal{N}}(a)$

And $I_{\mathcal{M}\cap\mathcal{N}}(1) = \max\{I_{\mathcal{M}}(1), I_{\mathcal{N}}(1)\}$

$\leq \max\{I_{\mathcal{M}}(a), I_{\mathcal{N}}(a)\}$

$= I_{\mathcal{M}\cap\mathcal{N}}(a)$

Therefore, $I_{\mathcal{M}\cap\mathcal{N}}(1) \leq I_{\mathcal{M}\cap\mathcal{N}}(a)$

And $F_{\mathcal{M}\cap\mathcal{N}}(1) = \max\{F_{\mathcal{M}}(1), F_{\mathcal{N}}(1)\}$

$\leq \max\{F_{\mathcal{M}}(a), F_{\mathcal{N}}(a)\}$

$= F_{\mathcal{M}\cap\mathcal{N}}(a)$

Therefore, $F_{\mathcal{M}\cap\mathcal{N}}(1) \leq F_{\mathcal{M}\cap\mathcal{N}}(a)$

ii) $T_{\mathcal{M}\cap\mathcal{N}}(b*c) = \max\{T_{\mathcal{M}}(b*c), T_{\mathcal{N}}(b*c)\}$

$\geq \min\{\min\{T_{\mathcal{M}}(a), T_{\mathcal{M}}(a*(b*c))\}, \min\{T_{\mathcal{N}}(a), T_{\mathcal{N}}(a*(b*c))\}\}$

$= \min\{\min\{T_{\mathcal{M}}(a), T_{\mathcal{N}}(a)\}, \min\{T_{\mathcal{M}}(a*(b*c)), T_{\mathcal{N}}(a*(b*c))\}\}$

$= \min\{T_{\mathcal{M}\cap\mathcal{N}}(a), T_{\mathcal{M}\cap\mathcal{N}}(a*(b*c))\}$

Therefore, $T_{\mathcal{M}\cap\mathcal{N}}(b*c) \geq \min\{T_{\mathcal{M}\cap\mathcal{N}}(a), T_{\mathcal{M}\cap\mathcal{N}}(a*(b*c))\}$

And $I_{\mathcal{M}\cap\mathcal{N}}(b*c) = \max\{I_{\mathcal{M}}(b*c), I_{\mathcal{N}}(b*c)\}$

$\leq \max\{\max\{I_{\mathcal{M}}(a), I_{\mathcal{M}}(a*(b*c))\}, \max\{I_{\mathcal{N}}(a), I_{\mathcal{N}}(a*(b*c))\}\}$

$= \max\{\max\{I_{\mathcal{M}}(a), I_{\mathcal{N}}(a)\}, \max\{I_{\mathcal{M}}(a*(b*c)), I_{\mathcal{N}}(a*(b*c))\}\}$

$= \max\{I_{\mathcal{M}\cap\mathcal{N}}(a), I_{\mathcal{M}\cap\mathcal{N}}(a*(b*c))\}$

Therefore, $I_{\mathcal{M}\cap\mathcal{N}}(b*c) \leq \max\{I_{\mathcal{M}\cap\mathcal{N}}(a), I_{\mathcal{M}\cap\mathcal{N}}(a*(b*c))\}$

And $F_{\mathcal{M}\cap\mathcal{N}}(b*c) = \max\{F_{\mathcal{M}}(b*c), F_{\mathcal{N}}(b*c)\}$

$\leq \max\{\max\{F_{\mathcal{M}}(a), F_{\mathcal{M}}(a*(b*c))\}, \max\{F_{\mathcal{N}}(a), F_{\mathcal{N}}(a*(b*c))\}\}$

$= \max\{\max\{F_{\mathcal{M}}(a), F_{\mathcal{N}}(a)\}, \max\{F_{\mathcal{M}}(a*(b*c)), F_{\mathcal{N}}(a*(b*c))\}\}$

$= \max\{F_{\mathcal{M}\cap\mathcal{N}}(a), F_{\mathcal{M}\cap\mathcal{N}}(a*(b*c))\}$

Therefore, $F_{\mathcal{M}\cap\mathcal{N}}(b*c) \leq \max\{F_{\mathcal{M}\cap\mathcal{N}}(a), F_{\mathcal{M}\cap\mathcal{N}}(a*(b*c))\}$

Hence, $\mathcal{M} \cap \mathcal{N}$ is a neutrosophic fuzzy bi-ideal of $\mathfrak{B}$

Corollary 3.4. Let $\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_n$ are neutrosophic fuzzy bi-ideal of $\mathfrak{B}$, then $\mathcal{M} = \bigcap_{i=1}^{n} \mathcal{M}i$ is also a neutrosophic fuzzy bi-ideal of $\mathfrak{B}$

Lemma 3.5. For all s, t $\in$ I and i be any positive integer, if s = t, then

i) $s^i \leq t^i$

ii) $[\min(s, t)]^i = \min(s^i, t^i)$

iii) $[\max(s, t)]^i = \max(s^i, t^i)$

Theorem 3.6. Let $\mathcal{N}$ be a neutrosophic fuzzy bi-ideal of $\mathfrak{B}$, then $\mathcal{N}^i = \{<a, T_{\mathcal{N}}^i(a), I_{\mathcal{N}}^i(a), F_{\mathcal{N}}^i(a)>: a \in \mathfrak{B}\}$ is a neutrosophic fuzzy bi-ideal of $\mathfrak{B}^i$, where i is any positive integer and $T_{\mathcal{N}}^i(a) = (T_{\mathcal{N}}(a))^i$, $I_{\mathcal{N}}^i(a) = (I_{\mathcal{N}}(a))^i$, $F_{\mathcal{N}}^i(a) = (F_{\mathcal{N}}(a))^i$

Proof

Let be a neutrosophic fuzzy bi-ideal of $\mathfrak{B}$. For any $a, b, c \in \mathfrak{B}$

i) $T_{\mathcal{N}}^i(1) = (T_{\mathcal{N}}(1))^i$
$\geq (T_{\mathcal{N}}(a))^i$
$= T_{\mathcal{N}}^i(a)$

Therefore, $T_{\mathcal{N}}^i(1) \geq T_{\mathcal{N}}^i(a)$

And $I_{\mathcal{N}}^i(1) = (I_{\mathcal{N}}(1))^i$
$\leq (I_{\mathcal{N}}(a))^i$
$= I_{\mathcal{N}}^i(a)$

Therefore, $I_{\mathcal{N}}^i(1) \leq I_{\mathcal{N}}^i(a)$

And $F_{\mathcal{N}}^i(1) = (F_{\mathcal{N}}(1))^i$
$\leq (F_{\mathcal{N}}(a))^i$
$= F_{\mathcal{N}}^i(a)$

Therefore, $F_{\mathcal{N}}^i(1) \leq F_{\mathcal{N}}^i(a)$

ii) $T_{\mathcal{N}}^i(b*c) = (T_{\mathcal{N}}(b*c))^i$
$\geq \min\{T_{\mathcal{N}}(a), T_{\mathcal{N}}(a*(b*c))\}\}^i$
$= \min\{(T_{\mathcal{N}}(a))^i, T_{\mathcal{N}}(a*(b*c))^i\}$
$= \min\{T_{\mathcal{N}}^i(a), T_{\mathcal{N}}^i(a*(b*c))\}$

Therefore, $T_{\mathcal{N}}^i(b*c) \geq \min\{T_{\mathcal{N}}^i(a), T_{\mathcal{N}}^i(a*(b*c))\}$

And $I_{\mathcal{N}}^i(b*c) = (I_{\mathcal{N}}(b*c))^i$
$\leq \max\{I_{\mathcal{N}}(a), I_{\mathcal{N}}(a*(b*c))\}\}^i$
$= \max\{(I_{\mathcal{N}}(a))^i, I_{\mathcal{N}}(a*(b*c))^i\}$
$= \max\{I_{\mathcal{N}}^i(a), I_{\mathcal{N}}^i(a*(b*c))\}$

Therefore, $I_{\mathcal{N}}^i(b*c) \leq \max\{I_{\mathcal{N}}^i(a), I_{\mathcal{N}}^i(a*(b*c))\}$

And $F_{\mathcal{N}}^i(b*c) = (F_{\mathcal{N}}(b*c))^i$
$\leq \max\{F_{\mathcal{N}}(a), F_{\mathcal{N}}(a*(b*c))\}\}^i$
$= \max\{(F_{\mathcal{N}}(a))^i, F_{\mathcal{N}}(a*(b*c))^i\}$
$= \max\{F_{\mathcal{N}}^i(a), F_{\mathcal{N}}^i(a*(b*c))\}$

Therefore, $F_{\mathcal{N}}^i(b*c) \leq \max\{F_{\mathcal{N}}^i(a), F_{\mathcal{N}}^i(a*(b*c))\}$

Hence $\mathcal{N}^i$ be a neutrosophic fuzzy bi-ideal of $\mathfrak{B}^i$

4. DIRECT PRODUCT OF NEUTROSOPHIC FUZZY BI IDEAL OF BS-ALGEBRAS

In this Section, we shall discuss with the direct product of neutrosophic fuzzy bi-ideals of BS-Algebras.

Definition 4.1. Let $\mathcal{M}$ and $\mathcal{N}$ be two neutrosophic fuzzy subsets of BS-algebra $\mathfrak{B}_1$ and $\mathfrak{B}_2$ respectively. Then the direct product of neutrosophic fuzzy subsets of BS-algebras $\mathfrak{B}_1$ and $\mathfrak{B}_2$ is defined by $\mathcal{M}x\mathcal{N}$: $\mathfrak{B}_1 \times \mathfrak{B}_2 \longrightarrow [0,1]$ such that $\mathcal{M}x\mathcal{N} = \{<(a,b), T_{\mathcal{M}x\mathcal{N}}(a,b), I_{\mathcal{M}x\mathcal{N}}(a,b), F_{\mathcal{M}x\mathcal{N}}(a,b)>: a \in \mathfrak{B}_1, b \in \mathfrak{B}_2\}$, where $T_{\mathcal{M}x\mathcal{N}}(a,b) = \min(T_{\mathcal{M}}(a), T_{\mathcal{N}}(b))$;

$I_{\mathcal{M}x\mathcal{N}}(a,b) = \max(I_{\mathcal{M}}(a), I_{\mathcal{N}}(b))$;

$F_{\mathcal{M}x\mathcal{N}}(a,b) = \min(F_{\mathcal{M}}(a), F_{\mathcal{N}}(b))$

Definition 4.2. Let $\mathcal{M}$ and $\mathcal{N}$ be two neutrosophic fuzzy subsets of BS-algebra $\mathfrak{B}_1$ and $\mathfrak{B}_2$ respectively. Then $\mathcal{M}x\mathcal{N}$ is a neutrosophic fuzzy bi-ideal of $\mathfrak{B}_1 \times \mathfrak{B}_2$ if it satisfies the following conditions

i) $T_{\mathcal{M}\times\mathcal{N}}(1,1) \geq T_{\mathcal{M}\times\mathcal{N}}(a_1, a_2)$; $I_{\mathcal{M}\times\mathcal{N}}(1,1) \leq I_{\mathcal{M}\times\mathcal{N}}(a_1, a_2)$; $F_{\mathcal{M}\times\mathcal{N}}(1,1) \leq F_{\mathcal{M}\times\mathcal{N}}(a_1, a_2)$;

ii) $T_{\mathcal{M}\times\mathcal{N}}((b_1, b_2)*(c_1, c_2)) \geq \min\{T_{\mathcal{M}\times\mathcal{N}}(a_1, a_2), T_{\mathcal{M}\times\mathcal{N}}((a_1, a_2)*((b, b_2)*(c_1, c_2)))\}$;

$I_{\mathcal{M}\times\mathcal{N}}((b_1, b_2)*(c_1, c_2)) \leq \max\{I_{\mathcal{M}\times\mathcal{N}}(a_1, a_2), I_{\mathcal{M}\times\mathcal{N}}((a_1, a_2)*((b, b_2)*(c_1, c_2)))\}$;

$F_{\mathcal{M}\times\mathcal{N}}((b_1, b_2)*(c_1, c_2)) \leq \max\{F_{\mathcal{M}\times\mathcal{N}}(a_1, a_2), F_{\mathcal{M}\times\mathcal{N}}((a_1, a_2)*((b, b_2)*(c_1, c_2)))\}$

Theorem 4.3. Let $\mathcal{M}$ and $\mathcal{N}$ be two neutrosophic fuzzy bi-ideals of BS-algebra $\mathfrak{B}_1$ and $\mathfrak{B}_2$ respectively. Then $\mathcal{M}x\mathcal{N}$ is a neutrosophic fuzzy bi-ideals of $\mathfrak{B}_1 \times \mathfrak{B}_2$

Proof

Let $\mathcal{M}$ and $\mathcal{N}$ be two neutrosophic fuzzy bi-ideals of BS-algebra $\mathfrak{B}_1$ and $\mathfrak{B}_2$ respectively.

Let $(a_1, a_2), (b, b_2), (c_1, c_2) \in \mathfrak{B}_1 \times \mathfrak{B}_2$. We have

i) $T_{\mathcal{M}\times\mathcal{N}}(1,1) = \min\{T_{\mathcal{M}}(1), T_{\mathcal{N}}(1)\}$
$\geq \min\{T_{\mathcal{M}}(a_1), T_{\mathcal{N}}(a_2)\}$
$= T_{\mathcal{M}\times\mathcal{N}}(a_1, a_2)$

Therefore, $T_{\mathcal{M}\times\mathcal{N}}(1,1) \geq T_{\mathcal{M}\times\mathcal{N}}(a_1, a_2)$

Again $I_{\mathcal{M}\times\mathcal{N}}(1,1) = \max\{I_{\mathcal{M}}(1), I_{\mathcal{N}}(1)\}$
$\leq \max\{I_{\mathcal{M}}(a_1), I_{\mathcal{N}}(a_2)\}$
$= I_{\mathcal{M}\times\mathcal{N}}(a_1, a_2)$

Therefore, $I_{\mathcal{M}\times\mathcal{N}}(1,1) \geq I_{\mathcal{M}\times\mathcal{N}}(a_1, a_2)$

Again $F_{\mathcal{M}\times\mathcal{N}}(1,1) = \max\{F_{\mathcal{M}}(1), F_{\mathcal{N}}(1)\}$
$\leq \max\{F_{\mathcal{M}}(a_1), F_{\mathcal{N}}(a_2)\}$
$= F_{\mathcal{M}\times\mathcal{N}}(a_1, a_2)$

Therefore, $F_{\mathcal{M}\times\mathcal{N}}(1,1) \geq F_{\mathcal{M}\times\mathcal{N}}(a_1, a_2)$

ii) Then $T_{\mathcal{M}\times\mathcal{N}}((b_1, b_2)*(c_1, c_2)) = T_{\mathcal{M}x\mathcal{N}}(b_1*c_1, b_2*c_2)$
$= \min\{T_{\mathcal{M}}(b_1*c_1), T_{\mathcal{N}}(b_2*c_2)\}$

$\geq \min[\min\{T_{\mathcal{M}}(a_1), T_{\mathcal{M}}(a_1*(b_1*c_1))\}, \min\{T_{\mathcal{N}}(a_2), T_{\mathcal{N}}(a_2*(b_2*c_2))\}]$

$= \min[\min\{T_{\mathcal{M}}(a_1), T_{\mathcal{N}}(a_2)\}, \min\{T_{\mathcal{M}}(a_1*(b_1*c_1)), T_{\mathcal{N}}(a_2*(b_2*c_2))\}]$

$= \min\{T_{\mathcal{M}x\mathcal{N}}(a_1, a_2), T_{\mathcal{M}x\mathcal{N}}(a_1*(b_1*c_1)), (a_2*(b_2*c_2))\}$

$= \min\{T_{\mathcal{M}\times\mathcal{N}}(a_1, a_2), T_{\mathcal{M}\times\mathcal{N}}((a_1, a_2)*((b, b_2)*(c_1, c_2)))\}$

Therefore, $T_{\mathcal{M}\times\mathcal{N}}((b_1, b_2)*(c_1, c_2)) \geq \min\{T_{\mathcal{M}\times\mathcal{N}}(a_1, a_2), T_{\mathcal{M}\times\mathcal{N}}((a_1, a_2)*((b, b_2)*(c_1, c_2)))\}$

And $I_{\mathcal{M}\times\mathcal{N}}((b_1, b_2)*(c_1, c_2)) = I_{\mathcal{M}\times}(b_1*c_1, b_2*c_2)$

$= \max\{I_{\mathcal{M}}(b_1*c_1), I_{\mathcal{N}}(b_2*c_2)\}$

$\leq \max[\max\{I_{\mathcal{M}}(a_1), I_{\mathcal{M}}(a_1*(b_1*c_1))\},$

$\max\{I_{\mathcal{N}}(a_2), I_{\mathcal{N}}(a_2*(b_2*c_2))\}]$

$= \max[\max\{I_{\mathcal{M}}(a_1), I_{\mathcal{N}}(a_2)\},$

$\max\{I_{\mathcal{M}}(a_1*(b_1*c_1)), I_{\mathcal{N}}(a_2*(b_2*c_2))\}]$

$= \max\{I_{\mathcal{M}\times\mathcal{N}}(a_1, a_2), I_{\mathcal{M}\times\mathcal{N}}(a_1*(b_1*c_1)), (a_2*(b_2*c_2))\}$

$= \max\{I_{\mathcal{M}\times\mathcal{N}}(a_1, a_2), I_{\mathcal{M}\times\mathcal{N}}((a_1, a_2)*((b, b_2)*(c_1, c_2)))\}$

Therefore, $I_{\mathcal{M}\times\mathcal{N}}((b_1, b_2)*(c_1, c_2)) \leq \max\{I_{\mathcal{M}\times\mathcal{N}}(a_1, a_2), I_{\mathcal{M}\times\mathcal{N}}((a_1, a_2)*((b, b_2)*(c_1, c_2)))\}$

And $F_{\mathcal{M}\times\mathcal{N}}((b_1, b_2)*(c_1, c_2)) = F_{\mathcal{M}\times}(b_1*c_1, b_2*c_2)$

$= \max\{F_{\mathcal{M}}(b_1*c_1), F_{\mathcal{N}}(b_2*c_2)\}$

$\leq \max[\max\{F_{\mathcal{M}}(a_1), F_{\mathcal{M}}(a_1*(b_1*c_1))\},$

$\max\{F_{\mathcal{N}}(a_2), F_{\mathcal{N}}(a_2*(b_2*c_2))\}]$

$= \max[\max\{F_{\mathcal{M}}(a_1), F_{\mathcal{N}}(a_2)\},$

$\max\{F_{\mathcal{M}}(a_1*(b_1*c_1)), F_{\mathcal{N}}(a_2*(b_2*c_2))\}]$

$= \max\{F_{\mathcal{M}\times\mathcal{N}}(a_1, a_2), F_{\mathcal{M}\times\mathcal{N}}(a_1*(b_1*c_1)), (a_2*(b_2*c_2))\}$

$= \max\{F_{\mathcal{M}\times\mathcal{N}}(a_1, a_2), F_{\mathcal{M}\times\mathcal{N}}((a_1, a_2)*((b, b_2)*(c_1, c_2)))\}$

Therefore, $F_{\mathcal{M}\times\mathcal{N}}((b_1, b_2)*(c_1, c_2)) \leq \max\{F_{\mathcal{M}\times\mathcal{N}}(a_1, a_2), F_{\mathcal{M}\times\mathcal{N}}((a_1, a_2)*((b, b_2)*(c_1, c_2)))\}$

5. HOMOMORPHISM OF NEUTROSOPHIC FUZZY BI IDEAL OF BS-ALGEBRAS

In this section, we discuss about homomorphism

Definition 5.1. Let $\mathfrak{B}_1$ and $\mathfrak{B}_2$ be two BS-algebras and $h: \mathfrak{B}_1 \longrightarrow \mathfrak{B}_2$ be a function. If $\mathcal{N}$ is a neutrosophic fuzzy set in $\mathfrak{B}_2$, then the preimage of $\mathcal{N}$ under h denoted by $h^{-1}(\mathcal{N})$ is the neutrosophic fuzzy set in $\mathfrak{B}_1$ is defined by

$h^{-1}(\mathcal{N}) = \{<(a), h^{-1}(T_{\mathcal{N}}(a)), h^{-1}(I_{\mathcal{N}}(a)), h^{-1}(F_{\mathcal{N}}(a))>: a \in \mathfrak{B}\}$,

where $h^{-1}(T_{\mathcal{N}}(a)) = T_{\mathcal{N}}(h(a))$; $h^{-1}(I_{\mathcal{N}}(a)) = I_{\mathcal{N}}(h(a))$;

$h^{-1}(F_{\mathcal{N}}(a)) = F_{\mathcal{N}}(h(a))$

Theorem 5.2. Let $h: \mathfrak{B}_1 \rightarrow \mathfrak{B}_2$ be an epimorphism of BS-algebras if $\mathcal{N}$ is a neutrosophic fuzzy bi-ideal of $\mathfrak{B}_2$, then the pre image of $\mathcal{N}$ under h is also a neutrosophic fuzzy bi-ideal of $\mathfrak{B}_1$.

Proof

Let $\mathcal{N}$ is a neutrosophic fuzzy bi-ideal of $\mathfrak{B}_2$. Let $a,b,c \in \mathfrak{B}_1$

Now, $h^{-1}(T_{\mathcal{N}}(1)) = T_{\mathcal{N}}(h(1))$

$\geq T_{\mathcal{N}}(h(a))$

$= h^{-1}(T_{\mathcal{N}}(a))$

Therefore $h^{-1}(T_{\mathcal{N}}(1)) \geq h^{-1}(T_{\mathcal{N}}(a))$

And $h^{-1}(I_{\mathcal{N}}(1)) = I_{\mathcal{N}}(h(1))$

$\leq I_{\mathcal{N}}(h(a))$

$= h^{-1}(I_{\mathcal{N}}(a))$

Therefore $h^{-1}(I_{\mathcal{N}}(1)) \leq h^{-1}(I_{\mathcal{N}}(a))$

And $h^{-1}(F_{\mathcal{N}}(1)) = F_{\mathcal{N}}(h(1))$

$\leq F_{\mathcal{N}}(h(a))$

$= h^{-1}(F_{\mathcal{N}}(a))$

Therefore $h^{-1}(F_{\mathcal{N}}(1)) \leq h^{-1}(F_{\mathcal{N}}(a))$

And, $h^{-1}(T_{\mathcal{N}}(b*c)) = T_{\mathcal{N}}(h(b*c))$

$= T_{\mathcal{N}}(h(b)*h(c))$

$\geq \min\{T_{\mathcal{N}}(h(a)), T_{\mathcal{N}}(h(a)*[h(b)*h(c)])\}$

$= \min\{T_{\mathcal{N}}(h(a)), T_{\mathcal{N}}(h(a*(b*c)))\}$

Therefore, $h^{-1}(T_{\mathcal{N}}(b*c)) \geq \min\{T_{\mathcal{N}}(h(a)), T_{\mathcal{N}}(h(a*(b*c)))\}$

And $h^{-1}(I_{\mathcal{N}}(b*c)) = I_{\mathcal{N}}(h(b*c))$

$= I_{\mathcal{N}}(h(b)*h(c))$

$\leq \max\{I_{\mathcal{N}}(h(a)), I_{\mathcal{N}}(h(a)*[h(b)*h(c)])\}$

$= \max\{I_{\mathcal{N}}(h(a)), I_{\mathcal{N}}(h(a*(b*c)))\}$

Therefore, $h^{-1}(I_{\mathcal{N}}(b*c)) \leq \max\{I_{\mathcal{N}}(h(a)), I_{\mathcal{N}}(h(a*(b*c)))\}$

And $h^{-1}(F_{\mathcal{N}}(b*c)) = F_{\mathcal{N}}(h(b*c))$

$= F_{\mathcal{N}}(h(b)*h(c))$

$\leq \max\{F_{\mathcal{N}}(h(a)), F_{\mathcal{N}}(h(a)*[h(b)*h(c)])\}$

$= \max\{F_{\mathcal{N}}(h(a)), F_{\mathcal{N}}(h(a*(b*c)))\}$

Therefore, $h^{-1}(F_{\mathcal{N}}(b*c)) \leq \max\{F_{\mathcal{N}}(h(a)), F_{\mathcal{N}}(h(a*(b*c)))\}$

Definition 5.3. Let $\mathfrak{B}_1$ and $\mathfrak{B}_2$ be two BS-algebras and $h: \mathfrak{B}_1 \rightarrow \mathfrak{B}_2$ be a homomorphism. Then $h(1) = 1$

Proof

Let $\mathfrak{B}_1$ and $\mathfrak{B}_2$ be two BS-algebras

Let $a \in \mathfrak{B}_1$ therefore $h(a) \in \mathfrak{B}_2$

Now $h(1)=h(a*a)=h(a)*h(a)=1*1=1$

Theorem 5.4. Let $h: \mathfrak{B}_1 \rightarrow \mathfrak{B}_2$ be a homomorphism of BS-algebras if $\mathcal{N}$ is a neutrosophic fuzzy bi-ideal of $\mathfrak{B}_1$, then $h(\mathcal{N})$ is a neutrosophic fuzzy bi-ideal of $\mathfrak{B}_2$.

Proof

Let $a_1, a_2, a_3 \in \mathfrak{B}_1$ and $b_1, b_2, b_3 \in \mathfrak{B}_2$ such that $h(a_1) = b_1$, $h(a_2) = b_2$, $h(a_3) = b_3$

Now, $T_{\mathcal{N}}(b_1) = T_{\mathcal{N}}(h(a_1))$

$= h^{-1}(T_{\mathcal{N}}(a_1))$

$\leq h^{-1}(T_{\mathcal{N}}(1))$

$= T_{\mathcal{N}}(h(1))$

$= T_{\mathcal{N}}(1)$

Therefore, $T_{\mathcal{N}}(b_1) \leq T_{\mathcal{N}}(1)$

And $I_{\mathcal{N}}(b_1) = I_{\mathcal{N}}(h(a_1))$

$= h^{-1}(I_{\mathcal{N}}(a_1))$

$\geq h^{-1}(I_{\mathcal{N}}(1))$

$= I_{\mathcal{N}}(h(1))$

$= I_{\mathcal{N}}(1)$

Therefore, $I_{\mathcal{N}}(b_1) \geq I_{\mathcal{N}}(1)$

And $F_{\mathcal{N}}(b_1) = F_{\mathcal{N}}(h(a_1))$

$= h^{-1}(F_{\mathcal{N}}(a_1))$

$\geq h^{-1}(F_{\mathcal{N}}(1))$

$= F_{\mathcal{N}}(h(1))$

$= F_{\mathcal{N}}(1)$

Therefore, $F_{\mathcal{N}}(b_1) \geq F_{\mathcal{N}}(1)$

ii) Again we have

$T_{\mathcal{N}}(b_2 * b_3) = T_{\mathcal{N}}(h(a_2) * h(a_3))$

$= h^{-1}(T_{\mathcal{N}}(a_2 * a_3))$

$\geq \min\{h^{-1}(T_{\mathcal{N}}(a_1)), h^{-1}(T_{\mathcal{N}}(a_1 * (a_2 * a_3)))\}$

$= \min\{T_{\mathcal{N}}(h(a_1)), T_{\mathcal{N}}(h(a_1 * (a_2 * a_3)))\}$

$= \min\{T_{\mathcal{N}}(h(a_1)), T_{\mathcal{N}}(h(a_1) * (h(a_2) * h(a_3)))\}$

$= \min\{T_{\mathcal{N}}(b_1), T_{\mathcal{N}}(b_1 * (b_2 * b_3))\}$

Therefore, $T_{\mathcal{N}}(b_2 * b_3) \geq \min\{T_{\mathcal{N}}(b_1), T_{\mathcal{N}}(b_1 * (b_2 * b_3))\}$

And $I_{\mathcal{N}}(b_2 * b_3) = I_{\mathcal{N}}(h(a_2) * h(a_3))$

$= h^{-1}(I_{\mathcal{N}}(a_2 * a_3))$

$\leq \max\{h^{-1}(I_{\mathcal{N}}(a_1)), h^{-1}(I_{\mathcal{N}}(a_1 * (a_2 * a_3)))\}$

$= \max\{I_{\mathcal{N}}(h(a_1)), I_{\mathcal{N}}(h(a_1 * (a_2 * a_3)))\}$

$= \max\{I_{\mathcal{N}}(h(a_1)), I_{\mathcal{N}}(h(a_1) * (h(a_2) * h(a_3)))\}$

$= \max\{I_{\mathcal{N}}(b_1), I_{\mathcal{N}}(b_1 * (b_2 * b_3))\}$

Therefore, $I_{\mathcal{N}}(b_2 * b_3) \leq \max\{I_{\mathcal{N}}(b_1), I_{\mathcal{N}}(b_1 * (b_2 * b_3))\}$

And $F_{\mathcal{N}}(b_2 * b_3) = F_{\mathcal{N}}(h(a_2) * h(a_3))$

$= h^{-1}(F_{\mathcal{N}}(a_2 * a_3))$

$\leq \max\{h^{-1}(F_{\mathcal{N}}(a_1)), h^{-1}(F_{\mathcal{N}}(a_1 * (a_2 * a_3)))\}$

$= \max\{F_{\mathcal{N}}(h(a_1)), F_{\mathcal{N}}(h(a_1 * (a_2 * a_3)))\}$

$= \max\{F_{\mathcal{N}}(h(a_1)), F_{\mathcal{N}}(h(a_1) * (h(a_2) * h(a_3)))\}$

$= \max\{F_{\mathcal{N}}(b_1), F_{\mathcal{N}}(b_1 * (b_2 * b_3))\}$

Therefore, $F_{\mathcal{N}}(b_2 * b_3) \leq \max\{F_{\mathcal{N}}(b_1), F_{\mathcal{N}}(b_1 * (b_2 * b_3))\}$

CONCLUSION

In this paper, the notion of Neutrosophic fuzzy bi-ideal of BS-algebras are introduced and studied their algebraic properties. We obtained the Cartesian product of neutrosophic fuzzy bi-ideal for BS-algebras. Finally, we studied how to deal with homomorphism.

REFERENCES

1. Ayesha Parveen, P. and Himaya Jaleela Begum, M. *Doubt Fuzzy Bi-ideal of BS-Algebras*, Journal of Emerging Technologies and Innovative Research, Vol.6(3), 20-24, 2019
2. Imai, Y. and Iseki, K. *On Axiom System of Propositional Calculi*, 15 Proc.Japan Academy, Vol. 42,19-22,1966.
3. Neggers, J. and Kim, H.S. *On B-algebras*, Math. Vensik, Vol.54, 21-29,2002.
4. Smarandache, F. *Neutrosophy, A new branch of Philosophy logic in multiple-valued logic*, An international journal, vol.8(3),2002, 297-384
5. Smarandache, F. *Neutrosophy and Neutrosophic logic*, First international Conference on Neutrosophy, Neutrosophic Logic set, Probability and Statistics, University of New Mexico, Gallup, NM 87301, USA (2002)
6. Smarandache, F. *Neutrosophic set - a generalization of the Intuitionistic fuzzy sets*, J. Pure Appl.Math, vol.24,2005, 287-297
7. Zadeh, L.A. *Fuzzy sets Information and control*, 8(1965), 338-353.