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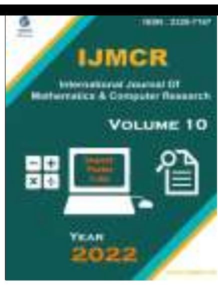

Fixed Point and Common Fixed Point Theorem for Multivalued Mappings for Banach Space

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ARTICLE INFO	ABSTRACT
Published Online: 26 December 2022 Corresponding Author: **Sandhya Singh**	In this Paper we will investigate some fixed point and common fixed point theorem for multivalued mappings of Banach space. We show that an extended form of many Known results taking multivalued mappings and inequities. The general form of a result proved by Banach Space and Convergence for multivalued mappings in fixed point and common fixed point which contains new generalized form of Theorem.

KEYWORDS: Banach space, Normed Banach space, fixed point, Common fixed point, uniformaly convex, multivalued mapping, fejér monotone.

1. INTRODUCTION

Let E be a non-empty compact Convex. subset of uniformal convex Banach Space X and T is a self map

Then $T: E \to E$ is called multivalued non-expansive mapping If

$$\|T_s - T_t\| \leqslant \| s, t \| \quad \forall\, S, t \in E$$

Since X is a unifomaly conver then every non-expansive mapping

$T: E \to E$ has a fixed point (see **Browder** [5] Kirk [1] [6] [7])gives the Comprehensive survey Concerning. A fixed point theorem for non-expainsive mappings.

and (Dwivedi, Bhardwaj, Shrivastava [13]) worked for Common fixed theorems in Banach space.

Here N is a set of all positive integers and $F(T)$ is a set of all fixed point of a mapping T.

Then $F(T) = \{S \in E: T_S = S\}$ and if $S_0 \in E$ then $\{S_n\}$ in E defined as

$$\begin{cases} S_{n+1} = (1-\alpha_n)T_{S_n} + \alpha_n T_{y_n} \\ t_n = (1-\beta_n)u_n + \beta_n T_{u_n} \\ u_n = (1-\delta_n)S_n + \delta_n T_{S_n} \end{cases} \qquad (\mathbf{1.1})$$

Where $\{\alpha_n\}, \{\beta_n\}$ and $\{\delta_n\}$ are sequence in (0,1)

we write some definitions Before start the main result.

2. PRELIMINARIES

Definition 2.1: A multivalued mapping $T: E \to CB(E)$ is Called non-expansive if

$$H(T_s, T_t) \leq \| S - t \| \cdot \forall\, S, t \in E$$

Definition 2.2: A sequence $\{S_n\}: n \in N$ in X is Called fejér monotone w.r. to subset E of X if

$$\|S_{n+1} - p\| \leq \|S_n - p\| \;\forall\, p \in E \text{ and } n \geq 1$$

Example 2.3: Suppose E is a non-empty subset of X then $T: E \to E$ is a quasi-non expansive mapping then the sequence $\{S_n\}$ defined of $S_{n+1} = T_n$ is fejér monotone w.r. to $F(I)$

Definition 2.4: Let E is a non-empty subset of X. $\{S_n\}$ is a fejér monotone sequence w.r. to E then -

(**1**) The sequence $\{S_n\}$ is Bounded

(**ii**) And for every $S \in E\{\|S_n - S\|\}$ is converges.

Definition 2. 5: [3] A Banach space X is satisfy the condition if For any sequence $\{S_n\}$ in X $S_n \to S \Rightarrow \lim_{n\to\infty}$ sup $\|S_n - S\| \leq \lim_{n\to\infty} \sup \|S_n - t\| \;\forall\, t \in E$ with $t \neq S$

Definition 2. 6: [15] Let E is a subset of X and A multivalued non-expansive mapping

$T: E \to CB(E)$ is satisfy the condition If $\exists$ a non decreasing function.

$$F: [0, \infty) \to [0, \infty) \text{ with } f(0) = 0$$
$$f(r) > 0 \;\forall\, r \in [0, \infty)$$

Such that $D(X, T_S) \geq f(D(X, F(T)) \forall\, S \in E$

Lemma 2. 7: [15] Suppose that X be a unifarmaly convex Banach space and

$$0 \leq p \leq x_n \leq q \leq 1 \;\forall n \in N$$

$\{S_n\}$ and $\{t_n\}$ are two sequences of X

Then

$$\lim_{n\to\infty} \sup\|S_n\| \le r$$
$$\lim_{n\to\infty} \sup\|t_n\| \le r$$

and also $\lim_{n\to\infty} \|x_n S_n + (1-x_n)t_n\| = r \ \forall r > 0$

Then

$$\lim_{x\to\infty} \| S_n - t_n \| = 0 \quad (\mathbf{2.1})$$

3. MAIN RESULT

Let X be a Normed Banach space and E be a non-empty closed and convex subset of X and T is. sef mapping

Then $T: E \to P(E)$ is multivalued mapping

Let $\{S_n\}$ is a sequence in $P(E)$ defined as

$$\begin{cases} S_{n+1} & = (1-\alpha_n)x_n + \alpha_n y_n \\ t_n & = (1-\beta_n)a_n + \beta_n z_n \\ u_n & = (1-\delta_n)s_n + \delta_n x_n \end{cases} \quad (\mathbf{3.1})$$

Where $x_n \in T_{S_n}, y_n \in T_{t_n}$ and $z_n \in T_{u_n}$ and $\{\alpha_n\},\{\beta_n\},\{\delta_n\}$ are sequences in (0,1)

Theorem 3.1. Let X be a uniformaly convex Banach space and E be a non-empty Closed Convex subset of X and T is a self map.

Then $T: E \to p(E)$ be a multivalued mapping.

such that $F(T) \neq \psi$ and P_T be a nonexpansive mapping

$$P_T P = \{P\} \ \forall \ P \in F(T)$$

The sequence $\{S_n\}$ defined in (3.1) Now we have to prove

$$\lim_{n\to\infty} D(S_n; T_{S_n}) = 0$$

Proof: By using definition 2.2

$\lim_{n\to\infty} \|S_n - p\|$ exist for $p \in F(T)$

Let

$$\lim_{n\to\infty} \|S_n - p\| = c \ge 0$$

if e = 0

$$\begin{aligned} D(S_n, T_{S_n}) &\le \|S_n - x_n\| \le \|S_n - p\| + \|x_n - p\| \\ &\le \|S_n - p\| + H(P_T S_n; P_T P) \\ D(S_n, T_{S_n}) &\leqslant \|S_n - P\| + \|S_n - p\| \\ &\leqslant (2) \| S_n - p\| \\ &\to 0 \ as \ n \to \infty \end{aligned}$$

Then $c = 0$ *if* $c \ge 0$

From (3.1)

$$\|S_{n+1} - p\| = \|(1-\alpha_n)x_n + \alpha_n y_n - p\| \quad (\mathbf{3.2})$$

After solving (3.2)

$$\begin{aligned} &\le (1-\alpha_n)\|x_n - p\| + \alpha_n\|y_n - p\| \\ &\le (1-\alpha_n)H(P_T S_n, P_T p) + \alpha_n H(P_T t_n, P_T p) \\ &\le (1-\alpha_n)\|S_n - p\| + \alpha_n\|t_n - p\| \end{aligned}$$

Using (2.1) we get

$$\|S_{n+1} - p\| \leqslant \|t_n - p\| \quad (\mathbf{3.3})$$

Similarl from (3.1)

$$\|t_n - p\| \le \|u_n - p\| \quad (\mathbf{3.4})$$

and

$$\|u_n - p\| \le \|S_n - p\| \quad (\mathbf{3.5})$$

(a). Taking sup lim on both side (3.5) we get

$$\lim_{n\to\infty} \sup\|u_n - p\| \le c \quad (\mathbf{3.6})$$

(b). Taking sup lim on both side (3.4) we get

$$\lim_{n\to\infty} \sup\|t_n - p\| \le \lim_{n\to\infty} \sup\|u_n - p\| \leqslant C \quad (\mathbf{3.7})$$

$$\begin{aligned} \text{Now } \lim_{n\to\infty} \sup\|x_n - p\| &\le \lim_{n\to\infty} \sup H(P_T S_n, P_T p) \\ &\le \lim_{n\to\infty} \sup\|S_n - p\| \end{aligned} \quad (\mathbf{3.8})$$

$$\lim_{n\to\infty} \sup\|x_n - p\| \le C \quad (\mathbf{3.9})$$

and using (3.7) we have

$$\lim_{n\to\infty} \sup\|y_n - p\| \le \lim_{n\to\infty} \sup\|h_n - p\|$$

$$\begin{aligned} \lim_{n\to\infty} \sup\|y_n - p\| &\le \lim_{n\to\infty} \sup H(P_T t_n, P_T p) \\ &\le \lim_{n\to\infty} \sup\|t_n - p\| \leqslant C \end{aligned} \quad (\mathbf{3.10})$$

Since we know that

$$\begin{aligned} &\lim_{n\to\infty} \|\alpha_n(x_n - p1) + (1-\alpha_n)(y_n - p)\| \\ &= \lim_{n\to\infty} \|S_{n+1} - p\| = c. \end{aligned}$$

It follows Lemma (2.7) we have

$$\lim_{n\to\infty} \|S_n - t_n\| = 0 \quad (\mathbf{3.11})$$

From 3.2 we have

$$\begin{aligned} \|S_{n+1} - p\| &= \|(1-\alpha_n)x_n + \alpha_n y_n - p\| \\ \Rightarrow \quad \|S_n - p\| &= \frac{\|S_n - p\| - \|S_{n+1} - p\|}{\alpha_n} + \|t_n - p\| \end{aligned} \quad (\mathbf{3.12})$$

Taking inf lim on both side 3.12 we get

$$\lim_{n\to\infty} \inf \| S_{n-p\|} = \lim_{n\to\infty} \inf \frac{\|S_n - p\| - \|S_{n+1} - p\|}{\alpha_n} + \|t_n - p\|$$

$$e \le \lim_{n\to\infty} \inf\|t_n - p\| \quad (\mathbf{3.13})$$

From (3.7) and (3.13) we get

$$\lim_{n\to\infty} \|t_n - p\| = c$$

It follows Lemma 2.7 we have

$$\lim_{n\to\infty} \|u_n - z_n\| = 0 \quad (\mathbf{3.14})$$

$$\begin{aligned} \text{Since } \|t_n - p\| &= \|(1-\beta_n)\mu_n + \beta_n z_n - p\| \\ &\le (1-\beta_n)\|u_n - p\| + \beta_n\|z_n - u_n\| \end{aligned}$$

Applying (3.14) we get

$$\|t_n - p\| \le \|u_n - p\| \quad (\mathbf{3.15})$$

Taking inf lim on both side (2.15) we get

$\lim_{n\to\infty} \inf\|t_n - p\| \le \lim_{n\to\infty} \inf \|u_n - p\|$

$$c \le \lim_{n\to\infty} \inf \|u_n - p\| \quad (\mathbf{3.16})$$

From (3.6) and (3.16)

$$\lim_{n\to\infty} \|u_n - p\| = C$$

It follows Lemma 2.7 we have

$$\lim_{n\to\infty} \|S_n - x_n\| = 0$$

Since

$$D(S_n, T_{S_n}) \le \|S_n - x_n\|$$

Hence

$$\lim_{n\to\infty} D(S_n, T_{S_n}) = 0 \quad (\mathbf{3.17})$$

$(\mathbf{3.17})$ is the proved of our theorem.

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