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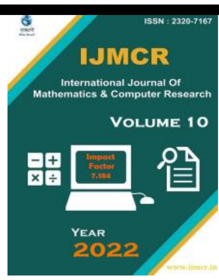

On *Q*-Derivations of *BE*-Algebras

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ARTICLE INFO	ABSTRACT
Published Online: **12 August 2022**	*BE*-algebra is an algebra $(X; *, 1)$ of type $(2,0)$ that satisfies the following axioms: $(BE1)$ $x * x = 1$, $(BE2)$ $x * 1 = 1$, $(BE3)$ $1 * x = x$, and $(BE4)$ $x * (y * z) = y * (x * z)$ for all $x, y, z \in X$. In this paper, the notion of q-derivation in *BE*-algebra is defined and its properties are discussed. Finally, the properties of the fixed set and kernel of a *q*-derivation in *BE*-algebra are identified based on their relation to subalgebra and its elements.

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KEYWORDS: BE-algebra; derivation; q-derivation; fixed set; kernel.

I. INTRODUCTION

The study of algebraic structures is growing with the discovery of new algebraic structures. Iseki [1] introduced the concepts of *BCI*-algebra and *BCK*-algebra. Various forms of generalization of *BCK*-algebra have been discussed by researchers. Among them is *BE*-algebra that has been discussed by H. S. Kim and Y. H. Kim [2]. *BE*-algebra is an algebra $(X; *, 1)$ of type (2, 0) that satisfies the following axioms: (*BE1*) $(x * x = 1$, (*BE2*) $(x * 1 = 1$, (BE3) $1 * x = x$, and (BE4) $x * (y * z) = y * (x * z)$ for all $x, y, z \in X$. Further studies of *BE*-algebra have been discussed by Kim [3]. Subsequently, Ahn and Han [4] introduced *BP*-algebra whose construction also deals with the concepts of *BCI*-algebra and *BCK*-algebra.

In the study of abstract algebra, derivation is a function that maps a set to itself based on a certain rule. The concept of derivation was first introduced in the study of ring and near ring [5], later, it has been applied to several other algebraic structures. Al-Shehrie [6] has discussed the concept of derivation in *B*-algebra. A mapping of *d* from *B*-algebra $(X; *, 0)$ to itself is said to be a left-right derivation ((*l*, *r*)-derivation) in *X* if for every *x*, *y* ∈ *X* meets

$$d(x * y) = (d(x) * y) \wedge (x * d(y))$$

and *d* is said to be right-left derivation ((*r*, *l*)-derivation) in *X* if

$$d(x * y) = (x * d(y)) \wedge (d(x) * y),$$

by define $x \wedge y = y * (y * x)$ for all $x, y \in X$. *d* is said to be derivation in *X* if it is (*l*, *r*)-derivation at once (*r*, *l*)-derivation in *X*.

The concept of derivation is also discussed in *BE*-algebra by Kim and Lee [7]. A self-map *d* of *BE*-algebra $(X; *, 1)$ is called a derivation in X if for every x, $y \in X$ meets

$$d(x * y) = (x * d(y)) \vee (d(x) * y).$$

by defining $x \vee y = (y * x) * x$ for all $x, y \in X$. Kim and Davvaz [8] discuss the concept of *f*-derivation in *BE*-algebra by involving an endomorphism. In addition, as a development of the concepts of derivation and *f*-derivation in *BE*-algebra, Kim in [9] and [10] also discusses the concepts generalized of derivation and generalized of *f*-derivation in *BE*-algebra. The concepts involve two self-maps in their definition.

The concept of *t*-derivation in *BE*-algebra has been discussed by Anhari et al. [11]. The construction of *t*-derivation in *BE*-algebra refers to the concept of *t*-derivation in *BP*-algebra [12], which begins by define a mapping $d_t(x) = x * t$ of *BE*-algebra $(X; *, 1)$, then define the concept of *t*-derivation in *BE*-algebra and determined its properties. In the paper, that has not found an example that satisfies the concept of *t*-derivation in *BE*-algebra,but the concept of *t*-derivation will satisfy the properties that have been given.

Another type of derivation that has been discussed by researchers is the concept of f_q-derivation in *BM*-algebra [13]. As with the definition of *t*-derivation, the construction of f_q-derivation in *BM*-algebra, also involves mapping similar to, but included a mapping d_t that is an endomorphism in *BM*-algebra. Gemawati et al. [14] also discuss the concept of f_q- derivation in another algebraic structure, namely BN_1-algebra.

Based on the concept of derivation in *BE*-algebra by Kim and Lee [7] and the concept of *t*-derivation in *BE*-algebra by Anhari et al. [11] discussed a new type of derivation as a form of development of the concept derivation in *BE*-algebra, which is called *q*-derivation. Then, based on the concept, the properties of *q*-derivation in *BE*-algebra are determined, as well as the properties of fixed set and kernel of *q*-derivation in *BE*-algebra.

II. PRELIMINARIES

In this section, several definitions are given that are needed to construct the main results of the study, namely the basic definitions and theories about *BE*-algebra, derivation in *BE*-algebra, and *t*-derivations in *BE*-algebra which all such concepts have been discussed in [2, 3, 7, 11].

Definition 2.1. [2] An algebra $(X; *, 1)$ of type $(2, 0)$ is said to be *BE*-algebra if it satisfies the following axioms:

(*BE1*) $x * x = 1$,

(*BE2*) $x * 1 = 1$,

(*BE3*) $1 * x = x$,

(*BE4*) $x * (y * z) = y * (x * z)$

for all $x, y, z \in X$.

Suppose $(X; *, 1)$ be a *BE*-algebra. Defined a relation $\leq$ on X by $x \leq y$ if and only if $x * y = 1$ for all $x, y \in X$.

Example 1. Let $R = \{1, a, b, c, d, 0\}$ be a set defined in Table 1.

Table 1. Cayley Table for $(R; *, 1)$

$*$	1	a	b	c	d	0
1	1	a	b	c	d	0
a	1	1	a	c	c	d
b	1	1	1	c	c	c
c	1	a	b	1	a	b
d	1	1	a	1	1	a
0	1	1	1	1	1	1

By Table 1 can be proven that $(R; *, 1)$ is a *BE*-algebra.

Definition 2.2. [2] Let $(X; *, 1)$ be a *BE*-algebra and F be a non-empty subset of X. F is said to be filter of X if

(*F1*) $1 \in F$,

(*F2*) $x \in F$ and $x * y \in F$ imply $y \in F$.

By Example 1, we obtain that $F_1 = \{1, a, b\}$ is a filter of R, whereas $F_2 = \{1, a\}$ is not a filter of X, because $a \in F_2$ and $a * b \in F_2$, but $b \notin F_2$.

Definition 2.3. [2] A *BE*-algebra $(X; *, 1)$ is said to be self-distributive if $x * (y * z) = (x * y) * (x * z)$ for all $x, y, z \in X$.

Example 2. Let $X = \{1, a, b, c, d,\}$ be a set defined in Table 2.

Table 2. Cayley table for $(X; *, 1)$

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	c	d
b	1	a	1	c	c
c	1	1	b	1	b
d	1	1	1	1	1

By Table 2, it can be proven that $(X; *, 1)$ is a *BE*-algebra satisfying self-distributive. Whereas, *BE*-algebra in Example 1 not self-distributive, due to $x = d$, $y = a$, and $z = 0$ obtained $d * (a * 0) = d * d = 1$, whereas $(d * a) * (d * 0) = 1 * a = a$.

Proposition 2.4. [7] Let $(X; *, 1)$ be a *BE*-algebra, then the following identity applies for all $x, y, z \in X$.

$$(P1) \ x * (y * x) = 1,$$

$$(P2) \ x * ((x * y) * y) = 1,$$

(*P1*) Let $(X; *, 1)$ be a self-distributive *BE*-algebra. If $x \leq y$, then $z * x \leq z * y$ and $y * z \leq x * z$.

The concept of derivation in *BE*-algebra has been discussed in [7]. Let $(X; *, 1)$ be a *BE*-algebra. We define $x \vee y = (y * x) * x$ for all $x, y \in X$.

Definition 2.5. [7] A self-map d of *BE*-algebra $(X; *, 1)$ is called a derivation in X if $d(x * y) = (x * d(y)) \vee (d(x) * y)$ for all $x, y \in X$.

Definition 2.6. [8] Suppose $(X; *, 1)$ is a *BE*-algebra. A self-map d of X is called regular if $d(1) = 1$.

The concept of a fixed set and kernel of derivation in *BE*-algebra has been discussed in [7]. Suppose $(X; *, 1)$ it is *BE*-algebra and d is a derivation in X. Defined a fixed set of d by

$$Fix_d(X) = \{x \in X \mid d(x) = x\}, \text{ for each } x \in X,$$

and kernel of d as

$$Kerd(X) = \{x \in X \mid d(x) = 1\}, \text{ for each } x \in X.$$

Definition 2.7. [3] Suppose $(X; *, 1)$ and $(Y; *, 1)$ are two *BE*-algebra. A mapping $f: X \to Y$ is called a homomorphism if $f(x * y) = f(x) * f(y)$ for all $x, y \in X$.

A homomorphism f is called an endomorphism if $f: X \to X$.

Definition 2.8. [11] Let $(X; *, 1)$ be a *BE*-algebra. A mapping d_t of X to itself is defined by $d_t\,(x) = x * t$ for all $t, x \in X$.

Definition 2.9. [11] Let $(X;*,1)$ BE-algebra. A mapping d_t of X to itself is called t-derivation in X if

$$d_t(x*y) = (x*d_t(y)) \vee (d_t(x)*y),$$

for all $x,y \in X$.

III. MAIN RESULT

In this section is defined the concept of q-derivation in BE-algebra based on the concept of derivation in BE-algebra [7] and t-derivation in BE-algebra [11]. Later, the properties of q-derivation in BE-algebra are investigated. At the end, a fixed set and kernel of q-derivation in BE-algebra are defined, as well as their properties.

Definition 3.1. Suppose $(X;*,1)$ is a BE-algebra. A mapping d_q of X to itself is defined by $d_q\ (x) = q*x$ for all $q,x \in X$.

Theorem 3.2. Suppose $(X;*,1)$ is a BE-algebra and d_q is a mapping of X to itself.

(i) d_1 is an identity function,

(ii) $x \leq d_q(x)$ for all $x \in X$,

(iii) $d_q(x*y) = x*d_q(y)$ for all $x,y \in X$.

Proof. Suppose $(X;*,1)$ is a BE-algebra.

(i) For $q=1$, from the axiom $BE3$ obtained $d_1(x) = 1*x = x$ for all $x \in X$. Hence, it is evident that d_1 is an identity function.

(ii) By axioms $BE4$, $BE1$, and $BE3$ for all $x \in X$ obtained

$$\begin{aligned} x*d_q(x) &= x*(q*x) \\ &= q*(x*x) \\ &= q*1 \\ x*d_q(x) &= 1. \end{aligned}$$

Since $x*d_q(x) = 1$, then $x \leq d_q(x)$ for all $x \in X$.

(iii) From axiom $BE4$, for all $x,y \in X$ obtained

$$\begin{aligned} d_q(x*y) &= q*(x*y) \\ &= x*(q*y) \\ d_q(x*y) &= x*d_q(y). \end{aligned}$$

Thus, $d_q(x*y) = x*d_q(y)$ for all $x,y \in X$.

Theorem 3. 3. Suppose $(X;*,1)$ be a BE-algebra self-distributive and d_q is a mapping of X to itself.

(i) $d_q(x)*y \leq x*d_q(y)$ for all $x,y \in X$,

(ii) d_q is an endomorphism of X.

Proof. Let $(X;*,1)$ is a BE-algebra self-distributive.

(i) From Proposition 2.4 (P3) and Theorem 3.2 (ii) obtained $d_q(x)*y \leq x*y \leq x*d_q(y)$ for all $x,y \in X$.

(ii) Since $(X;*,1)$ is a BE-algebra self-distributive, then for all $x,y \in X$ obtained

$$\begin{aligned} d_q(x*y) &= q*(x*y) \\ &= (q*x)*(q*y) \\ d_q(x*y) &= d_q(x)*d_q(y). \end{aligned}$$

Since $d_q: X \to X$ and it satisfy $d_q(x*y) = d_q(x)*d_q(y)$ for all $x,y \in X$, then d_q is an endomorphism of X .

Furthermore, based on the definition d_q and definition of derivation in BE-algebra, the concept of q-derivation in BE-algebra is defined.

Definition 3.4. Let $(X;*,1)$ be a BE-algebra. A mapping d_q of X to itself is called q-derivation in X if

$$d_q(x*y) = (x*d_q(y)) \vee (d_q(x)*y),$$

for all $x,y \in X$.

Example 3. Suppose $H = \{1,2,3\}$ is a set defined in Table 3.

Table 3: Cayley's table for $(H;*,1)$

$*$	1	2	3
1	1	a	b
2	1	1	b
3	1	a	1

Then, $(H;*,1)$ is a BE-algebra and d_q is a q-derivation in H.

The following is given the nature of the existence of q-derivation in BE-algebra which states that there is one d_q in BE-algebra, that is, which is always a d_1 is 1-derivation in BE-algebra.

Theorem 3. 5. If $(X;*,1)$ is a BE-algebra, then d_1 is 1-derivation in X.

Proof. Suppose $(X;*,1)$ be a BE-algebra. By axiom $BE3$ obtained $d_1(x*y) = 1*(x*y) = x*y$ for all $x,y \in X$. Then, from axioms $BE3$ and $BE1$ obtained

$$\begin{aligned} \big(x*d_1(y)\big) \vee (d_1(x)*y) &= (x*(1*y)) \vee ((1*x)*y) \\ &= (x*y) \vee (x*y) \\ &= \big((x*y)*(x*y)\big)*(x*y) \\ &= 1*(x*y) \\ \big(x*d_1(y)\big) \vee (d_1(x)*y) &= x*y\,. \end{aligned}$$

Thus, $d_1(x*y) = \big(x*d_1(y)\big) \vee (d_1(x)*y)$ for all $x,y \in X$. Hence, d_1 is 1-derivation in X.

Furthermore, the properties of q-derivation in BE-algebra are given.

Theorem 3.6. Let $(X;*,1)$ is a BE-algebra. If d_q is a q-derivation in X, then

(i) $d_q(x) = d_q(x) \vee x$ for all $x \in X$.

(ii) $d_q\left(x*d_q(x)\right) = d_q\big(d_q(x)*x\big)$ for all $x \in X$.

Proof. Let $(X;*,1)$ BE-aljabar and d_q is q-derivation in X.

(i) By axioms $BE2$ and $BE3$, for all $x \in X$ obtained

$$\begin{aligned} d_q(x) &= d_q(1*x) \\ &= (1*d_q(x)) \vee (d_q(1)*x) \\ &= d_q(x) \vee \big((q*1)*x\big) \\ &= d_q(x) \vee (1*x) \\ d_q(x) &= d_q(x) \vee x. \end{aligned}$$

(ii) From axioms $BE1$ and $BE3$, for all $x \in X$ we have

$$\begin{aligned} & d_q\left(x*d_q(x)\right) \\ &= \left(x*d_q\left(d_q(x)\right)\right) \vee \left(d_q(x)*d_q(x)\right) \\ &= \left(x*d_q\left(d_q(x)\right)\right) \vee 1 \\ &= (1*(x*d_q(d_q(x))))*(x*d_q(d_q(x))) \end{aligned}$$

$$= \left(x * d_q\left(d_q(x)\right)\right) * (x * d_q\left(d_q(x)\right))$$
$$d_q\left(x * d_q(x)\right) = 1.$$

On the other hand, from the axioms *BE1* and *BE2* obtained

$$d_q(d_q(x) * x) = (d_q(x) * d_q(x)) \vee (d_q\left(d_q(x)\right) * x)$$
$$= 1 \vee (d_q\left(d_q(x)\right) * x)$$
$$= \left(\left(d_q\left(d_q(x)\right) * x\right) * 1\right) * 1$$
$$d_q(d_q(x) * x) = 1.$$

Therefore, $d_q\left(x * d_q(x)\right) = d_q(d_q(x) * x)$ for all $x \in X$.

Furthermore, definition of regular of d_q is given.

Definition 3.7. Let $(X; *, 1)$ is a *BE*-algebra. A mapping d_q of X to itself is called regular if $d_q(1) = 1$.

Example 4. In *BE*-algebra $H = \{1, 2, 3\}$ given in Example 3, it is obtained that $d_1(1) = 1 * 1 = 1$, $d_2(1) = 2 * 1 = 1$, $d_3(1) = 3 * 1 = 1$ which states that $d_q(1) = 1$ for all $q \in H$. Hence, d_q is a regular in H.

In the following Theorem 3.8, it is stated that every q-derivation in *BE*-algebra is a regular.

Theorem 3.8. Let $(X; *, 1)$ be a *BE*-algebra. If d_q is a q-derivation in X, then d_q is regular.

Proof. Let $(X; *, 1)$ *BE*-aljabar. Since d_q is a q-derivation in X, by axioms *BE1*, *BE4*, and *BE2* for all $x, y \in X$ obtained

$$d_q(1) = d_q(x * x)$$
$$= \left(x * d_q(x)\right) \vee \left(d_q(x) * x\right)$$
$$= (x * (q * x)) \vee ((q * x) * x)$$
$$= (q * (x * x)) \vee ((q * x) * x)$$
$$= (q * 1) \vee ((q * x) * x)$$
$$= 1 \vee \left((q * x) * x\right)$$
$$= \left[\left((q * x) * x\right) * 1\right] * 1$$
$$d_q(1) = 1.$$

Thus, d_q is regular in X.

Let $(X; *, 1)$ is a *BE*-aljabar and d_q is a q-derivation in X. Defined a fixed set by $Fix_{d_q}(X) = \{x \in X \mid d_q(x) = x\}$. Then, defined kernel of d_q by $Kerd_q = \{x \in X \mid d_q(x) = 1\}$.

Example 5. Let $(H; * ,1)$ is a *BE*-algebra given in Example 3. In the example, it has been obtained that d_q is q-derivation in H. Therefore, the fixed set of d_q is $Fix_{d_1}(H) = \{1,2,3\}$, $Fix_{d_2}(H) = \{1\}$, $Fix_{d_3}(H) = \{1\}$, and $Kerd_1 = \{1\}$, $Kerd_2 = \{1,2\}$, $Kerd_3 = \{1,2,3\}$.

Some properties of q-derivation in *BE*-algebra given in Theorem 3.9.

Theorem 3. 9. Let $(X; *, 1)$ is a *BE*-algebra and d_q is a q-derivation in X.

(i) If $x, y \in Fix_{d_q}(X)$, then $x \vee y \in Fix_{d_q}(X)$ for all $x, y \in X$.

(ii) If $x \in Fix_{d_q}(X)$, then $\left(d_q \circ d_q\right)(x) = x$ for all $x \in X$.

Proof. Let $(X; *, 1)$ be a *BE*-aljabar and d_q is a q-derivation in X.

(i) Since $x \in Fix_{d_q}(X)$, then $d_q(x) = x$ for every $x \in X$. Then, from the axioms *BE1* and *BE3* obtained

$$d_q(x \vee y) = d_q\left((y * x) * x\right)$$
$$= \left[(y * x) * d_q(x)\right] \vee \left[d_q(y * x) * x\right]$$
$$= [(y * x) * x] \vee [((y * x) \vee (y * x)) * x]$$
$$= [(y * x) * x] \vee [(y * x) * x]$$
$$= (y * x) * x$$
$$d_q(x \vee y) = x \vee y.$$

So, it is evident that $x \vee y \in Fix_{d_q}(X)$.

(ii) Suppose $x \in Fix_{d_q}(X)$, then $d_q(x) = x$. So that

$$\left(d_q \circ d_q\right)(x) = d_q\left(d_q(x)\right) = d_q(x) = x.$$

Hence, $\left(d_q \circ d_q\right)(x) = x$ for all $x \in X$.

The following property states that the fixed set and kernel of q-derivation in *BE*-algebra are subalgebras.

Theorem 3. 10. Suppose $(X; *, 1)$ is a *BE*-algebra and d_q is a q-derivation in X.

(i) $Fix_{d_q}(X)$ is a subalgebra.

(ii) $Kerd_q$ is a subalgebra.

Proof. Suppose $(X; *, 1)$ is a *BE*-aljabar.

(i) From the axiom *BE2* obtained $d_q(1) = q * 1 = 1$, then $1 \in Fix_{d_q}(X)$. So, $Fix_{d_q}(X)$ is a non-empty set. Let $x, y \in Fix_{d_q}(X)$, then $d_q(x) = x$ and $d_q(y) = y$. Since d_q is a q-derivation in X and by axioms *BE1* and *BE3* we get

$$d_q(x * y) = \left(x * d_q(y)\right) \vee (d_q(x) * y)$$
$$= (x * y) \vee (x * y)$$
$$d_q(x * y) = x * y.$$

Then, $x * y \in Fix_{d_q}(X)$. Therefore, $Fix_{d_q}(X)$ is a subalgebra of X.

(ii) From the axiom *BE2* obtained $d_q(1) = q * 1 = 1$, then $1 \in Kerd_q$. So, $Kerd_q$ is a non-empty set. Let $x, y \in Kerd_q$, then $d_q(x) = 1$ and $d_q(y) = 1$. Since d_q is a q-derivation in X, and by axioms *BE1*, *BE2*, and *BE3* we have

$$d_q(x * y) = \left(x * d_q(y)\right) \vee (d_q(x) * y)$$
$$= (x * 1) \vee (1 * y)$$
$$= 1 \vee y$$
$$= (y * 1) * 1$$
$$d_q(x * y) = 1.$$

Then, $x * y \in Kerd_q$. Therefore, $Kerd_q$ is a subalgebra of X.

Furthermore, given the kernel properties of q-derivation in *BE*-algebra.

Theorem 3.11. Let $(X; *, 1)$ is a *BE*-algebra and d_q is q-derivation in X.

(i) If $x \in Kerd_q$, then $x \vee y \in Kerd_q$ for all $y \in X$.

(ii) If $y \in Kerd_q$, then $x * y \in Kerd_q$ for all $x \in X$.

Proof. Let $(X; *, 1)$ be a *BE*-aljabar and d_q is a *q*-derivation in X.

(i) Since $x \in Kerd_q$, then $d_q(x) = 1$. By axiom *BE2* we obtain

$$\begin{aligned} d_q(x \vee y) &= d_q((y * x) * x) \\ &= [(y * x) * d_q(x)] \vee [d_q(y * x) * x] \\ &= [(y * x) * 1] \vee [d_q(y * x) * x] \\ &= 1 \vee [d_q(y * x) * x] \\ &= [(d_q(y * x) * x) * 1] * 1 \\ d_q(x \vee y) &= 1. \end{aligned}$$

Hence, if $x \in Kerd_q$, then $x \vee y \in Kerd_q$ for all $y \in X$.

(ii) Let $y \in Kerd_q$, then $d_q(y) = 1$. By axiom *BE2* obtained

$$\begin{aligned} d_q(x * y) &= (x * d_q(y)) \vee (d_q(x) * y) \\ &= (x * 1) \vee (d_q(x) * y) \\ &= 1 \vee (d_q(x) * y) \\ &= ((d_q(x) * y) * 1) * 1 \\ d_q(x * y) &= 1. \end{aligned}$$

Thus, if $y \in Kerd_q$, then $x * y \in Kerd_q$ for all $x \in X$.

IV. CONCLUSION

In this paper, the concept of *q*-derivation in *BE*-algebra is defined as a development of the concept of derivation in *BE*-algebra. The definition of *q*-derivation in *BE*-algebra begins with defining a mapping d_q that is a self-map in *BE*-algebra. Later, it was proved that d_1 it is a 1-derivation in *BE*-algebra, as well as in obtaining other properties of *q*-derivation. Finally, the properties of the fixed set and kernel of *q*-derivation in *BE*-algebra are obtained based on its elements, and it is obtained that the fixed set and the kernel of *q*-derivation in *BE*-algebra is a subalgebra.

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