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Maximum Principle for Optimal Control of COVID-19 Spread

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ABSTRACT

Covid-19 is a disease caused by a new corona virus that has spread throughout the world and become a pandemic. It is classified as an infectious disease that can be transmitted from human to human through droplets. So that we need controls to reduce the spread of Covid-19. The optimal control that will be carried out this work is self-precaution, treatment and quarantined that will be applied to the dynamical modelling of Covid-19 spread using the Pontryagin's Maximum Principle (PMP) to find out the optimal solution for the control. According to this principle the optimal control, corresponding optimal state, and adjoint function must minimize the Hamiltonian function. PMP converts the optimal control problem into a multipoint boundary value problem. That is, the optimality condition results in control. The optimal control variable, corresponding state and adjoint can be computed by solving an Ordinary Differential Equation system. The control strategies is aimed to reduce covid-19 transmission. Numerical results show the effectiveness of the control strategies in reducing Covid-19 spread. It is found that self-precaution more effective than treatment and quarantined.

KEYWORDS: covid-19, optimal control, pontryagin's maximum principle, self precaution, treatment, quarantined

I. INTRODUCTION

An outbreak of severe respiratory pneumonia recently emerged in China related to the 2019 novel coronavirus (Covid-19) caused by the SARS-CoV-2 virus. symptoms in people infected with the virus experience mild to moderate respiratory distress [1]. This disease mostly affects the elderly and people who have underlying medical conditions such as cardiovascular disease, diabetes, chronic respiratory disease, or cancer will develop serious illnesses [2-6]. Covid-19 is a disease that can be transmitted from person to person through droplets on infected people [7-9].

Coronavirus has become a serious global phenomenon in recent years, thus negatively impacting the world's health and economy. Currently, scientists are conducting research to obtain drugs and vaccines to prevent the spread of quantitative and qualitative information about the etiology of this disease is very important for us in the formation of mathematical models [10]. Based on data released by the official website of the World Health Organization (WHO) in early June 2022, there are more than 528 million confirmed cases of infection, with more than 6 million deaths worldwide. In Indonesia, as of June 3, 2022, there were 6,056,017 confirmed cases of COVID-19 with 156,604 deaths, which were reported by WHO [11].

Mathematical modeling plays an important role in describing the Coronavirus Disease 2019 (COVID-19) outbreak using Ordinary Differential Equations (ODE) and Fractional Differential Equations [12]. Several mathematical models were developed Several useful mathematical models have been formulated in some endemic disease transmission. Compartment models and real cases are more effective in providing valuable information about a particular disease outbreak. The dynamics of the Covid-19 pandemic use different compartments, namely susceptible, exposed, asymptomatic infective, symptomatic infective, hospitalization, and recovery. Global stability analysis was carried out using the Lyapunov method when the basic reproduction number was greater than one [13,14].

Further, dynamical modeling studies can be of great help in understanding the natural history of this new disease, treatment and vaccine efficacy [15]. The control of the COVID-19 outbreaks using the SEIRV model was formulated and analyzed using optimal control with preventive measures, surface disinfection and medical measures to minimize the number of individuals exposed and infected by considering implementation costs. PMP is used to determine the optimal strategy needed to reduce disease [16,17,18]. Several researchers have published the application of the optimal control theory of COVID 19 used for pandemic surveillance, four controls are introduced i.e preventive, detection of asymptomatic cases, detection of symptomatic cases, and vaccination. Optimal strategy in scenarios that involve administration of vaccines, increased vaccine supply, and limited combinations of interventions [19-23]. Furthermore, the mathematical model is extended to the optimal control problem by including control variables in the model as surgical face masks, social distancing, self-isolation, sensitization and awareness of disease transmission, as well as control including reducing the cost infrastructure such as testing facilities, which can lead to detection fast cases infected with Covid [24-29].

Optimal control theory in epidemiology is useful for understanding how spread strategies by applying a combination of isolation, quarantine, vaccination and treatment controls are often needed to reduce infectious diseases. Optimization strategies have been used in several COVID-19 studies [30, 31]. The COVID-19 deterministic mathematical model was formulated to extensively investigate optimal control as a preventive measure to reduce the transmission and spread of COVID-19 [32-34]. So it is very important to apply optimal control theory to determine strategies in controlling the spread of Covid-19 disease.

Fitriyani, et. al. [35], studied the STQIR model for the Covid-19 outbreak and proved its stability in several analytic. It is theoretically proven that dynamic transmission depends on basic reproduction numbers so that its epidemiological relevance has also been proven. However, this model has not been given control to prevent the spread of Covid-19. Therefore, the purpose of this paper is to develop a mathematical model [35] by adding three controls variables, namely self-prevention, treatment, and quarantine to reduce the transmission of COVID-19 while minimizing the functional objective. Self-prevention (the use of hand sanitizer, medical masks, physical distancing) to reduce the infection probability, treatment to accelerate recovery, and quarantine to prevent disease transmission. Further we will determine the effectiveness of the control strategies. To verify the proposed model, we estimated the parameters values base on data from Central Java, Indonesia.

The work is organized as follows, in the Section 1 we give introduction consist of scientific background, importance, and objectives. Proposed dynamical model with control variables is presents in the Section 2. The optimal conditions and analysis of the optimal control model by using Pontryagin's Maximum Principle are described in Section 3. Section 4 presents numerical simulations of the optimal control and control effectiveness analysis. Next, conclusions are given in the last section.

II. COVID-19 SPREAD DYNAMICAL MODEL WITH CONTROL

In this section, we develop a dynamical model with control based on the pandemic situation. The human population (size N) is divided into five class, i.e susceptible individuals (S), exposed individuals (T), quarantine individuals (Q), infected individuals (I), and recovered individuals (R). Consider mathematical model without control [29] as follows.

$$S(t)=\pi+ST(\kappa\rho_1-\beta r)-S\beta(1-r)-S\tau$$
$$T(t)=ST(\beta r-\kappa\rho_1)-T\kappa\rho_2-T\kappa(1-\rho_1-\rho_2)-T\tau$$
$$Q(t)=S\beta(1-r)+T\kappa(1-\rho_1-\rho_2)-Q\gamma_1-Q(\mu+\tau)$$
$$I(t)=T\kappa\rho_2-I\gamma_2-I(\mu+\tau)$$
$$R(t)=Q\gamma_1+I\gamma_2-R\tau$$

where the initial condition $S(0)\geq 0, T(0)\geq 0$ $Q(0)\geq 0, I(0)\geq 0, R(0)\geq 0$ with parameters π is recruitment rate of S class, β is infection rate, r is rate of people become probable, κ is rate of people become suspect, ρ_1 is rate at which traced people back to susceptible class, ρ_2 is rate at which traced individuals become infected, γ_1 is recovery rate of quarantined class, γ_2 is recovery rate of infected class, μ is death rate due to the infection, τ is natural death rate.

We proposed the developed mathematical model with control as follows

$$\begin{aligned}
\dot{S}(t)&=\pi+ST(1-u_1(t))(\kappa\rho_1-\beta r)-S(1-u_1(t))\beta(1-r)-S\tau\\
\dot{T}(t)&=ST(1-u_1(t))(\beta r-\kappa\rho_1)-T\kappa\rho_2-T\kappa(1-\rho_1-\rho_2)-T\tau\\
\dot{Q}(t)&=S(1-u_1(t))\beta(1-r)+T\kappa(1-\rho_1-\rho_2)-Qu_3(t)-Q(\mu+\tau)\\
\dot{I}(t)&=T\kappa\rho_2-Iu_2(t)-I(\mu+\tau)\\
\dot{R}(t)&=Qu_3(t)+Iu_2(t)-R\tau
\end{aligned}\tag{2.2}$$

where $u_1(t)$ is control variable for self precaution, $u_2(t)$ is control variable for covid-19 treatment and $u_3(t)$ is control variable for quarantined.

III. PONTRYAGIN MAXIMUM PRINCIPLE FOR OPTIMAL CONTROL

In this section, we derive the maximum principle for optimal control problem (2.2) and (3.1). We proposed the necessary conditions for optimal controls. Let objective functional corresponding with control variables and the dynamic model (2.2)

$$\begin{aligned}
J(u_1,u_2,u_3)=\min\int_0^T\Big[&A_1T(t)+A_1T(t)+A_1T(t)\\
&+\frac{1}{2}\left(w_1u_1^2(t)+w_2u_2^2(t)+w_3u_3^2(t)\right)\Big]dt
\end{aligned}\tag{3.1}$$

where A_1, A_2, A_3 is the weight of traced, quarantined, and infected sub-population. While w_1u_1, w_2u_2, w_3u_3 is cost function of u_1, u_2 and u_3.

The conditions to determine the optimal control that minimizes the objective functional can be found by using Pontryagin's maximum principle [36,37]. The principle of the PMP method is to transform the model (2.2) that minimizes the objective functional (3.1) into a problem that minimizes the Hamiltonian function. Consider Halmitonian function (H) as follows

$$H(x,u,\varphi,t) = A_1T(t) + A_2Q(t) + A_3I(t) + \frac{1}{2}\left(w_1u_1^2(t) + w_2u_2^2(t) + w_3u_3^2(t)\right) + \varphi_1\left(\frac{dS}{dt}\right) + \varphi_2\left(\frac{dT}{dt}\right) + \varphi_3\left(\frac{dQ}{dt}\right) + \varphi_4\left(\frac{dI}{dt}\right) + \varphi_5\left(\frac{dR}{dt}\right) \quad (3.2)$$

The goal of objective functional is to minimize the number of infected individuals and the cost of control measures. Then define the optimal control u_1^*, u_2^*, u_3^* and solutions S^*, T^*, Q^*, I^*, R^* of system (2.2) for minimizing $J(u_1, u_2, u_3)$ over U,

with $U := \left\{\left(u_1^*, u_2^*, u_3^*\right) | \, 0 \le u_i(t) \le 1, i = 1,2,3, t \in (0, t_f)\right\}$

Theorem 1. There exist optimal control $u^* = \left(u_1^*, u_2^*, u_3^*\right) \in U$ minimizing $J(u_1, u_2, u_3)$. Then $\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5$ are adjoin variables that satisfies:

$$\begin{aligned}
\frac{d\varphi_1}{dt} &= \varphi_1\left(T(1-u_1(t))\beta(1-r) + \tau - T(1-u_1(t))(\kappa\rho_1 - \beta r)\right) \\
&\quad - \varphi_2\left(T(1-u_1(t))(\kappa\rho_1 - \beta r)\right) - \varphi_3 T(1-u_1(t))\beta(1-r) \\
\frac{d\varphi_2}{dt} &= -A_1 - \varphi_1\left(S(1-u_1(t))(\kappa\rho_1 - \beta r) - S(1-u_1(t))\beta(1-r)\right) \\
&\quad - \varphi_2\left(S(1-u_1(t))(\beta r - \kappa\rho_1) - \kappa\rho_2 - \kappa(1-\rho_1-\rho_2) - \tau\right) \\
&\quad - \varphi_3\left(S(1-u_1(t))\beta(1-r) + \kappa(1-\rho_1-\rho_2) - \varphi_4(\kappa\rho_2)\right) \\
\frac{d\varphi_3}{dt} &= -A_2 - \varphi_3\left(u_3(t) - (\mu+\tau)\right) - \varphi_5\left(u_3(t)\right) \\
\frac{d\varphi_4}{dt} &= -A_3 + \varphi_4\left(-u_2(t) - (\mu+\tau)\right) - \varphi_5\left(u_2(t)\right) \\
\frac{d\varphi_5}{dt} &= \tau
\end{aligned} \quad (3.3)$$

Hamilton function will be minimized by transversality condition,

$$\varphi_1(t_f) = \varphi_2(t_f) = \varphi_3(t_f) = \varphi_4(t_f) = \varphi_5(t_f) = \varphi(t_f) = 0 \quad (3.4)$$

and the control u_1^*, u_2^* and u_3^* satisfy the optimality condition:

$$\begin{aligned}
u_1^*(t) &= \min\left\{u_{1\max}, \max\left(0, \frac{1}{w_1}\left(\varphi_2\left(S^*T^*(\beta r - \kappa\rho_1)\right) + \varphi_3\left(S^*T^*\beta(1-r)\right)\right.\right.\right. \\
&\quad \left.\left.\left. - \varphi_1\left(S^*T^*\beta(1-r) - (\kappa\rho_1 - \beta r)\right)\right)\right)\right\} \\
u_2^*(t) &= \min\left\{u_{2\max}, \max\left(0, \frac{1}{w_2} I^*(\varphi_4 - \varphi_5)\right)\right\} \\
u_3^*(t) &= \min\left\{u_{3\max}, \max\left(0, \frac{1}{w_3} Q^*(\varphi_3 - \varphi_5)\right)\right\}
\end{aligned} \quad (3.5)$$

Proof. The adjoint system (3.3) is found by partially differentiating the Hamiltonian function (3.2) with respect to the associating state variables S, T, Q, I and R as follows.

$$\begin{aligned}
\frac{d\varphi_1}{dt} &= -\frac{\partial H}{\partial S} = \varphi_1\left(T(1-u_1(t))\beta(1-r) + \tau - T(1-u_1(t))(\kappa\rho_1 - \beta r)\right), \\
&\quad - \varphi_2\left(T(1-u_1(t))(\beta r - \kappa\rho_1)\right) - \varphi_3 T(1-u_1(t))\beta(1-r),
\end{aligned} \quad (3.6)$$

$$\begin{aligned}
\frac{d\varphi_2}{dt} &= -\frac{\partial H}{\partial T} = -A_1 - \varphi_1\left(S(1-u_1(t))(\kappa\rho_1 - \beta r) - S(1-u_1(t))\beta(1-r)\right), \\
&\quad - \varphi_2\left(S(1-u_1(t))(\beta r - \kappa\rho_1) - \kappa\rho_2 - \kappa(1-\rho_1-\rho_2) - \tau\right), \\
&\quad - \varphi_3\left(S(1-u_1(t))\beta(1-r) + \kappa(1-\rho_1-\rho_2) - \varphi_4(\kappa\rho_2)\right), \\
\frac{d\varphi_3}{dt} &= -\frac{\partial H}{\partial Q} = -A_2 - \varphi_3\left(u_3(t) - (\mu+\tau)\right) - \varphi_5\left(u_3(t)\right), \\
\frac{d\varphi_4}{dt} &= -\frac{\partial H}{\partial I} = -A_3 + \varphi_4\left(-u_2(t) - (\mu+\tau)\right) - \varphi_5\left(u_2(t)\right), \\
\frac{d\varphi_5}{dt} &= -\frac{\partial H}{\partial R} = \tau,
\end{aligned}$$

with condition (3.4), the characterization of the controls of (3.5) are obtined by solving u_1^*, u_2^*, u_3^* from the equations,

$$\begin{aligned}
0 &= \frac{\partial H}{\partial u_1} = w_1u_1^* + \varphi_1\left(S^*T^*\left(\beta(1-r) - (\kappa\rho_1 - \beta r)\right)\right) \\
&\quad - \varphi_2\left(S^*T^*(\beta r - \kappa\rho_1)\right) - \varphi_3\left(S^*T^*\beta(1-r)\right) \\
0 &= \frac{\partial H}{\partial u_2} = w_2u_2^* + I^*(\varphi_4 - \varphi_5) \\
0 &= \frac{\partial H}{\partial u_3} = w_3u_3^* + Q^*(\varphi_3 - \varphi_5)
\end{aligned} \quad (3.7)$$

Thus, we find

$$\begin{aligned}
u_1^* &= \frac{1}{w_1}\left(\varphi_2\left(S^*T^*(\beta r - \kappa\rho_1)\right) + \varphi_3\left(S^*T^*\beta(1-r)\right)\right. \\
&\quad \left.\left(-\varphi_1\left(S^*T^*\beta(1-r) - (\kappa\rho_1 - \beta r)\right)\right)\right) \\
u_2^* &= \frac{1}{w_2} I^*(\varphi_4 - \varphi_5) \\
u_3^* &= \frac{1}{w_3} Q^*(\varphi_3 - \varphi_5)
\end{aligned} \quad (3.8)$$

If $\frac{\partial H}{\partial u_i} < 0$ at t, then $u_i^*(t) = 0$, for $i = 1,2,3$ and if $\frac{\partial H}{\partial u_i} > 0$ at t, we take $u_i^*(t) = 1$

Therefore, we can write the optimal control u_1^*, u_2^*, u_3^* such as:

$$\begin{aligned}
u_1^*(t) &= \min\left\{u_{1\max}, \max\left(0, \frac{1}{w_1}\left(\varphi_2\left(S^*T^*(\beta r - \kappa\rho_1)\right) + \varphi_3\left(S^*T^*\beta(1-r)\right)\right.\right.\right. \\
&\quad \left.\left.\left. - \varphi_1\left(S^*T^*\beta(1-r) - (\kappa\rho_1 - \beta r)\right)\right)\right)\right\} \\
u_2^*(t) &= \min\left\{u_{2\max}, \max\left(0, \frac{1}{w_2} I^*(\varphi_4 - \varphi_5)\right)\right\} \\
u_3^*(t) &= \min\left\{u_{3\max}, \max\left(0, \frac{1}{w_3} Q^*(\varphi_3 - \varphi_5)\right)\right\}
\end{aligned} \quad (3.9)$$

■

IV. NUMERICAL ANALYSIS AND DISCUSSION

In this section, we consider the method to solve the COVID-19 control system (2.2), numerically using the Runge Kutta 4 method which has been extensively explored to solve the system optimality of the optimal control model. The optimality system results and the control parameter system are numerically obtained by Matlab R2015a. For design numerical scheme the optimal control strategies, we used parameter values that were fitted to the COVID-19 data Central Java Province, Indonesia as follows

$\pi = 0.063; \beta = 0.601; r = 0.07; \kappa = 0.02057; \rho_1 = 0.037462;$
$\rho_2 = 0.1227; \gamma_1 = 0.037462\gamma_2 = 0.028272; \mu = 0.0059; \tau = 0.033121$
final time $(t_f) = 100$ days. We use the initial values as follows
$S(0) = 21887966; T(0) = 49690; Q(0) = 10340; I(0) = 6892;$
$R(0) = 11328; w_1 = 0.2; w_2 = 1; w_3 = 0.2$

In the following figures, we present the control variables for self-precaution, covid-19 treatment and quarantined. At the beginning of the pandemic, the transmission of the disease (R_0) was high, so susceptible and traced individuals reached a peak in a short time of 0-10 days, 150,000 and 2,500,000 susceptible individuals were infected as shown in Figures (1) and (2). The proposed control parameters help control and reduce the spread of the disease. Figures (3) and (4) show a sloping curve from day 10 to 100, this is due to the application of control parameters to infected and quarantined individuals. Due to the reduction of infected sub population, the number of recovered sub population increased which is shown in Figure (5).

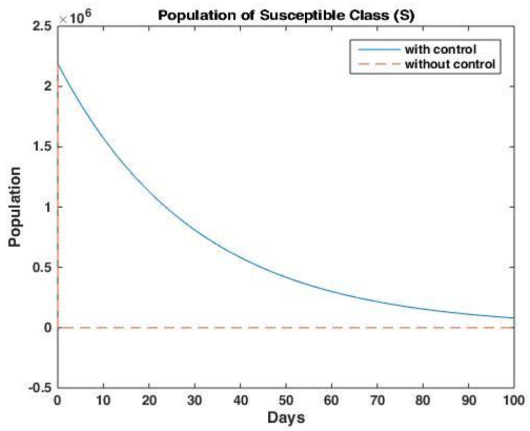

Figure 1. The effect on susceptible class with and without controls.

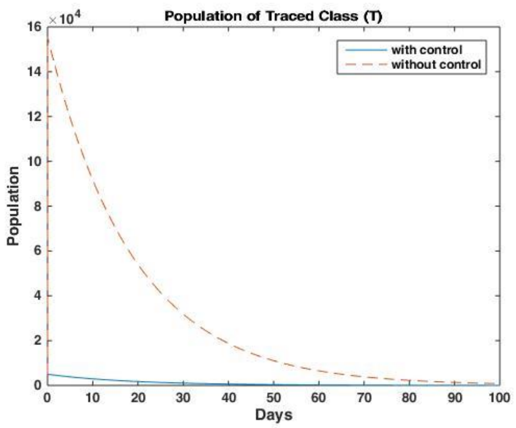

Figure 2. The effect on traced class with and without controls.

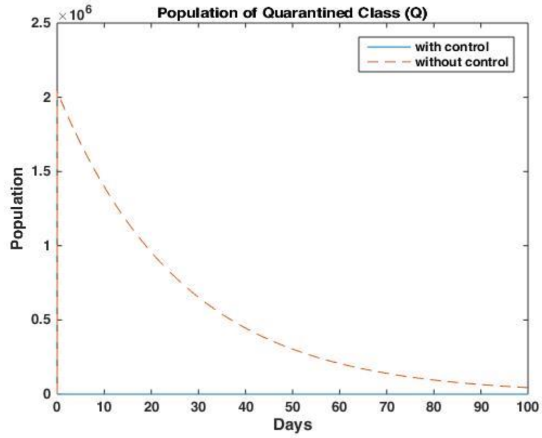

Figure 3. The effect on quarantined class with and without controls.

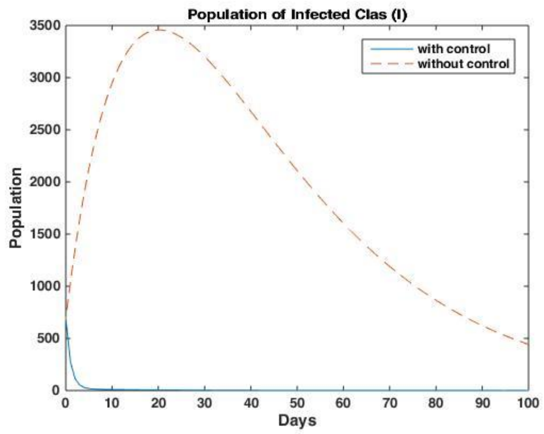

Figure 4. The effect on infected class with and without controls.

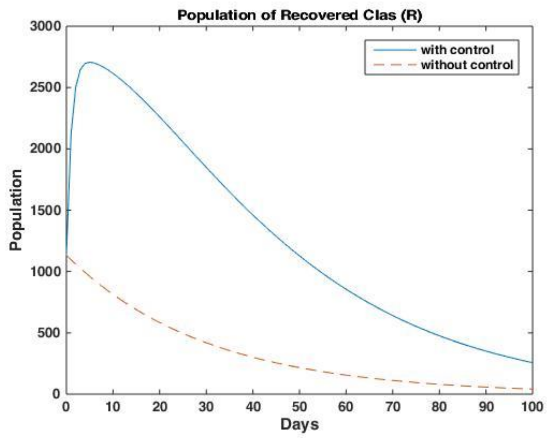

Figure 5. The effect on recovered class with and without controls.

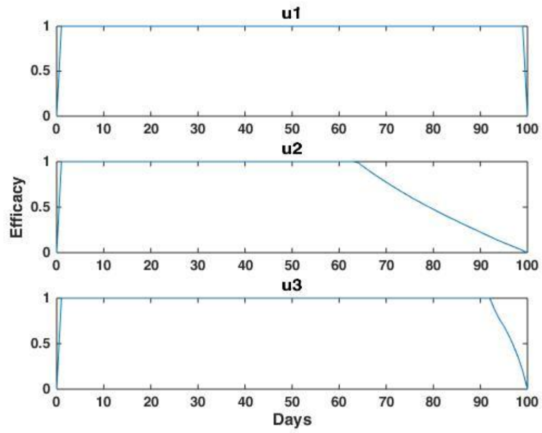

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Figure 6. The Fig the intervention of Optimal Control of self-precaution, covid-19 treatment and quarantined.

Figure 6 shows the optimal profile of the control strategy effective rate. It shows the optimal control of u_1 is at upper bound for almost 100 days means that the optimal effort is given in the self precaution. Then we got the optimal control of u_2 is at upper bound for 65 days and then decreasing. And for u_3 is at upper bound for day 1 until early 90. This shows that the control provision in the form of self precaution is more effective that the other controls.

V. CONCLUSION

In this paper we have proposed a dynamical model of Covid-19 spread with implementation of control strategies, namely self-precaution, treatment and quarantine. We found the value of optimal control by using Pontryagin's maximum principle method. The necessary conditions for optimality control have been proven. To verify the proposed model, numerical simulations are given. From the simulation results, we obtained that giving control is in the form of self-precaution, treatment and quarantine in the system can reduce the transmission of COVID-19. The effectiveness of the controller at approximately 0 to 100 days can reduce the number of individuals infected with COVID-19, so that the transmission of COVID-19 can be suppressed and increase the number of recovered individuals and be able to minimize the costs required to provide control.

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REFERENCES

1. WHO https://www.who.int/health-topics/coronaviru s#tab=tab_12021, Accessed, 3 June 2022
2. C. Xie et al. Comparison Of Different Samples For 2019 Novel Coronavirus Detection By Nucleic Acid Amplification Tests. International Journal of Infectious Diseases 93: 264–267. 2020.
3. C. Sohrabi, Alsafi, N. O'Neill, M. Khan, A. Kerwan, A. Al-Jabir and R. Agha, "World Health Organization Declares Global Emergency: a review of thr 2019 novel coronavirus (COVID-19)," Int J Surg, p. 76:71, 2020.
4. E. B. Postnikov, "Estimation of COVID-19 dynamics "on a back-of-envelope": Does the simplest SIR model provide quantitative parameters and predictions?," ScienceDirect, 2020.
5. L. Wang., Y. Wang., Liu Qq, "A review of the 2019 novel coronavirus (COVID-19) based on current evidence," Int J Antimicrob Agents, 2020.
6. He F, Deng Y and Li W, "Coronavirus disease 2019 (COVID-19): what we know?," J Med Virol, 2020.
7. [7] C. Yang and J. Wang, "A mathematical model for the novel coronavirus epidemic in Wuhan, China," Mathematical Biosciences and Engineering., vol. 17, no. 3, pp. 2708-2724, 2020.
8. N. Kapata et al. Is Africa prepared for tackling the COVID-19 (SARS-CoV-2) epidemic. Lessons from past outbreaks, ongoing pan-African public health efforts, and implications for the future / International Journal of Infectious Diseases 93: 233–236. 2020.
9. Z. Hu, "Evaluation and prediction of the COVID-19 variations at different input population and quarantine strategies, a case study in Guangdong province, China," International Journal of Infectious Diseases, 2020.
10. Zamir, M., Shah, Z., Nadeem, F. et. al. Non-Pharmaceutical Interventions for Optimal Control of COVID-19. Computer Methods and Programs in Biomedicine 196: 105642. 2020.
11. https://covid19.who.int/region/searo/country/id, Accessed, 3 June 2022
12. R. Din., E. A. Algehyne. Mathematical analysis of COVID-19 by using SIR model with convex incidence rate. Results in Physics 23:103970. 2021.
13. P. Riyapan., S. E. Shuaib., A Intarasit. A Mathematical Model of COVID-19 Pandemic: A Case Study of Bangkok, Thailand. Computational and Mathematical Methods in Medicine Vol 2021: 6664483. 2021.
14. S. Annas, "Stability analysis and numerical simulation of SEIR model for pandemic COVID-19 spread in Indonesia," Chaos. Solitons. Fractals, 2020.

15. B. Chhetri., V. M. Bhagat, S. Muthusamy, V. S. Ananth., D. K. K. Vamsi., C. B. Sanjeevi. Time Optimal Control Studies on COVID-19 Incorporating Adverse Events of the Antiviral Drugs. Comput. Math. Biophys. 9:214–241. 2021
16. L. L. Obsu and S. F. Balcha. Optimal control strategies for the transmission risk of COVID-19. Journal Of Biological Dynamics. Vol. 14, No. 1, 590–607. 2020.
17. S. Wang and Z. Wu. Maximum Principle for Optimal Control Problems of Forward-Backward Regime-Switching Systems Involving Impulse Controls. Mathematical Problems in Engineering. Vol 2015: 892304. 2015.
18. C. D. S. Estadilla, J. Uyheng., E. P. L. Tuprio., T. R. Teng., J. M. R. M. Macalalag., R. J. E. Estuar. Impact of vaccine supplies and delays on optimal control of the COVID-19 pandemic: mapping interventions for the Philippines. Infect Dis Poverty. 10:107. 2021.
19. [19] A. A. Khan., S. Ullaha., R. Amin. 2022. Optimal control analysis of COVID-19 vaccine epidemic model: a case study. Eur. Phys. J. Plus. 137:156.
20. S. I. Araz. Analysis of a Covid-19 model: Optimal control,stability and simulations. Alexandria Engineering Journal. 60: 647-658. 2021.
21. J.K.K. Asamoah, M.A. Owusu and Z. Jin et al. Global stability and cost-effectiveness analysis of COVID-19 considering the impact of the environment: using data from Ghana. Chaos, Solitons and Fractals 140: 110103. 2020.
22. Olivares, A.; Staffetti, E. Optimal Control Applied to Vaccination and Testing Policies for COVID-19. Mathematics, 9, 3100. 2021.
23. S. S. Chaharborj., S. S. Chaharborj., J H Asl. P. S. Phang. Controlling of pandemic COVID-19 using optimal control theory. Results in Physics. 26: 104311. 2021.
24. N. K. Goswami., B. Shanmukha. 2021. Dynamics of COVID-19 Outbreak and Optimal Control Strategies: A Model-Based Analysis. Adv Syst Sci Appl. 04:65–86.
25. J. M. Macalisang., M. L. Caay., J. P. Arcede, and R. L. Caga-anan. Optimal Control for a COVID-19 Model Accounting for Symptomatic and Asymptomatic. Comput. Math. Biophys. 8:168–179; 2020.
26. N. Moussouni and M. Aliane. Optimal control of COVID-19 An International Journal of Optimization and Control: Theories & Applications. Vol.11, No.1, pp.114-122. 2021.
27. M. M. Morato., S. B. Bastos., D. O. Cajueiro., J.E. N. Rico. An optimal predictive control strategy for COVID-19 (SARS-CoV-2) social distancing policies in Brazil. Annual Reviews in Control. 50: 417-431. 2020.
28. C. Tsay., F. Lejarza., M.A. Stadtherr., M Baldea. Modeling, state estimation, and optimal control for the US COVID 19 outbreak. Scientific Reports. 10:1071. 2020.
29. M. Zamir., T. Abdeljawad., F. Nadeem., A. Wahid., A. Yousef. 2021. An optimal control analysis of a COVID-19 model. Alexandria Engineering Journal. Vol 60: 2875-2884.
30. Perkins, T. A., España G. 2020. Optimal Control of the COVID-19 Pandemic with Non-pharmaceutical Interventions. Bulletin of Mathematical Biology, 82:118.
31. Silva, C. J., Cruz, C., Torres, D. F. M., Muñuzuri, A. P., Carballosa, A., Area, I., Nieto.J.J., Pinto, R. F., Passadouro. R., E. S. Santos., Abreu.W., Mira. J. 2021. Optimal control of the COVID-19 pandemic: controlled sanitary deconfnement in Portugal. Scientifc Reports, 11:3451.
32. Madubueze, C. E., Dachollom, S., Onwubuya, I. O. 2020. Controlling the Spread of COVID-19: Optimal Control Analysis. Computational and Mathematical Methods in Medicine, Vol 2020, 6862516.
33. A. Dipo, M. Z, Ndii and B. M. Samiadji, "Optimal Control on COVID-19 eradication in Indonesia under the Effect of Community awareness.," Mathematical Biosciences and Engineering., 2020.
34. Sugiyanto and Muchammad Abrori, "A mathematical Model of the Covid-19 Cases in Indonesia (Under and Without Lockdown Enforcement)," BIOMEDICH, vol. 9, pp. 15-19, 2020.
35. U. A. Fitriani, Widowati, Sutimin, and P. S. Sasongko, "Mathematical modeling and analysis of COVID-19 transmission dynamics in Central Java Province, Indonesia," J. Phys. Conf. Ser., vol. 1943(1), 2021.
36. A. A. Agrachev et al., Control Theory from the Geometric Viewpoint. Springer-Verlag Berlin Heidelberg 2004.
37. A. V. Dmitruk and A. M. Kaganovich. 2011. Maximum Principle For Optimal Control Problems With Intermediate Constraints. Computational Mathematics and Modeling, Vol. 22, No. 2.