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Few Applications of (r*g*)* Closed Sets in Topological Spaces

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ABSTRACT: **In this paper we introduce new types of spaces , (r*g*)* closure and (r*g*)* interior and study some of its properties**.

Key words : (r*g*)*closed set ,(r*g*)* open set. (r*g*)* closure and (r*g*)* interior.

Mathematics Subject Classification : 54A05

1.INTRODUCTION

N Levine [7] introduced the class of g closed sets. Many authors introduced several generalized closed sets. The Authors [10] have already introduced (r*g*)* closed sets and investigated some of their properties. Applying these sets, some New Spaces Like (r*g*)*$T_{1/2}$, (r*g*)* T_c and (r*g*)* $T_{1/2}{}^{\#}$ spaces are introduced and some of their properties are investigated. Also (r*g*)* closure and (r*g*)*interior and their basic properties are investigated.

2. PRELIMINARIES:

2.1: A subset A of a space X is called

(1) a preopen set if A $\subseteq$ int (cl(A)) and a pre closed set if cl(int(A)) $\subseteq$ A.

(2) a semi-open set if A $\subseteq$ cl(int(A)) and a semi-closed set if (int(cl(A) $\subseteq$ A.

(3) A semi-preopen set (β _open) if A $\subseteq$ cl(int(cl(A))) and a semi- preclosed set (β _closed) if int (cl(int(A))) $\subseteq$ A.

Definition:2.2: A subset A of a space X is called

1. A generalized closed (g closed) [7] set if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open.
2. A semi generalized closed (briefly sg - closed) [5] if scl(A) $\subseteq$ U whenever (A) $\subseteq$ U and U is semiopen in X.
3. A generalized semi closed (briefly gs - closed) [2] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
4. A g* closed [11] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open.
5. A g# closed [12] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α g open.
6. A r*g*closed set [9] if rcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g- open.

Definition 2.3: A Topologiccal space (x,τ) is said to be

1. A $T_{1/2}$ space [11] if every g closed set in it is closed.
2. A semi $T_{1/2}$ [5] if every sg closed set in it is semi closed.
3. A semi pre $T_{1/2}$ [1] pace if every gsp closed set in it is semi pre closed.

4. A $T_{1/2}$* space[11] if every g* closed set in it is closed.
5. A * $T_{1/2}$ space[11] if every g closed set in it is g* closed.
6. T_b space [2] if every gs closed set in it is closed.
7. T_c [11] space if every gs closed set in it is g* closed.
8. $T_{1/2}$# [12] space if every #g closed set in it is closed.
9. $^{\#}T_{1/2}$ [13] space if every g closed set in it is $^{\#}$g closed.

Definition 2.4: A Space (X,τ) is called (r*g*)*$T_{1/2}$ space if every (r*g*)* closed set in it is closed.

Example 2.5: Let X={a,b,c} τ ={φ, X,{a},{b},{a,b}}

Here (r*g*)* C (X, τ) = C (X, τ)

Hence (X, τ) is a (r*g*)* $T_{1/2}$ space.

Example 2.6 :Let X={a,b,c} τ_1 ={φ, X,{a},{a,c}}

Here {a} is {r*g*)* closed set but not closed.

Hence (X, τ_1) is not a (r*g*)* $T_{1/2}$ space.

Theorem 2.7: If (X, τ) is a (r*g*)* $T_{1/2}$ space then every singleton set of X is either r*g* closed or open.

Proof: Let x∈ X and suppose that {x} is not a r*g* closed set of (X, τ). Then X – {x} is not a r*g* open set of (X ,τ) .Therefore

X is the only r*g* open set of (X, τ) containing X –{x} and hence X –{x} is(r*g*)* closed set of (X, τ). Since (X, τ) is a (r*g*)* $T_{1/2}$ Space every (r*g*)* closed set is closed. Hence X - {x} is closed hence {x} is open.

Theorem 2.8: Every (r*g*)* $T_{1/2}$ Space is $T_{1/2}$*.

Let X be (r*g*)* $T_{1/2}$. Let A ∈ (X ,τ) be g*closed. By 3.5 [10] every g* closed set is (r*g*)* closed. But in (X ,τ) , Every (r*g*)* closed set is closed. Which implies A is closed. Hence (X ,τ) is $T_{1/2}$*.

The Converse need not be true. Every $T_{1/2}$ * space need not be (r*g*)* $T_{1/2}$.

Example 2.9: Let X={a,b,c} and τ ={φ.X,{a}}

Closed sets are φ,X,{b,c}. (r*g*)*closed sets are φ, X,{b},{c},{b, c},{a,c},{a,b}

g*closed sets are φ,X,{b,c}. Here every g*closed set closed. Therefore (X ,τ) is a T1/2* space. But {a,c} is (r*g*)*closed but not closed. Therefore (X ,τ) is not a (r*g*)*T1/2 space.

Theorem 2.10 : If (X, τ) is both * $T_{1/2}$ and (r*g*)* $T_{1/2}$ then (X, τ) is a $T_{1/2}$ space.

Proof: Let A be a g closed set. Since (X, τ) is *$T_{1/2}$, A is a g* closed set. But by 3.5[10]

A is a (r*g*)* closed set. Since in a (r*g*)*$T_{1/2}$ Space Every (r*g*)* closed set is closed , hence A is closed. Hence (X, τ) is a $T_{1/2}$ space.

Now we show that (r*g*)*$T_{1/2}$ ness is independent of semi $T_{1/2}$ ness.

Result 2.11 : (r*g*)$T_{1/2}$ ness is independent of semi $T_{1/2}$ ness as it can be seen from the next examples.

Example 2.12: X={a,b,c}τ ={ φ,X,{a,b},{b}} closed sets are { φ,X,{c},{a,c}}

Semi open sets are φ ,X,{b},{a,b},{b,c} .Semi closed φ,X,{a,c},{{c},{a}

(r*g*)* closed sets φ ,X,{c},{b,c},{a,c} Sg closed sets { φ,X,{a},{c},{a,c}}

Here (X, τ) is not a (r*g*) $T_{1/2}$ space.But Every sg closed set is semi closed. Hence (X, τ) is semi $T_{1/2}$ space.

Example 2.13 : X={a,b,c} and τ={ φ, X,{a},{b},{a,b},{b,c}}

Closed sets are { φ,X, {a},{c},{a,c},{b,c}. Semi closed sets are φ, X,{a},{b},{b,c}

Sg closed sets are φ ,X,{a},{b},{c},{a,b},{b,c}. (r*g*)* closed sets are φ , X,{a},{c},{a,c},{b,c}. Here(X, τ) is (r*g*)$T_{1/2}$ but not semi$T_{1/2}$.

Hence (r*g*)*$T_{1/2}$ ness is independent of semi $T_{1/2}$ ness.

Definition 2.14 : A space is called (r*g*)* T_c space if every (r*g*)* closed set in it is g* closed.

Example: X = {a,b,c} τ = { φ,X,{a,b},{b}}

Here (r*g*)* C (X ,τ)=g* C(X ,τ)

(X ,τ) is (r*g*)* T_c space.

Theorem 2.15 : Every (r*g*)* $T_{1/2}$ space is (r*g*)*T_c space.

Proof: Let X be (r*g*)* $T_{1/2}$ space. Let A be a (r*g*)* closed set then it is closed.

But every closed set is g* closed. Hence X is (r*g*)*T_c space.

Theorem 2.16: If X is both $T_{1/2}$ and (r*g*)* Tc then (X ,τ) ia *$T_{1/2}$

Proof: Let (X ,τ) be both $T_{1/2}$ and (r*g*)* Tc. Let A $\subset$ (X ,τ) be g closed. Since X is $T_{1/2}$ A is closed , A is (r*g*)* closed. Since

X is also (r*g*)* Tc , A is g* closed => (X ,τ) is *$T_{1/2}$.

Theorem2.17 : If (X , τ) is (r*g*)* $T_{1/2}$ and Tc then the Space then (X , τ) is (r*g*)* T_c

Let A be (r*g*)* closed in X. Then A is closed. But every closed set is gs closed and since the space is T_c A is g* closed. Hence X is

(r*g*)* T_c

Theorem 2.18: If X is (r*g*)* T_c and T_b then X is (r*g*)*$T_{1/2}$.

Let A be (r*g*)* closed. Then A is g* closed But every g* closed set is gs closed

Since X is T_b, A is closed. Hence X is (r*g*)*$T_{1/2}$.

Definition 2.19: A space (X , τ) is called (r*g*)* $T_{1/2}^{\#}$ space if every (r*g*)* closed set in it is $g^{\#}$ closed **.**

Theorem 2.20: If X is $T_{1/2}^{\#}$ and (r*g*)* $T_{1/2}^{\#}$ Then X is (r*g*)* $T_{1/2}$.

Proof: Let A be (r*g*)* closed. Since X is (r*g*)* $T_{1/2}{}^{\#}$ .A is $g^{\#}$ closed. But every $g^{\#}$ closed set is $^{\#}g$ closed. Since X is $T_{1/2}{}^{\#}$ A is closed. Hence X is (r*g*)* $T_{1/2}$

3. (R*G*)* CLOSURE AND (R*G*)* INTERIOR.

Definition 3.1: Let X be a Topological space. Let A be a subset of X. (r*g*)* closure of A is defined as the intersection of all (r*g*)* closed sets containing A. That is

(r*g*)* cl (A) = $\cap$ { F / F is (r*g*)* closed A$\subseteq$ F}

Theorem:3.2

If A and B are subsets of X, then

1) (i) (r*g*)* cl (X) = X, (ii) (r*g*)*cl(φ) = φ
2) A $\subset$ (r*g*)* cl (A)
3) If B is any (r*g*)* closed set containing A then (r*g*)* cl (A) $\subset$ B
4) If A $\subset$B then (r*g*)* cl (A) $\subset$ (r*g*)* cl (B)
5) (r*g*)* cl ((r*g*)*) cl (A)) = (r*g*)*cl (A)

Proof:

1) (i) X is the only (r*g*)* closed set containing X => (r*g*)* cl (X) = X.
(ii) (r*g*)* cl (φ) = intersection of all (r*g*)*sets containing φ = $\varphi \cap$ (r*g*)*closed sets containing φ = φ
2) Follows from the definition of (r*g*)* closure of A.
3) Let B be any (r*g*)* closed set containing A. Since (r*g*)* cl (A) is the intersection of all (r*g*)* closed sets containing A
(r*g*)* cl (A) is contained in every (r*g*)* closed set containing A. Hence (r*g*)* cl (A) $\subset$ B
4) Let A $\subset$ B . Now (r*g*)* cl (B) = $\cap$ {F: F is(r*g*)* closed and B $\subset$ F}. If B $\subset$ F then by (3) (r*g*)* cl (B) $\subset$ F, Where F is (r*g*)* closed. But A $\subset$ B $\subset$F => (r*g*)*cl(A) $\subset$ F. Now (r*g*)* cl (A) $\subset$ $\cap$ {F : F is (r*g*)* closed B$\subset$ F} = (r*g*)cl(B).

Hence (r*g*)* cl (A) $\subset$ (r*g*)* cl (B).

5) Let A $\subset$X By definition, (r*g*)* cl (A) = $\cap$ {F : F is (r*g*)* closed and A $\subset$ F}
We know that (r*g*)* cl (A) $\subseteq$ F when A $\subset$ F
Since F is (r*g*)* closed containing (r*g*)* cl (A), (r*g*)* cl ((r*g*)*cl(A)) $\subset$ F
Hence (r*g*)* cl ((r*g*)* cl (A)) $\subset$ $\cap$ {F:F is (r*g*)*closed A $\subset$ F}= (r*g*)*cl (A).

Theorem 3.3: Let A $\subset$ X. If A is (r*g*)* closed then (r*g*)* cl (A) = A

Proof: Let A be (r*g*)* closed. Since A is (r*g*)* closed by (3), (r*g*)* cl (A) $\subset$ A. But always A $\subset$ (r*g*)* cl (A)

Hence (r*g*)* cl (A) = A

Theorem 3.4 : If A and B are subsets of X then

(r*g*)* cl(A$\cup$B) = (r*g*)*cl(A)$\cup$(r*g*)*cl (B)

Proof: A $\subset$ A $\cup$ B and B $\subset$ A $\cup$ B == > (rs*g*)* cl (A) $\subset$ (r*g*)* cl (A$\cup$ B) and

(r*g*)* cl (B) $\subset$ (r*g*)* cl (A $\cup$ B)

∴ $((r^*g^*)^*\ cl\ (A)\ \bigcup\ (r^*g^*)^*\ cl\ (B))\ \subset\ (r^*g^*)^*\ cl\ (A\ \bigcup\ B)$--------------------------(1)

Further $A \subset (r^*g^*)^*\ cl\ (A)$, $B \subset (r^*g^*)^*\ cl\ (B)$

$A \bigcup B \subset (r^*g^*)^*\ cl\ (A)\ \bigcup\ (r^*g^*)^*\ cl\ (B)$. The right hand side being a union of two (r*g*)*closed sets is (r*g*)*closed and contains $A \bigcup B$.

But $(r^*g^*)^*\ cl\ (A \bigcup B)$ is the smallest (r*g*)* closed set containing $A \bigcup B$

$(r^*g^*)^*\ cl\ (A \bigcup B)\ \subset\ (r^*g^*)^*\ cl\ (A)\ \bigcup\ (r^*g^*)^*\ cl\ (B)$------------------------------------(2)

From (1) & (2)

$(r^*g^*)^*\ cl\ (A \bigcup B)\ =\ (r^*g^*)^*\ cl\ (A)\ \bigcup\ (r^*g^*)^*\ cl\ (B)$

Theorem 3.5:

If A and B are subsets of X then $(r^*g^*)^*\ cl\ (A \cap B)\ \subset\ (r^*g^*)^*\ cl\ (A)\ \cap\ (r^*g^*)^*\ cl\ (B)$

Proof: $A \cap B \subset A$, $A \cap B \subset B$

$(r^*g^*)^*\ cl\ (A \cap B)\ \subset (r^*g^*)^*\ cl\ (A)$

$(r^*g^*)^*\ cl\ (A \cap B)\ \subset\ (r^*g^*)^*\ cl\ (B)$

⇨ $(r^*g^*)^*\ cl\ (A \cap B)\ \subset (r^*g^*)^*\ cl\ (A) \cap\ (r^*g^*)^*\ cl\ (B)$

Theorem 3.6: Let $x \in X$. $x \in (r^*g^*)^*\ cl\ (A)$ iff every (r*g*)* open set containing intersects A .

Proof: Let $x \in (r^*g^*)^*\ cl\ (A)$. Let V be a (r*g*)* open set containing x.

TPT $V \cap A \neq \varphi$

If $V \cap A = \varphi$ then $A \subset X - V$

Since V is (r*g*)* open X – V (r*g*)*closed. Since $x \in (r^*g^*)^*\ cl\ (A)$

$x \in X- V ==> x \notin V$ which is a contradiction.

Conversely suppose $V \cap A \neq \varphi$

TST $x \in (r^*g^*)^*\ cl\ (A)$. If not there exists a (r*g*)* closed set F containing A such that

$x \notin F$. Now X – F is (r*g*)* open and $(X - F) \cap A = \varphi$ Which is a contradiction. Therefore $x \in (r^*g^*)^*\ cl\ (A)$.

Theorem 3.7: If $A \subset X$ then $(r^*g^*)^*\ cl\ (A)\ \subset\ cl\ (A)$.

Proof: $cl\ (A) = \cap \{ F / F \text{ is closed } A \subset F\}$

But every closed set is (r*g*)* closed . ∴ F is (r*g*)* closed => $(r^*g^*)^*\ cl\ (A) \subset F$

∴ $(r^*g^*)^*\ cl\ (A)\ \subset\ \cap \{ F / F \text{ is closed } A \subset F\} = cl\ (A)$.

$(r^*g^*)^*\ cl\ (A)\ \subset\ cl\ (A)$.

Definition 3.8: Let X be a Topological space. Let A be a subset of X. (r*g*)* interior of A is defined as the union of all (r*g*)* open sets contained in A.

Theorem 3.9: Let A and B be subsets of X.

Then 1) (r*g*)* int (φ) = φ , (r*g*)* int(X) = X

2) If B is any (r*g*)* open set contained in A then B ⊂ (r*g*)* int (A)

3) If A ⊂ B then (r*g*)* int (A) ⊂ (r*g*)* int (B)

4) (r*g*)* int ((r*g*)* int(A)) = (r*g*)* int (A)

Theorem 3.11: If a subset A of X is (r*g*)* open then (r*g*)* int (A) = A

Theorem 3.12: If A and B are subsets of X then

(r*g*)* int (A) ⋃ (r*g*)* int (B) ⊂ (r*g*)* int (A⋃ B)

Theorem 3.11 If A & B are subsets of X

Then (r*g*)* int (A ∩ B) ⊂ (r*g*)* int (A) ∩ (r*g*)* int (B)

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