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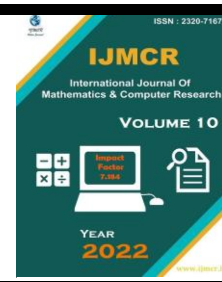

Computing of Existence and Uniqueness Solutions for Differential Equations in Metric Spaces by Using MATLAB

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ABSTRACT

A metric space is a set along with a measurement on the set, A metric actuates topological properties like open and shut sets, which lead to the investigation of more theoretical topological spaces. It also has many applications in functional analysis. The aim of this work is design and develop highly efficient algorithms that provide the existence of unique solutions to the differential equation in metric spaces using MATLAB. The quality algorithm was used and developed to solve the differential equation in metric spaces. For accurate results. The proposed model contributed to providing an integrated computer solution for all stages of the solution starting from the stage of solving differential equations in metric space and the stage of displaying and representing the results graphically in the MATLAB program

KEYWORDS: Metric Spaces, Existence , Uniqueness, Differential Equations , MATLAB.

1. INTRODUCTION

Functional analysis needs will be a champion among those imperative foundations about cutting edge associated science in the latest many years, from those standards What's more mathematical outcome for differential equations[1], A metric space is recently a situated X prepared for a work d from claiming two variables which measure that separation between points: $d(x, y)$ may be the separation of the middle of two focuses x Furthermore y to X. And that lead to making conditions for function d, and developed a general concept of distance that covers distances between numbers, vectors, sequences, functions, sets Also significantly additional. Inside this principle, clouded plan also demonstrates effects around joining Furthermore coherence once also for now. The reason for this single section to created that fundamental theory of metric spaces. In the fact sections, and help with some of the provisions of the theory[2].

In addition to its intrinsic mathematical interest, the theory of ordinary differential equations has extensive applications in the natural sciences, notably physics, as well as other fields. The existence and uniqueness of a solution to a first-order differential equation, given a set of initial conditions, is one of the most fundamental results of ODE. In his textbook on the subject [3, 4],The motivation for defining and investigating differential equations in metric spaces comes from many sources. Probably the most important are mutational equations, which appeared during the investigation of the evolution of sets (tubes) in vector spaces[5].

2. EXISTENCE AND UNIQUENESS SOLUTIONS FOR DIFFERENTIAL EQUATIONS IN METRIC SPACES

Theorem 2.1[3] (The Existence Theorem) Assume the right-hand side v of the differential equations$x = v(t, x)$is consistently differentiable in a neighborhood of the point $(t_0, x_0) \in R \times R^n$. Then, at that point, there is a neighborhood of the point $t_0$to such an extent that an answer of the differential equations is characterized around here with the underlying equations$\emptyset(t_0) = x$, where x is any point adequately near x_0. In addition, this arrangement relies constantly upon the underlying point x .

Theorem 2.2[3](The Uniqueness Theorem) Given the above conditions, there is just a single conceivable answer for some random introductory point, as in all potential arrangements are equivalent in the area viable.

Definition 2.3[6] A metric space is a set A together with a function $\rho: A \times A \rightarrow R, \rho(x, y)$ beingcalled the distance from x to y, such that

1. $\rho(x;\ y) > 0\ if\ x \neq y, and\ \rho(x;\ x) = 0;$
2. $\rho(x;\ y) = \rho(y;\ x)\ for\ all\ x;\ y\ 2\ A;$
3. $\rho(x;\ z) \leq \rho(x;\ y) + \rho(y;\ z)\ for\ all\ x;\ y;\ z \in A.$

Example 2.4[7] Let $M = [0,1)$ and let the metric on M be the Euclidian metric. Define$x \perp y\ if\ xy \in \{x, y\}$. M is not complete but it is O-complete. Let $x \perp y$ and $xy = x. If\ \{x_k\}$is an arbitrary Cauchy O-sequence in M, then there exists a subsequence $\{x_{k_n}\}$ of $\{x_k\}$for which $x_{k_n} = 0$ for all n. It follows that $\{x_{k_n}\}$ converges to a $x \in M$. On the otherhand, we know that every Cauchy sequence with a convergent subsequence is convergent.It follows that $\{x_k\}$ is convergent.

Let $f : M \rightarrow M$ be a mapping defined by $f(x) = \frac{x}{2}$ $if\ x \in Q \cap M\ and\ f(x) = 0\ if x \in Q^c \cap M.$

We have the following cases:

case 1) $x = 0$ and $y \in Q \cap M$. Then $f(x) = 0$ and $f(y) = \frac{y}{2}$.

case 2) $x = 0$ and $y \in Q \cap M$. Then $f(x) = f(y) = 0$.

This implies that $f(x)f(y) = f(x)$. Hence $f\ is\ \perp$ $-$preserving.Also, this implies that $|f(x) - f(y)| \leq \frac{1}{2}|x - y|$. Hence $f\ is\ \perp$$-$contraction. But f is nota contraction. To see this, for each $\lambda < 1, |f(\frac{1}{2}) - f(\frac{\sqrt{3}}{4})| > \lambda\, |\frac{1}{2} - \frac{\sqrt{3}}{4}|$

$If\ \{x_n\}$ is an arbitrary O-sequence in M such that $\{x_n\}$ converges to $x \in M$. Since f is$\perp-$contraction, for each $n \in N$ we have

$$|f(x_n) - f(x)|_{-}\frac{1}{2}|x_n - x|.$$

As n goes to infinity, f is $\perp$-continuous. But it can be easily seen that f is not continuous.We can now state the main theoretical result of [8]. Sufficient conditions under which any mapping on an orthogonal metric space will have a unique fixed point are given in the theorem.

Theorem 2.5[7] Let $(M, \rho, \perp)$ be an O-complete metric space (not necessarily complete metric space) and $0 < \lambda < 1$. Let $f : M \rightarrow M$ be $\perp-$continuous, $\perp-$contraction(with Lipschitz constant _) and $\perp-$preserving, then f has a unique fixed point x in M. Also, f is a Picard operator, that is, $lim\ f^n(x) = x^*$ for all $x \in M$.

3. EXISTENCE AND UNIQUENESS OF INTEGRODIFFERENTIAL EQUATIONS IN METRIC SPACES

Existence of unique common solution of the Urysohn integral equations of Volterra-Fredholm type:

$$x(t) = \int_0^t k_1\left(t, s, x(s)\right)ds + \int_0^t h_1\left(t, s, x(s)\right)ds + g_1(t)$$

(1)

$$x(t) = \int_0^t k_2\left(t, s, x(s)\right)ds + \int_0^t h_2\,(t, s, x(s))ds + g_2(t)$$

(2)

Definition3.1[9-11]. The Caputo derivative of order α, for a function $x : [0,1) \rightarrow R$ can be written as

$$\frac{d^\alpha x(t)}{d^\alpha t} = 1/\Gamma(1-\alpha)\int_0^t x'(s)/\,(t-s)^\alpha, 0 < \alpha \leq 1$$

(3)

$where\ x'(s) = dx\ (s\ )/ds$

If x is an abstract function with values in X, then the integrals and derivatives which appear in (2) are taken in Bochner's sense.

Definition3.1[12] A pair (S, T) of self-mappings X is said to be weakly compatible if they commute at their coincidence point $(i.e. ST\ x = TS\ x$ whenever $S\ x = T\ x). A$ point $y \in X$ is called point of coincidence of a family $Tj, j = 1, 2, ...,$of self-mappings on X if there exists a point $x \in X$ such that $y = T\ j\ x$ for all $j = 1, 2, ...$ We need thefollowing lemma for further discussion:

Lemma 3.2 [13] Let (X, d) be a complete cone metric space and P be an order cone. Let $S, T, F: X \rightarrow X$ be suchthat $S(X) \cup T(X) \subset F(X)$. Assume that the following conditions hold:

(i) $d(Sx, Ty) \leq \alpha d(Fx, Sx) + \beta d(Fy, Ty) + \gamma d(Fx, Fy), for\ all\ x, y \in X$, with $x \neq y$, where α, β, γ are non-negative real numbers with $\alpha + \beta + \gamma < 1$.

(ii) $d(Sx, Tx) < d(Fx, Sx) + d(Fx, Tx), for\ all\ x \in X, whenever\ Sx \neq Tx.$

$If\ F(X)\ or\ S(X) \cup T(X)$ is a complete subspace of X, then S, T and f have a unique point of coincidence.

Moreover, $if\ (S, F)$ and (T, F) are weakly compatible, then S, T and F have a unique common fixed point.

We list the following hypotheses for our convenience:

(H1)Assume that for all $t, s \in [a, T]$.

$$Fx(t) = \int_0^t k_1\left(t, s, x(s)\right)ds + \int_0^T h_1\left(t, s, x(s)\right)ds$$

$$Gx(t) = \int_0^t k_2\left(t, s, x(s)\right)ds + \int_0^T h_2\,(t, s, x(s))ds$$

$$(|Fx(t) - Gy(t) + g_1(t) - g_2(t)|, \alpha|Fx(t) - Gy(t) + g_1(t) - g_2(t)|),$$

(H2)$\leq \alpha(|Fx(t) + g_1(t) - x(t)|, p(|Fx(t) + g_1(t) - x(t)|) + \beta(|Gx(t) + g_2(t) - y(t)|, p(|Gx(t) + g_2(t) - y(t)|),$

where $\alpha + \beta + \gamma < 1$, for every $x, y \in Z$ with $\neq y$ and $t \in$

J.

H3) Whenever $Fx + g_1 \neq Gx + g_2$

$$\sup_{t\in J}(|Fx(t) - Gx(t) + g_1(t) - g_2(t)|, \alpha|Fx(t) - Gx(t) + g_1(t) - g_2(t)|),$$

$$\sup_{t\in J}\alpha(|Fx(t) + g_1(t) - x(t)|, p|Gx(t) + g_1(t) - x(t)|) + \beta(|Fx(t) + g_2(t) - x(t)|, p|Gx(t) + g_2(t) - x(t)|)$$

, for every $x \in Z$.

Theorem 4.3[9] Assume that hypotheses (H1)–(H3) hold. Then the integral equations (2)–(3) have a unique common solution x on [a, b].

Proof: Define $S, T : Z \to Z$ by $S(x) = Fx + g_1$ and $T(x) = Gx + g_2$. Using hypotheses, we have

$$(|S\,x(t) - Ty(t)|, \alpha\,|S\,x(t) - Ty(t)|) \leq \alpha\,(|S\,x(t) - x(t)|, p\,|S\,x(t) - x(t)|) + \beta(|Ty(t) - y(t)|, p\,|Ty(t) - y(t)|) + \gamma(|x(t) - y(t)|, p\,|x(t) - y(t)|)$$

For every $x, y \in Z$ and $x \neq y$. Hence

$$(||\,S - T||_\infty, ||\,S - T||_\infty \leq \alpha(||\,Sx - x||_\infty \leq \alpha(||\,Sx - x||_\infty, p||\,Sx - x||_\infty)\,\beta(||\,Tx - x||_\infty, p||\,Tx - x||_\infty)$$

For every $x \in Z$.By lemma 4.2 , if f is the identity map on Z, the Urysohn integral equations (2), (3) have a unique comm solution. This completes the proof of theorem 4.3.

4. PROPOSED SOLUTIONS BY MATLAB

We given some comprise solution presented a general existence and uniqueness result in pervious sections, by prove existence theorems. study the in existence and uniqueness solutions for differential equations in metric spaces and existence and uniqueness of integrodifferential equations in metric spaces and give some remarks and examples and the main result from each algorithm.

4.1 Algorithm 1.

```
%EXISTENCE AND UNIQUENESS SOLUTIONS FOR DIFFERENTIAL EQUATIONS IN METRIC SPACES
clear all
clc
symsQMf(x)f(y)ff(x,y)tlandaf3xyWxnQc
M= linspace(0,1,10)
Q= linspace(0,2,10)
f(x) =  x/2
S=intersect(Q,M)
U=union(Q,M)
% Qc= minus(U,Q)
if (x==0) & ((y>=0.5) & (y<=1))
   f(x)=0
   f(y) =  y/2
else
  (x==0) & ((y>=0.5) & (y<=1))
     f(x)=0
     f(y) =0
end
f(x )= f(x)*f(y)
if abs(f(x)-f(y)) <= (0.5*(abs(x - y)))
xn=length(S)
else
    result = "non exest and uniqu solution"
end
 x=2
for k=1:xn

  R(k)= abs(f(k) - f(x))-0.5*(abs(k - x))
end
   R
hold on
plot(M,Q,'b')
plot(S,R,'r')
hold off
```

Represent the solution graphically:

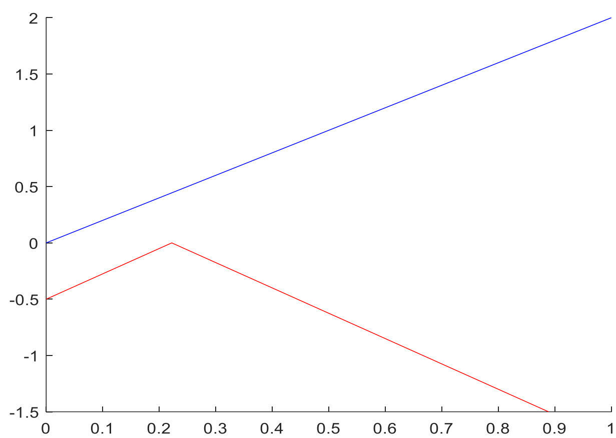

Figure 1: Existence and Uniqueness of Solutions for Differential Equation in Metric Space

4.2 Algorithm 2.

```
% Existence and Uniqueness of Integrodifferential Equations in Metric Spaces
clear all
clc
symsdawdtd2wtP1P2dtsdsIx0LXkx(s)w(t)w(t,u)dtx0(u)x(t)
M= linspace(0,1,10);
I= linspace(0,pi,10);
((t>=0) & (t<=1));
((u>=0) & (u<=pi));
w(t,u)= x0(u);
L=2;
```

```
X=(L^2)*M
xn=length(X)
 S= 0.5*(1+t)
for k=1:xn
f(k)=int(S,[0 k])
end
hold on
plot(f,X,'b')
plot(M,I,'r')
hold off
```

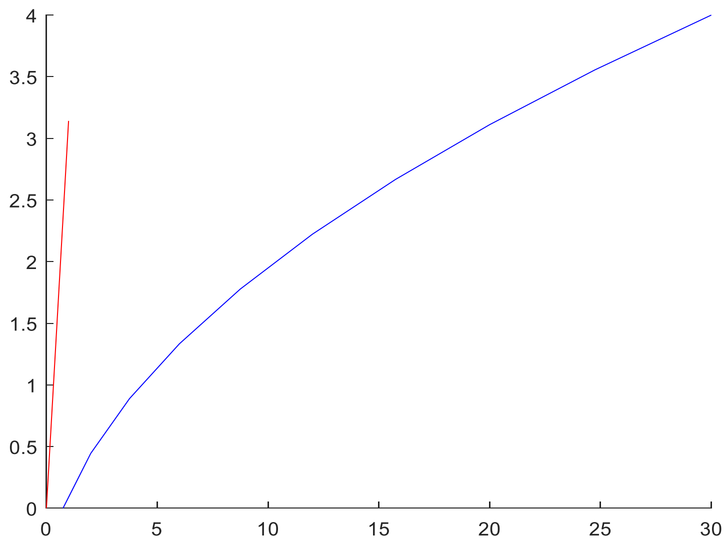

Figure 2: Existence and Uniqueness of Integro differential Equations in Metric Spaces.

5. RESULTS AND DISCUSSION

5.1 Results:

In the wake of executing every one of the means portrayed in the past segments, the model is assessed, and the end-product are examined. Results got from the proposed model naturally during execution, and testing interaction to get exact outcomes by utilized MATLAB by applying This is to rehash the trial tow times utilizing the qualities referenced in the past area for various examples of differential equation in metric spaces, for example, existence and uniqueness of solutions for differential equation in metric space and existence and uniqueness of integrodifferential equations in metric spaces, as every examination brought about a bend of designs. The mean was determined for exactness to get brilliant precision because of redundancy, preparing, and confirmation in every model.

5.2 Discussion

The exhibition of the computer solution arrangement model for existence and uniqueness solutions of differential equations in metric spaces was assessed in this review through the illustrations introduced in the past areas to confirm its legitimacy. This review utilized another strategy to address of existence and uniqueness solutions of differential equations in metric spaces by making amazing algorithms for results and address consequently in illustrations dissimilar to different examinations that zeroed in on a specific application, such as fixed points for multivalued weighted mean contractions in asymmetric generalized metric space, etc. This study achieved incredible progress in settling many kinds of differential equations in metric spaces. Yet, it neglected to coordinate these arrangements into the application in the issues of our regular routines, as it was fulfilled distinctly with the arrangement and addressed it graphically. This issue was settled by expanding the arrangement of more than one computer model so every utensil discovers the arrangement that fits the issue in our day to day routine (like the issues of the modern, horticultural, creature, and monetary creation) and hence the model accomplished an exactness of over 95% and the presence of one of a kind answers for the differential equations in metric spaces.

6. CONCLUSION

We presented an existence and uniqueness solutions of differential equation in metric spaces to solve and presented several differential equation examples by using Matlab programming language. models depend on the assortment of two sorts of differential equations from various sources with applied in all stages: arrangement stage results stage, and introduced graphically. The number of models and lemma for every class of differential equations in the proposed model was applied to force full algorithms to arrive at high precision of results. When entering a solve existence and uniqueness solutions of differential equation in metric spaces, it is tried by a contrasting with the past study with acquiring it consequently the exactness of its test is contrasted with the review's outcomes, the model outcomes showed fantastic effectiveness with 95% precision in existence and uniqueness solutions of differential equations in metric spaces.

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References

1. C. Clason, *Introduction to Functional Analysis*. Springer Nature, 2020.
2. T. L. Lindstrøm, *Spaces: An Introduction to Real Analysis*. American Mathematical Soc., 2017.
3. J. BUCHANAN, "THE EXISTENCE AND UNIQUENESS OF SOLUTIONS TO DIFFERENTIAL EQUATIONS," 2010.

4. V. I. Arnold, "Ordinary Differential Equations. Translated and Edited by Richard A. Silverman," ed: the MIT press, 1998.
5. J. Tabor, "Differential equations in metric spaces," *Mathematica Bohemica,* vol. 127, no. 2, pp. 353-360, 2002.
6. E. MacIndoe, "Analysis of deterministic and stochastic HIV models," 2019.
7. M. E. Gordji and H. Habibi, "Existence and uniqueness of solutions to a first-order differential equation via fixed point theorem in orthogonal metric space," *Facta Univ. Ser. Math. Inform,* vol. 34, pp. 123-135, 2019.
8. M. E. Gordji, M. Ramezani, M. De La Sen, and Y. J. Cho, "On orthogonal sets and Banach fixed point theorem," *Fixed Point Theory,* vol. 18, no. 2, pp. 569-578, 2017.
9. A. H. Ibrahim, "Existence and Uniqueness of Continuous Solutions of Fractional Mixed Type Integrodifferential Equations in Cone-Metric Spaces," *Mathematical Theory and Modeling,* vol. 5, no. 2, pp. 1-5, 2015.
10. I. Podlubny, *Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications*. Elsevier, 1998.
11. K. S. Miller and B. Ross, *An introduction to the fractional calculus and fractional differential equations*. Wiley, 1993.
12. G. Jungck and B. E. Rhoades, "Fixed points for set valued functions without continuity," *Indian Journal of pure and applied mathematics,* vol. 29, pp. 227-238, 1998.
13. M. Arshad, A. Azam, and P. Vetro, "Some common fixed point results in cone metric spaces," *Fixed point theory and applications,* vol. 2009, pp. 1-11, 2009.