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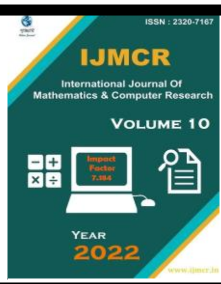

Counting the Orbits of Γ_1 − Non-Deranged Permutation Group

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ARTICLE INFO	ABSTRACT
Published Online: 02 January 2022 Corresponding Author: **Suleiman, I.**	In this work we employ a combinatorial process to establish the intransitivity of a non-deranged permutation group $G_p^{\Gamma_1}$. This provide for further study of some properties of the group thus, revealing that the average fixed points of every ω_i equals the number of orbits of $\omega_i \in G_p^{\Gamma_1}$. The study also deduce that the sum coefficients of cycle index polynomial of $G_p^{\Gamma_1}$ is unity.

KEYWORDS: Aunu permutation (ω_i), Permutation group, Burnside's Lemma, Cycle index polynomial, Orbits, Stabilizer and Group action.

1. INTRODUCTION

Arrangement of items or symbols under a prescribed rule is called pattern. Aunu pattern studied by [1] is a permutation pattern that is unique in its method of generation which eventually form a subgroup of the symmetry group S_n. The permutation group being a non-deranged permutation was represented in two line form by [2]. The evolution of the pattern into a group can be seen in [1, 3, 4] and [5]. Other interesting work on the structure ω_i such as pattern avoidance, permutation statistics, designs, codding and topology are observed in [6,7,8,9,10,11] and [12,13].

[13] uses this pattern and provide a structure $|\omega_i + \omega_j| mod p$ on which [14] generate a non-deranged permutation group for some prime $p \geq 5$.

A cross-section literature of the pattern reveal study of the intransitivity nature of the pattern via an algebraic approach as seen in [8]. As such a call for combinatorial test.

Herein, we employ some combinatorial approach to study the intransitivity nature of the pattern. This we achieved through the use of Burnside's Lemma and cycle index polynomial.

2. TERMINOLOGY

Definition 1 *(Permutation)***.** A *permutation* π on a set $X = \{1,2, \dots, n\}$ is the bijection $\pi(i): X \to X$. it is represented in two line notion as:

$$\pi(i) = \begin{pmatrix} 1 & 2 & \cdots\cdots & n \\ \pi(1) & \pi(2) & \cdots\cdots & \pi(n) \end{pmatrix}$$

and in one line notation as:

$$\pi(i) = (\pi(1), \pi(2), \dots, \pi(n))$$

for $i = 1,2, \dots, n$.

Example. Let $X = (123)$. Then, $\pi = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$.

Permutation usually occur in two forms. Permutations with one or more points fixed and permutations with no point fixed. Permutation that fixes no point is called deranged permutation while non-derangement permutations are permutations with a fixed point.

Example. $\pi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ (Deranged permutation).

$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$ (Non-deranged permutation).

Definition 2 *(Group)***.** Let G be a nonempty set and $*$ be a binary operation on G such that

$* : G \times G \longrightarrow G$.

Then $(G, *)$ is a group if it satisfies the following axioms:

1. G is associative. That is, for all a,b,c $\in G$, $a * (b * c) = (a * b) * c$.
2. G has an identity id. That is, for all a ∈ G, id∗a = a∗id = a.
3. G has an inverse. That is, for any a ∈ G, there exist $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} *$a=id.

The permutation group $G_p^{\Gamma_1} = \{\{\omega_i\} \, p \leq i \leq p-1\}$ where

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & \cdots\cdots & p \\ 1 & (i+1)_{mp} & (1+2i)_{mp} \cdots\cdots & & (1+(p-1)i)_{mp} \end{pmatrix}$$

is the generating function of the Aunu pattern over an arbitrary set Ω for some $p \geq 5$ is called gamma-one non-deranged permutation group.

Definition 3 ***(Cycle index of permutation group).*** Is the average of the cycle index monomial of all permutations ω_i in $G_p^{\Gamma_1}$. $Z(G_p^{\Gamma_1}) = \frac{1}{|G_p^{\Gamma_1}|}\sum_{\omega \in G_p^{\Gamma_1}} \prod_{k-1}^{n} a_k^{jk(\omega)}$.

Definition 4 *(Group action).* Let G be a group. And let X be a set, we say that G acts on the set X if there exist a map:

$\pi : G \times X \longrightarrow X$

$\pi : (g, x) \longrightarrow gx$ so that;

i. $ex = x \; \forall \; x \in X$ and e the identity in G.

ii. $g(hx) = (gh)x \; \forall \; g, h \in G$ and $\forall \; x \in X$.

Definition 5 *(Subgroup).* Let G be a group and H a non-empty subset of G. Then H is a subgroup of G iff for each a ∈ H and b ∈ H then ab^{-1} ∈ H.

Definition 6 *(Orbit).* Let G be a permutation group on a set X. For each x∈X, the orbit of x in G can be defined as orbitG(x) = {g(x): g ∈ G}.

Definition 7 *(Stablizer).* Let G be a permutation group on a set X, for each x∈X. Let StabG(x) = {g∈G: g(x) = x}. We call stabG(x) the stablizer of x in G.

Definition 8 *(Transitive).* The group action is transitive if it has only one orbit.

Definition 9. A permutation group G on a set X is primitive if G act transitivity on X and G preserves no non-trivial partition of X.

3. METHODOLOGY

In other to achieve the aim of this research work the following were employed:

1. The use of Burnside's Lemma: To establish the transitivity of the group $G_p^{\Gamma_1}$ by computing the orbits of the group.
2. Cycle index polynomial: The cycle index polynomial of the group $G_p^{\Gamma_1}$ was constructed from which the cycle index polynomial properties were deduced.

4. RESULTS

Remark. By definition the stabilizer of $G_p^{\Gamma_1}$ is the set $\text{Stab}(X) = \{\omega_i(x) = x : \omega_i \in G_p^{\Gamma_1}\}$

Example. Let $G_p^{\Gamma_1} = \{\omega_1, \omega_2, \omega_3, \ldots, \omega_{p-1}\}$ acts on a set $\Gamma_1 = \{1,2,3,4,5\}$.

For p=5, the stabilizer of $G_5^{\Gamma_1}$ can be computed as follows:

$$\text{Stab}(1) = \{\omega_i \in G_p^{\Gamma_1}: \omega_i(1) = 1\} = \{e, (2354), (2453), (25)(34)\}$$

$$\text{Stab}(2) = \{\omega_i \in G_p^{\Gamma_1}: \omega_i(2) = 2\} = \{e\}$$

$$\text{Stab}(3) = \{\omega_i \in G_p^{\Gamma_1}: \omega_i(3) = 3\} = \{e\}$$

$$\text{Stab}(4) = \{\omega_i \in G_p^{\Gamma_1}: \omega_i(4) = 4\} = \{e\}$$

$$\text{Stab}(5) = \{\omega_i \in G_p^{\Gamma_1}: \omega_i(5) = 5\} = \{e\}$$

The total fix point of $\omega_i \in G_5^{\Gamma_1}$ is 8.

For p=7, the stabilizer of $G_7^{\Gamma_1}$ is as follows:

$$\text{Stab}(1) = \{e, (235)(467), (243756), (253)(467), (265734), (27)(36)(45)\}$$

$$\text{Stab}(2) = \text{Stab}(3) = \text{Stab}(4) = \text{Stab}(5) = \text{Stab}(6) = \text{Stab}(7) = \{e\}$$

The total fix point of $\omega_i \in G_7^{\Gamma_1}$ is 12.

For p = 11, the stabilizer of $G_{11}^{\Gamma_1}$ is as follows:

Stab(1) = {e,(235961110847),(241065)(378119),(256104)(291187),(264510)(311798), (274810116953), (286341157109), (291075114368), (210546)(389711), (211)(310)(49)(58)(67)}.

$\text{Stab}(2) = \text{Stab}(3) = \text{Stab}(4) = \ldots = \text{Stab}(11) = \{e\}$.

The total fixed point of $G_{11}^{\Gamma_1}$ is 20.

Corollary 1. The total fix points of $\omega_i \in G_p^{\Gamma_1} = 2(p-1)$.

Corollary 2. If $G_p^{\Gamma_1}$ is a Γ_1 −non deranged permutation group then $F(\omega) \neq 0$ for all $\omega \in G_p^{\Gamma_1}$.

Proof. By the non-derangement property of $G_p^{\Gamma_1}$ there is a fix point in any arbitrary $\omega \in G_p^{\Gamma_1}$.

Proposition. For any $x \in X$ the set Stab(x) is a subgroup of $G_p^{\Gamma_1}$.

Proof. To show Stab(x) is a subgroup of $G_p^{\Gamma_1}$ we need to show the following:

1. The existence of identity.
2. Existence of inverse, i.e if g(x) = x, $g^{-1}(x) = x$.
3. Closure exists, if g(x) =x and h(x) = x, the g (h(x)) = x.

Thus;

1. From our definition of group action. The identity element of $G_p^{\Gamma_1}$ belongs to $\text{Stab}_{G_p^{\Gamma_1}}(x)$.
2. If g ∈ $\text{Stab}_{G_p^{\Gamma_1}}(x)$ then g(x) = x and so $g^{-1}(g(x)) = g^{-1}(x)$. Also $g^{-1}(g(x)) = (g^{-1}g)x = e.x = x$. Hence $g^{-1}(x) = x$ which implies $g^{-1} \in \text{Stab}_{G_p^{\Gamma_1}}(x)$.
3. Lastly, Assume that g,h ∈ $\text{Stab}_{G_p^{\Gamma_1}}(x)$ then gh(x) = g(h(x)) = g(x) = x.

So gh ∈ $\text{Stab}_{G_p^{\Gamma_1}}(x)$ is closed under the group operation of $G_p^{\Gamma_1}$

Lemma 1. [2] The order of Γ_1-non deranged permutation group $G_p^{\Gamma_1}$ is p-1.

Theorem. [2] Let $G_p^{\Gamma_1}$ be a Γ_1-non deranged permutation group (p≥5 and p is prime) and $\omega_1 = e$, then the character χ of the permutation matrix of ω_i in GL(p, ℝ) is

$$\chi(\omega_i) = \begin{cases} 1 \; if \; \omega_i \neq \omega_1 \\ p \; if \; \omega_i = \omega_1 \end{cases}$$

where i=1,2,...,p-1.

Lemma 2. Suppose $G_p^{\Gamma_1}$ is a Γ_1-non deranged permutation group then for every $\omega_i \in G_p^{\Gamma_1}$.

$$F(\omega_i) = \begin{cases} 1 \; if \; \omega_i \neq \omega_1 \\ p \; if \; \omega_i = \omega_1 \end{cases}$$

Proof. The identity element fixes all points and by the cardinality of the set; $|\Gamma| = p$. So $\mathrm{F(e)} = p$. Similarly for every $\omega \neq e$ in $G_p^{\Gamma_1}$ is a non derangement at 1 only, hence $F(\omega_i) = 1$ if $i \neq 1$ this completes the proof.

Lemma 3. Let $G_p^{\Gamma_1}$ be a Γ_1-non permutation group acting on a set Γ_1. Then the average number of fix points of $\omega \in G_p^{\Gamma_1}$ is $\frac{1}{|G_p^{\Gamma_1}|}\sum_{\omega \in G_p^{\Gamma_1}} F(\omega_i) = 2$.

Proof. From Lemma 1 $|G_p^{\Gamma_1}| = p - 1$. Also by Lemma 2 the $\mathrm{F(e)} = p$ implies there are p-2 $\omega_i \in G_p^{\Gamma_1}$ with $F(\omega_i) = 1$. Then;

$$\frac{1}{|G_p^{\Gamma_1}|}\sum_{\omega \in G_p^{\Gamma_1}} F(\omega_i) = \frac{1}{\mathrm{p}-1}(\mathrm{p} + (\mathrm{p}-2)) = \frac{\mathrm{p}+\mathrm{p}-2}{\mathrm{p}-1}$$
$$= \frac{2(\mathrm{p}-1)}{\mathrm{p}-1} = 2.$$

Remark. This prove that the average number of fixed points of $\omega_i \in G_p^{\Gamma_1}$ is the number of partition of Γ_1 which equals the number orbits of $G_p^{\Gamma_1}$.

Theorem (*Orbit stabilizer theorem)*. Let G be a finite group of permutation of a set S, then for any x ∈ X

$$|\mathrm{G}| = |\mathrm{Orb_G(x)}||\mathrm{Stab_G(x)}|$$

and is called orbit stabilizer theorem.

Example. Let $G_5^{\Gamma_1}$ be a Γ_1-non deranged permutation group acting on X = {1,2,3,4,5}.

$G_5^{\Gamma_1} = \{\omega_1, \omega_2, \omega_3,, \omega_4\}$, $G_5^{\Gamma_1} = \{\mathrm{e}, (1)(2354), (1)(2453), (1)(25)(34)\}$

$$\mathrm{Stab}(1) = \{\omega_i \in G_p^{\Gamma_1}\colon \omega_i(1) = 1\}$$
$$= \{\mathrm{e}, (2354), (2453), (25)(34)\}$$
$$\mathrm{Stab}(2) = \{e\}$$
$$\mathrm{Stab}(3) = \{e\}$$
$$\mathrm{Stab}(4) = \{e\}$$
$$\mathrm{Stab}(5) = \{e\}$$

$$\mathrm{Orb}(1) = \{1\}$$
$$\mathrm{Orb}(2) = \{2345\}$$
$$\mathrm{Orb}(3) = \{2345\}$$
$$\mathrm{Orb}(4) = \{2345\}$$
$$\mathrm{Orb}(5) = \{2345\}$$

Hence, $|\mathrm{G}| = \left|\mathrm{Orb}_{G_p^{\Gamma_1}}(\mathrm{x})\right|\left|\mathrm{Stab}_{G_p^{\Gamma_1}}(\mathrm{x})\right| = 2 \times 2 = 4.$

This shows that our result satisfies the Orbit-Stabilizer Theorem.

5. CYCLE INDEX POLYNOMIAL OF Γ_1-NON DERANGED PERMUTATION GROUP

Definition 10. The cycle index of a permutation group G is the average of the cycle index monomials of all the permutation g ∈ $G_p^{\Gamma_1}$.

Definition 11. Let G be a permutation group acting on a set X = {1, 2, 3,…,}. Each permutation g ∈ G can be decomposed into a product of disjoint cycles a_k where $1 \leq k \leq n$, for each k from 1 to n, we let $J_{\mathrm{k(w)}}$ to be the number of cycle of length k. Then the cycle index of G denoted by Z(G) is the polynomial in variable $a_1, a_2.., a_n$ defined by

$$Z(G_p^{\Gamma_1}) = \frac{1}{|G|}\sum_{g \in G}\prod_{k=1}^{n} a_k^{J_{\mathrm{k(w)}}} \quad (1)$$

Let $G_p^{\Gamma_1}$ be a Γ_1-non deranged permutation group acting on $\Gamma = \{1, 2, 3, .., \mathrm{p}\}$. We compute the cycle index polynomial of $G_5^{\Gamma_1}$ for $p \geq 5$, where p is prime for $G_5^{\Gamma_1}$

$G_5^{\Gamma_1} = \{\omega_1, \omega_2, \omega_3,, \omega_4\}$

$$\omega_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} = (1)(2)(3)(4)(5)$$

for $k = 1$, $J_1(\omega_1) = 5$, $\omega_1 \coloneqq a_1^5$.

$$\omega_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 2 & 4 \end{pmatrix} = (1)(2354)$$

for $k = 1$, $J_1(\omega_2) = 1$.

and

for $k = 4$, $J_4(\omega_2) = 1$, $\omega_2 \coloneqq a_1 a_4$.

$$\omega_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 2 & 5 & 3 \end{pmatrix} = (1)(2453)$$

for $k = 1$, $J_1(\omega_3) = 1$.

and

for $k = 4$, $J_4(\omega_3) = 1$, $\omega_3 = a_1 a_4$.

$$\omega_4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 4 & 3 & 2 \end{pmatrix} = (1)(25)(34)$$

for $k = 1$, $J_1(\omega_4) = 1$.

and

for $k = 2$, $J_2(\omega_4) = 2$, $\omega_4 = a_1 a_2^2$.

$G_5^{\Gamma_1}$

$= \{(1)(2)(3)(4)(5), (1)(2354), (1)(2453), (1)(25)(34)\}$

$= \{a_1^5, a_1 a_4, a_1 a_4, a_1 a_2^2\}$

This implies that;

$Z(G_5^{\Gamma_1}) = \frac{1}{|G_5^{\Gamma_1}|}\sum_{\omega \in G_5^{\Gamma_1}}\prod_{k=1}^{5} a_k^{J_{\mathrm{k}(\omega)}} = \frac{1}{4}(a_1^5 + a_1 a_2^2 + 2a_1 a_4)$.

where

a_1^5 implies five cycle of length one occurring together.

$a_1 a_2^2$ implies two cycle of length 2 and one cycle of length 1 occurring together.

$2a_1 a_4$ implies two cycles of length 1 and length 4 occurring together in two different pair.

For $G_7^{\Gamma_1}$

$$Z(G_7^{\Gamma_1}) = \frac{1}{6}(a_1^7 + 2a_1 a_3^2 + 2a_1 a_6 + a_1 a_2^3)$$

For $G_{11}^{\Gamma_1}$

$$Z(G_{11}^{\Gamma_1}) = \frac{1}{10}(a_1^{11} + 4a_1a_5^2 + 4a_1a_{10} + a_1a_2^5)$$

Corollary 3. The cycle index polynomial of $\omega_4 \in G_p^{\Gamma_1}$ satisfies $\frac{1}{p-1}\sum_{i=1}^{k} k.J_k = p$.

Example. Given cycle index polynomial of $Z(G_5^{\Gamma_1}) = \frac{1}{4}(a_1^5 + a_1a_2^2 + 2a_1a_4)$.

to show that

$$\frac{1}{p-1}\sum_{k=1}^{p} k.J_k = p$$

where k represent the length of a cycle and J_k as the number of cycle of length k we consider

$$Z(G_5^{\Gamma_1}) = \frac{1}{4}(a_1^5 + a_1a_2^2 + 2a_1a_4)$$
$$= \frac{1}{4}\big(1.5 + (1.1 + 2.2) + 2(1.2 + 1.4)\big)$$
$$= 5$$

Example. Given cycle index polynomial of

$$Z(G_7^{\Gamma_1}) = \frac{1}{6}(a_1^7 + 2a_1a_3^2 + 2a_1a_6 + a_1a_2^3)$$

to show that

$$\frac{1}{p-1}\sum_{k=1}^{p} k.J_k = p$$

we consider

$$Z(G_7^{\Gamma_1}) = \frac{1}{6}(a_1^7 + 2a_1a_3^2 + 2a_1a_6 + a_1a_2^3)$$
$$= \frac{1}{6}(1.7 + 2(1.1 + 2.3) + 2(1.1 + 6.1) + 1.1 + 2.3) = 7$$

Hence the above examples satisfies the Corollary $\frac{1}{p-1}\sum_{k=1}^{p} k.J_k = p$ for $p \geq 5$ and p is prime.

Corollary 4. The sum of coefficient of terms of cycle index of Γ_1-non deranged permutation group is unity.

Example. Given $G_5^{\Gamma_1} = \frac{1}{4}(a_1^5 + a_1a_2^2 + 2a_1a_4)$ to show that the sum of coefficients of the terms of $G_5^{\Gamma_1}$ is unity, we consider

$$G_5^{\Gamma_1} = \frac{1}{4}(a_1^5 + a_1a_2^2 + 2a_1a_4) = \frac{1}{4}(1 + 1 + 2) = 1$$

Example. Given $Z(G_7^{\Gamma_1}) = \frac{1}{6}(a_1^7 + 2a_1a_3^2 + 2a_1a_6 + a_1a_2^3)$ to show that $Z(G_7^{\Gamma_1})$ satisfies Corollary 4 we consider

$$Z(G_7^{\Gamma_1}) = \frac{1}{6}(a_1^7 + 2a_1a_3^2 + 2a_1a_6 + a_1a_2^3)$$
$$= \frac{1}{6}(1 + 2 + 2 + 1) = 1$$

Hence the above examples indicate that the Corollary 4 holds.

Corollary 5. Let e be the identity element of $G_7^{\Gamma_1}$ the cycle structure of e as an element of $G_7^{\Gamma_1}$ is always a_1^p.

Example. Given $e \in G_5^{\Gamma_1}$ as $e = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$ to show that the cycle structure of e is a_1^5 we consider

$$e = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} = (1)(2)(3)(4)(5) = a_1^1a_1^1a_1^1a_1^1a_1^1 = a_1^5.$$

Example. Given $e \in G_7^{\Gamma_1}$ as $e = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{pmatrix}$ to show that the cycle structure of e is a_1^7 we consider

$$e = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{pmatrix} = (1)(2)(3)(4)(5)(6)(7) = a_1^7$$

CONCLUSION

The present study on Γ_1-non deranged permutation group is sufficient to conclude that $G_p^{\Gamma_1}$ is intransitive. Also, we established that the orbits of $G_p^{\Gamma_1}$ partition the Γ into exactly two sets by using Burnside's lemma (orbits counting theorem on combinatorial structure).

RECOMMENDATION

In this research the combinatorial study of $G_p^{\Gamma_1}$ is not exhaustive we therefore recommend further study on combinatorial properties of $G_p^{\Gamma_1}$ such as generating the general function of cycle index polynomial of $G_p^{\Gamma_1}$.

CONFLICT OF INTEREST

The authors of this article declare absence of conflict of interest in this work and that the work has never been published by any journal.

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