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The Study of Equilibrium Strategies of the Observable Markovian Queue with Redundant Server for Balking and Delayed Repair

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ABSTRACT
Queueing system is one of the real applications which is used to establishing the relationship between customers and server for providing service facility. In Wang and zhang [9], The equilibrium threshold balking strategies are analyzed for fully observable and partially observable m/m/1 queue with server breakdown and delayed repair. By the observation and state of server, when customer arrive in the system, he/she decide whether to join or balk the queue. In this paper we consider equilibrium strategy markovian queue with presence of redundant server for condition of balking and delayed repair. By the redundant server, system can improve service quality. Customers may not lose their time for service. In this paper, we calculate the stationary distribution of queue size of queueing system. With the help of markove chain approach and system cast analysis. We calculated equilibrium threshold strategy and equilibrium social benefit for fully observable and partially observable with redundant server for server breakdown and delayed repair.

KEYWORDS: Balking, Equilibrium threshold, Equilibrium social benefit, markovian queue, observable queue

INTRODUCTION

The study of customer's behavior in queueing theory is a very active research area for new research in many different direction. There are unlimited possibilities between customers and service providers for determine stretegic behavior. Economou and Kanta [3] studied the equilibrium balking strategies in the observable M/M/1 queue with an unreliable server and repairs. They considered the system with two different levels of information: (1) fully observable case. The customers can observe both the queue length and the state of the server upon arrival; (2) almost observable case. The customers can observe only the queue length and the state of the server is unobservable. Based on the available information, the customers decide whether to balk the system. In system, breakdown of server is require repair process, are very common in practice, especially in manufacturing industries, communication networks, any among others. However, in many real-life situations due to non-availability of the repair facility it may not be easy to start the repair process immediately and therefore the system may delay the repair time

Wang and zhang [9] studied the equilibrium analysis of the observable queues with balking and delayed repair. In this paper, we investigate such a queueing system based on Wang ang zhang [9] but there are some difference in the modal here we use redundant server in markovian queue. Under this assumption, we study the equilibrium threshold balking strategies for both of the fully and almost observable markovian queue with redundant server for condition of balking and delayed repair. In our model we minimize the waiting time of customer with help of the redundant server in M/M/1. If main server goes in breakdown state then working process is not affected because of the redundant server. Customers moves the redundant server and customer is served without any delay. In the case, when each server fails, service facility stops and system enters in repair state. Since repair process also takes some time, it also constitutes some delay. In this situation customers face delay in service. But with the help of redundant server overall reliability of the system increases so customers do not balk from the system. Redundant server is an extra server used in our model so that the system provides a reliable working facility to the customer.

Brief Review

In The literature Naor [18]is the first who study the effect of information on the strategy of customer behavior in queueing systems. Naor [18] studied an M/M/1 queueing model with a linear reword-cost structure in the fully observable case. It is assumed that an arriving customer observes the number of customers and then decided whether to join or balk the queue. Edelson and Hildebrand [15] investigated Naor's model by assuming that there is no information on the queue length for an arriving customer.

Burnetas and Economou [1] first presented several Markovian queues with setup times and four precision levels of system information and analyzed the customers' equilibrium strategies. Economou and Kanta [3] studied the equilibrium balking strategies in the observable *M* / *M* /1 queueing system with an unreliable server and repairs. Some works have incorporated server vacation policies, such as Guo and Hassin [17] and Sun et al. [27]. Moreover, there are many papers that deal with the economic analysis of the balking behavior of customers in variants of the *M* / *M* /1 queue in [2], Economou and Manou discussed a Markovian clearing queueing system that operates in an alternating environment.

Guo and Zipkin [16] considered a queue with balking under three levels of delayed information: no information, system occupancy, and exact waiting time. They showed how to compute the key performance measures in the three systems and compare them. Hassin [21] analyzed the effect of information and uncertainty on profits in an unobservable single server queueing system. He obtained explicit answers to the effect of information about the system's parameters on the server profits and system's overall welfare. Hassin and Haviv [22] dealt with the economic analysis of a queueing model with priorities in which two priority levels can be purchased and obtain all of the Nash equilibrium strategies (pure or mixed) of the threshold type. The fundamental results about various observable models can be found in the comprehensive monographs of Hassin and Haviv [23] and Stidham [25] with extensive bibliographical references.

the aim of this paper is to study the equilibrium behavior of customers in the context of both fully observable and partially observable markovian queue with redundant server an unreliable server and delayed repairs.

The paper is organized as follows. Descriptions of the model and price structure are given in Section 2. In Section 3, the equilibrium strategies for fully observable and partially observable queues are identified and the equilibrium social benefit for partially observable case are derived. Finally, in Section 5, some conclusions are given.

2. MODAL DESCRIPTION

We investigate the same model discussed in Wang and Zhang [9] but there is some difference in our model here we use redundant sever. We consider the fully observable and partialy observable M / M /1 queueing system with an infinite waiting room where customers arrive according to a Poisson process with intensity λ and customers are served at a rate of μ. The server has an exponential lifetime with failure rate 2ξ when he is working. Once the server fails it will not experience an exponential delayed time to activate the repair process. Because all customers load of system is transfer on the redundant server. Working process is continuing. In this situation fail server is going to repair process. But in this interval of time redundant server fails. Then delayed time is exponentially distributed with parameter δ. During the delay time, the server stops providing service to arriving customers and waits for repair facility to begin the repair process. The repair time is assumed to be exponentially distributed with parameter θ. In other words, the repair process may not be started immediately when the server fails due to non-availability of the repair facility. The repair delayed time is introduced as the time interval between the epoch of server breakdown and the beginning of repair process. We realize that the repair time has two stages and hence it is not memoryless. We describe the state of the system at time t by a pair (N(t), I(t)), where L (t) records the number of customers in the system and I (t) denotes the state of the server (s_1: working state s_2: working state (redundant server); d: delayed period; r: under repair). The stochastic process {(N(t), I(t)), t ≥ 0} is a two-dimensional continuous-time Markov chain

$$q_{(n,i)(n+1,i)} = \lambda \quad ,n \geq 0 \ ; i = r, s_1, s_2, d;$$
$$q_{(n,i)(n-1,i)} = \mu \quad ,n \geq 1 \ ; i = s_1, s_2;$$
$$q_{(n,s_1)(n,s_2)} = 2\xi \quad n \geq 0;$$
$$q_{(n,s_2)(n,d)} = \xi \quad ,n \geq 0 \ ;$$
$$q_{(n,d)(n,r)} = \alpha \quad ,n \geq 0 \ ;$$
$$q_{(n,r)(n,s_1)} = \theta \quad ,n \geq 0 \quad ;$$

As in Economou and Kanta [3], we assume that the customers are allowed to decide whether to join or balk upon their arrival based on the information they have. After service, every customer receives a reward of R units. This may reflect his satisfaction or the added value of being served. On the other hand, there exists a waiting cost of C units per time unit when the customers remains in the system including the time of waiting in queue and being served. Customers are risk neutral and maximize their expected net benefit. Their decisions are irrevocable that retrials of balking customers and reneging of entering customers are not allowed. Each customer can observe the number of customers ahead of him upon his arrival. In this paper, we consider two information cases regarding whether the customers observe also the state of the server or not, which are called fully observable and partially observable cases in the literature.

3. EQUILIBRIUM THRESHOLD STRATEGIES

In this section, we shall obtain the threshold equilibrium strategy in two cases mentioned above. In the fully observable case where customers are known both the state of the server I(t) and the number of the present customer N(t) at arrival time t, a pure threshold strategy is specified by a $(n_e(\mathrm{r}),\ n_e(s_1),\ n_e(s_2), n_e(\mathrm{d}))$ and the balking strategy has the form 'While arriving at time t, observe (N(t), I(t)); enter if N(t) $\leq n_e$(I(t)) – 1 and balk otherwise'. In the partially observable case, a pure threshold strategy is specified by a single number n_e and has the form 'While arriving at time t, observe N(t); enter if such that a customer who observes the system at state (N(t), I(t)) upon his arrival enters if N(t) $\leq n_e$ – 1 and balks otherwise.

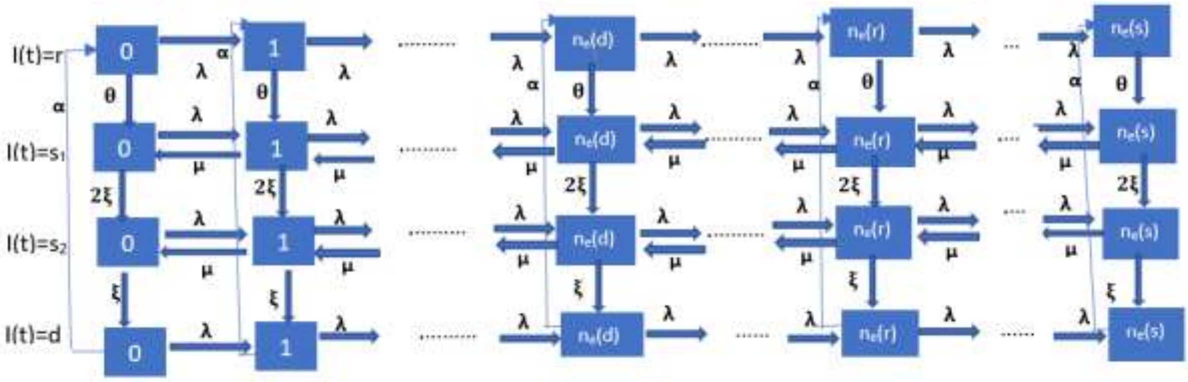

Fig.1. Transition rate diagram for the $(n_e(\mathrm{r}),\ n_e(s_1),\ n_e(s_2), n_e(\mathrm{d}))$ equilibrium strategy in fully observable queue with redundant server with breakdown and delayed repair

3.1 Fully observable queue

Fig 1 illustrates the dynamics of the queueing system in the fully observable case, where customers are informed both the state of the server I(t) and the number of the present customer N(t) at arrival time t. And we have the following result.

Theorem 1. In the fully observable M/M/1 queue with breakdowns and delayed repairs there exist a thresholds:

$$(n_e(\mathrm{r}),\ n_e(s_1),\ n_e(s_2), n_e(\mathrm{d})) = \left(\left\lfloor\frac{(R\theta-C)\mu\alpha}{C(\alpha\theta+\alpha\xi+\xi\theta)}\right\rfloor, \left\lfloor\frac{R\theta\mu\alpha}{C(\alpha\theta+\alpha\xi+\xi\theta)}\right\rfloor, \left\lfloor\frac{R\theta\mu\alpha}{C(\alpha\theta+\alpha\xi+\xi\theta)}\right\rfloor, \left\lfloor\frac{(R\alpha\theta-C\theta-C\alpha)\mu}{C(\alpha\theta+\alpha\xi+\xi\theta)}\right\rfloor\right) \quad (1)$$

such that a customer who observes the system at state (N(t), I(t)) upon his arrival enters if N(t) $\leq n_e$(I(t)) – 1 and balks otherwise.

Proof. It is obvious that for an arriving customer, his expected net reward if he enters is:

$$S(n, i) = R - CT(n, i) \quad (2)$$

where T(n, i) denotes his expected mean sojourn time given that he finds the system at state ((N(t), I(t)) upon his arrival.

Here we assume that time taken by both server to serve a customer is same i.e. both servers are identical. so $T(n, s_1) = T(n, s_2)$ we have the following equations:

$$T(n, r) = \frac{1}{\theta} + T(n, s_1) \qquad n=0,1,2,3\ldots\ldots \quad (3)$$

$$T(0, s_1) = \frac{\mu}{\mu+\xi}\frac{1}{\mu} + \frac{\xi}{\mu+\xi}T(0, d) \quad (4)$$

$$T(n, s_1) = \frac{\mu}{\mu+\xi}\frac{1}{\mu} + \frac{\mu}{\mu+\xi}T(n-1, s_1) + \frac{\xi}{\mu+\xi}T(n, d) \qquad n=1,2,3\ldots\ldots \quad (5)$$

$$T(n, d) = \frac{1}{\alpha} + T(n, r) \qquad n=0,1,2,3\ldots\ldots \quad (6)$$

Solving the system of (3) and (6) for n = 0 along with (4) we obtain T(0, r), T(0, s) and T(0,d).

$$T(0, s_1) = \frac{\mu}{\mu+\xi}\frac{1}{\mu} + \frac{\xi}{\mu+\xi}\left[\frac{1}{\alpha} + \frac{1}{\theta} + T(0, s_1)\right]$$

$$T(0, s_1) = \left(1 + \frac{\xi}{\alpha} + \frac{\xi}{\theta}\right)\frac{1}{\mu} \quad (7)$$

From Eqs. (3) and (6) we can obtain:

$$T(n, d) = \frac{1}{\alpha} + \frac{1}{\theta} + T(n, s_1) \qquad n=0,1,2,3\ldots\ldots \quad (8)$$

By plugging (8) in (5)we have

$$T(n, s_1) = T(n-1, s_1) + \left(1 + \frac{\xi}{\alpha} + \frac{\xi}{\theta}\right)\frac{1}{\mu} \quad (9)$$

using T(0,1) and Eq (7) we get:

$T(n, s_1) = (n+1)\left(1+\frac{\xi}{\alpha}+\frac{\xi}{\theta}\right)\frac{1}{\mu}$ n=0,1,2…….. (10)

By plugging (10) in (3) we have:

$T(n,r) = (n+1)\left(1+\frac{\xi}{\alpha}+\frac{\xi}{\theta}\right)\frac{1}{\mu}+\frac{1}{\theta}$ n=0,1,2…….. (11)

Solving Eq. (6) by using (11), we obtain:

$T(n,d) = (n+1)\left(1+\frac{\xi}{\alpha}+\frac{\xi}{\theta}\right)\frac{1}{\mu}+\frac{1}{\theta}+\frac{1}{\alpha}$ n=0,1,2…….. (12)

If S(n, i) > 0, i.e. if the reward for service greater then the expected cost for waiting, the customer prefers to enter.

And if S(n, i) = 0,

i.e., if the reward is equal to the cost he is indifferent between entering and balking.

By solving S(n, i) ≥ 0 for n, using Eqs. (2) and (10), (11), (12), we obtain that the customer arriving at time t decides to enter if and only if $n \le n_e$(I(t)) – 1

where (n_e(r), $n_e(s_1)$, $n_e(s_2)$, n_e(d)) is given by (1).

This strategy is preferable, independently of what the other customers do, i.e. it is a weakly dominant strategy

For the stationary analysis of the system, note that if all customers follow the threshold strategy in (1) the system follows a Markov chain with state space restricted to Qfo = {(n, i)j0 6 n 6 ne(s), i = r, s,d} and identical transition rates. The corresponding stationary distribution {p(n, i): (n, i) 2 Qfo} is obtained as the unique positive normalized solution of the following system of balance equations:

$(\lambda+\theta)P(0, r) = \alpha P(0, d)$ n=0 (13)

$(\lambda+\theta)P(n, r) = \lambda P(n-1, r) + \alpha P(n, d)$ n=1,2,….. $n_e(r)$-1 (14)

$\theta P(n_e(r), r) = \lambda P(n_e(r)-1, r) + \alpha P(n_e(r), d)$ n= $n_e(r)$ (15)

$\theta P(n, r) = \alpha P(n, d)$ n= $n_e(r)$+1,…. $n_e(s_1)$ or $n_e(s_2)$ (16)

$(\lambda+2\xi)P(0, s_1) = \mu P(1, s_1) + \theta P(0, r)$ (17)

$(\lambda+2\xi+\mu)P(n, s_1) = \mu P(n+1, s_1) + \theta P(n, r) + \lambda P(n-1, s_1)$ n=1,2,….. $n_e(s_1)$-1 (18)

$(\mu+2\xi)P(n_e(s_1), s_1) = \lambda P(n_e(s_1)-1, s_1) + \theta P(n_e(s_1), r)$ (19)

$(\lambda+\xi)P(0, s_2) = \mu P(1, s_2) + 2\xi P(0, s_1)$ (20)

$(\lambda+\xi+\mu)P(n, s_2) = \mu P(n+1, s_2) + 2\xi P(n, s_1) + \lambda P(n-1, s_2)$ n=1,2,….. $n_e(s_2)$-1 (21)

$(\mu+\xi)P(n_e(s_2), s_2) = \lambda P(n_e(s_2)-1, s_2) + 2\xi P(n_e(s_2), s_1)$ (22)

$(\lambda+\alpha)P(0, d) = \xi P(0, s_2)$ (23)

$(\lambda+\alpha)P(n, d) = \lambda P(n-1, d) + \xi P(n, s_2)$ n=1,2,….. $n_e(d)$-1 (24)

$\alpha P(n_e(d), d) = \lambda P(n_e(d)-1, d) + \xi P(n_e(d), s_2)$ (25)

$\alpha P(n, d) = \xi P(n, s_2)$ n=$n_e(d)$+1,…. $n_e(s_1)$or $n_e(s_2)$ (26)

From eqs (16),(18),(21)and (26) we have

$\mu^2 P(n+2, s_2) - \mu(2\lambda+3\xi+2\mu)P(n+1, s_2) + \{(\lambda+2\xi+\mu)(\lambda+\xi+\mu)+2\lambda\mu+2\xi^2\}P(n, s_2) - \lambda(2\lambda+3\xi+2\mu)P(n-1,s_2) + \lambda^2 P(n-2,s_2)=0$ (27)

Which is a four order difference equation $x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$ with solution $\eta_i (i = 1,2,3,4)$therefore we can set

$P(n, s_2) = \sum_{i=0}^{4} b_{1i}\eta_i^n$ n= $n_e(r)$+1,…., $n_e(s_2)$-1 (28)

where b_{1i} i=1,2,3,4 are constant to be determined by using normalization condition

From eqs (21),

$P(n, s_1) = \sum_{i=0}^{4} b_{2i}\eta_i^n$ n= $n_e(r)$+1,…., $n_e(s_1)$-1 (29)

Where $b_{2i} = \frac{1}{2\xi}\{(\lambda+\xi+\mu) - \mu\rho_i - \frac{\lambda}{\rho_i}\}b_{1i}$

Put the value of eqs (28) in (26) and (16) then we have

$P(n, d) = \frac{\xi}{\alpha}\sum_{i=0}^{4} b_{1i}\eta_i^n$ n= $n_e(r)$+1,…., $n_e(s_2)$-1 (30)

$P(n, r) = \frac{\xi}{\theta}\sum_{i=0}^{4} b_{1i}\eta_i^n$ n= $n_e(r)$+1,…., $n_e(s_2)$-1 (31)

From eqs (16), (19), (22)and (26) we have

$\mu(\mu+3\xi)P(n_e(s_2), s_2) - (2\mu+3\xi)\lambda P(n_e(s_2)-1, s_2) + \lambda^2 P(n_e(s_2)-2, s_2) = 0$

Which is second order difference equation with solution

$P(n_e(s_2), s_2) = P_1 = c_1\left(\frac{\lambda}{\mu}\right)^{n_e(s_2)} + c_2\left(\frac{\lambda}{\mu+3\xi}\right)^{n_e(s_2)}$ (32)

Plugging eq(32) in eq(22),

$P(n_e(s_1), s_1) = \left(\frac{(\xi+\mu)}{2\xi}\right)P_1 + \frac{1}{2\xi}\sum_{i=0}^{4} b_{1i}\eta_i^{n_e(s_2)-1}$ (33)

From eq(26) $P(n_e(s_2), d) = \frac{\xi}{\alpha} P_1$ (34)

From eq(16) $P(n_e(s_2), r) = \frac{\xi}{\theta} P_1$ (35)

The normalization equation

$$\sum_{n=0}^{n_e(s)} P(n,r) + \sum_{n=0}^{n_e(s)} P(n,s_1) + \sum_{n=0}^{n_e(s)} P(n,s_2) + \sum_{n=0}^{n_e(s)} P(n,d) = 1$$

Here $n_e(s)$ indicates $n_e(s_1)$ or $n_e(s_2)$

the probability of balking is equal to

$$\sum_{n=n_e(r)}^{n_e(s)} P(n,r) + P(n_e(s_1), s_1) + P(n_e(s_2), s_2) + \sum_{n=n_e(d)}^{n_e(s)} P(n,d)$$

the social benefit per time unit when all customers follow the threshold policies $(n_e(r),\ n_e(s_1),\ n_e(s_2), n_e(d))$ given in theorem 1 equals:

$$SB_{fo} = \lambda\left[1- \sum_{n=n_e(r)}^{n_e(s)} P(n,r) - P(n_e(s_1), s_1) - P(n_e(s_2), s_2) - \sum_{n=n_e(d)}^{n_e(s)} P(n,d)\right] - C \sum_{n=0}^{n_e(s)} n(P(n,r) + P(n,s_1) + P(n,s_2) + P(n,d)) \quad (36)$$

3.2 Partially observable queue

In this section we proceed to the partially observable case where the arriving customers observe the number of customers upon arrival, but not the state of the server. The transition diagram is depicted in Fig. 2. The corresponding equilibrium strategies within the class of pure threshold strategies will be studied. To this end, it is necessary to obtain the stationary distribution of the system when the customers follow a given pure threshold strategy. We have the following preliminary result.

Lemma 1. In the partially observable M/M/1 queue with redundant server with breakdowns and delayed repairs where the customers enter the system according to a threshold strategy 'while arriving at time t, observe N(t); enter if $N(t) \le n_e(I(t)) - 1$ and balk otherwise',

the stationary distribution $(P(n, i): (n, i) \epsilon \{0,1, 2, \dots, n_e\} \times \{r, s_1, s_2, d\})$ is given as follows

$P(n, r) = \sum_{i=0}^{6} u_i \rho_i^n$ n= 0,1,2,………………… n_e -1 (37)

$P(n, s_1) = \sum_{i=0}^{6} w_i \rho_i^n$ n= 0,1,2,………………… n_e -1 (38)

$P(n, s_2) = \sum_{i=0}^{6} v_i \rho_i^n$ n= 0,1,2,………………… n_e -1 (39)

$P(n, d) = \sum_{i=0}^{6} c_i \rho_i^n$ n=0,1,2………………… n_e -1 (40)

$P(n_e, r) = \frac{(\lambda+2\xi)(\lambda+\xi)}{\theta(\lambda+3\xi)}\left[\sum_{i=1}^{6}\left(w_i + u_i + c_i + \frac{\xi}{\lambda+\xi} v_i\right)\rho_i^{n_e-1}\right] - \frac{\lambda}{\theta}\sum_{i=0}^{6} w_i \rho_i^{n_e-1}$ (41)

$P(n_e, s_1) = \frac{\lambda+\xi}{\lambda+3\xi}\left[\sum_{i=1}^{6}\left(w_i + u_i + c_i + \frac{\xi}{\lambda+\xi} v_i\right)\rho_i^{n_e-1}\right]$ (42)

$P(n_e, s_2) = = \frac{(\lambda+2\xi)(\lambda+\xi)}{\xi(\lambda+3\xi)}\left[\sum_{i=1}^{6}\left(w_i + u_i + c_i + \frac{\xi}{\lambda+\xi} v_i\right)\rho_i^{n_e-1}\right] - \frac{\lambda}{\xi}\sum_{i=0}^{6}(w_i + u_i + c_i)\rho_i^{n_e-1}$ (43)

$P(n_e, d) = \frac{(\lambda+2\xi)(\lambda+\xi)}{\alpha(\lambda+3\xi)}\left[\sum_{i=1}^{6}\left(w_i + u_i + c_i + \frac{\xi}{\lambda+\xi} v_i\right)\rho_i^{n_e-1}\right] - \frac{\lambda}{\alpha}\sum_{i=0}^{6}(w_i + u_i)\rho_i^{n_e-1}$ (44)

Where

$v_i = \frac{(\lambda+\alpha)\rho_i - \lambda}{\xi\rho_i}$ i=1,2….6 (45)

$w_i = \frac{v_i}{2\xi}\ (\lambda + \xi + \mu - \mu\rho_i - \frac{\lambda}{\rho_i})$ i=1,2…….6 (46)

$u_i = \frac{w_i}{\theta}\ (\lambda + 2\xi + \mu - \mu\rho_i - \frac{\lambda}{\rho_i})$ i=1,2…….6 (47)

$$a = \frac{-(\lambda+\alpha)\{(\lambda+\theta)(2\lambda+3\xi+2\mu)\mu+\mu^2\lambda\} - \mu^2(\lambda+\theta)\lambda}{\mu^2(\lambda+\theta)(\lambda+\alpha)} \quad (48)$$

$$b = \frac{\lambda(\lambda+\theta)(2\lambda+3\xi+2\mu)\mu+\mu^2\lambda^2+(\lambda+\alpha)(\lambda+\theta)\{(\lambda+2\xi+\mu)(\lambda+\xi+\mu)+2\mu\lambda\}+(\lambda+\alpha)(2\lambda+3\xi+2\mu)\mu\lambda - 2\alpha\xi^2\theta}{\mu^2(\lambda+\theta)(\lambda+\alpha)} \quad (49)$$

$$c = \frac{-\lambda[(\lambda+\alpha)\{(\lambda+\theta)(2\lambda+3\xi+2\mu) + (\lambda+2\xi+\mu)(\lambda+\xi+\mu) + 2\mu\lambda\} + (\lambda+\theta)\{(\lambda+2\xi+\mu)(\lambda+\xi+\mu) + 2\mu\lambda\} + \lambda\mu(2\lambda+3\xi+2\mu)]}{\mu^2(\lambda+\theta)(\lambda+\alpha)} \quad (50)$$

$$d = \frac{\lambda^2[(\lambda+\alpha)(3\lambda+3\xi+2\mu+\theta) + (\lambda+\theta)(2\lambda+3\xi+2\mu)+(\lambda+2\xi+\mu)(\lambda+\xi+\mu) + 2\mu\lambda]}{\mu^2(\lambda+\theta)(\lambda+\alpha)} \quad (51)$$

$e = \frac{-\lambda^3(4\lambda+3\xi+2\mu+\theta+\alpha)}{\mu^2(\lambda+\theta)(\lambda+\alpha)}$ (52)

$f = \frac{\lambda^4}{\mu^2(\lambda+\theta)(\lambda+\alpha)}$ (53)

and c_i(i = 1,2,3,4,5,6) are determined by using normalization condition and ρ_i (i = 1,2,3,4,5,6) are six roots of equation $x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$.

Proof :

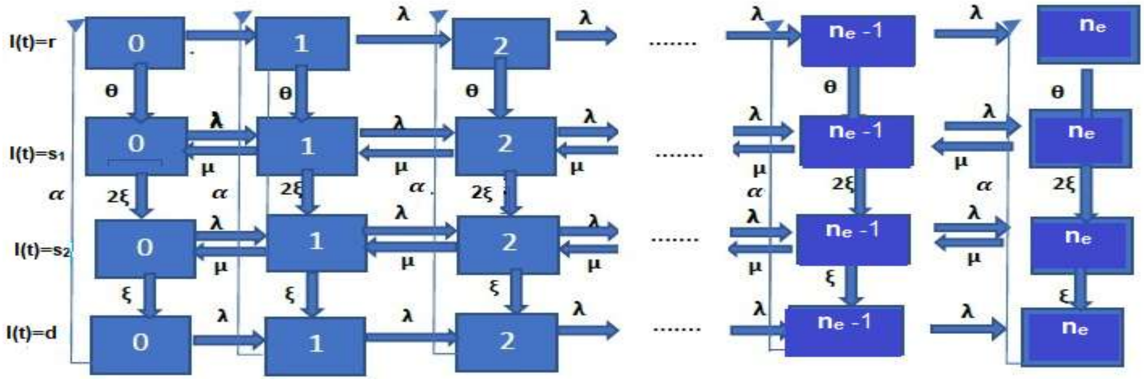

Fig.2. Transition rate diagram for the n_e equilibrium strategy in partially observable queue with redundant server with breakdown and delayed repair

The stationary distribution (P(n, i)) is obtained by using the balance equations:

$(\lambda+\theta)P(0, r) = \alpha P(0, d)$ (54)
$(\lambda+\theta)P(n, r) = \lambda P(n-1, r) + \alpha P(n, d)$ (55)
$\theta P(n_e, r) = \lambda P(n_e-1, r) + \alpha P(n_e, d)$ (56)
$(\lambda+2\xi)P(0, s_1) = \mu P(1, s_1) + \theta P(0, r)$ (57)
$(\lambda+2\xi+\mu)P(n, s_1) = \mu P(n+1, s_1) + \theta P(n, r) + \lambda P(n-1, s_1)$ (58)
$(\lambda+2\xi)P(n_e, s_1) = \lambda P(n_e-1, s_1) + \theta P(n_e, r)$ (59)
$(\lambda+\xi)P(0, s_2) = \mu P(1, s_2) + 2\xi P(0, s_1)$ (60)
$(\lambda+\xi+\mu)P(n, s_2) = \mu P(n+1, s_2) + 2\xi P(n, s_1) + \lambda P(n-1, s_2)$ (61)
$(\lambda+\xi)P(n_e, s_2) = \lambda P(n_e-1, s_2) + 2\xi P(n_e, s_1)$ (62)
$(\lambda+\alpha)P(0, d) = \xi P(0, s_2)$ (63)
$(\lambda+\alpha)P(n, d) = \lambda P(n-1, d) + \xi P(n, s_2)$ (64)
$\alpha P(n_e, d) = \lambda P(n_e-1, d) + \xi P(n_e, s_2)$ (65)

From eq (55), (58), (61) and (64) we obtain:

$\mu^2(\lambda+\theta)(\lambda+\alpha)P(n+2, d) - [(\lambda+\alpha)\{(\lambda+\theta)(2\lambda+3\xi+2\mu)\mu+\mu^2\lambda\} + \mu^2(\lambda+\theta)\lambda]P(n+1, d) + [\lambda(\lambda+\theta)(2\lambda+3\xi+2\mu)\mu+\mu^2\lambda^2+(\lambda+\alpha)(\lambda+\theta)\{(\lambda+2\xi+\mu)(\lambda+\xi+\mu)+2\mu\lambda\}+(\lambda+\alpha)(2\lambda+3\xi+2\mu)\mu\lambda - 2\alpha\xi^2\theta]P(n, d) - \lambda[(\lambda+\alpha)\{(\lambda+\theta)(2\lambda+3\xi+2\mu)+(\lambda+2\xi+\mu)(\lambda+\xi+\mu)+2\mu\lambda\} + (\lambda+\theta)\{(\lambda+2\xi+\mu)(\lambda+\xi+\mu)+2\mu\lambda\}+\lambda\mu(2\lambda+3\xi+2\mu)]P(n-1,d) + \lambda^2[(\lambda+\alpha)(3\lambda+3\xi+2\mu+\theta)+(\lambda+\theta)(2\lambda+3\xi+2\mu)+(\lambda+2\xi+\mu)(\lambda+\xi+\mu)+2\mu\lambda]P(n-2, d) - \lambda^3(4\lambda+3\xi+2\mu+\theta+\alpha)P(n-3, d) + \lambda^4 P(n-4, d) = 0$

$x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$

which is a six-order difference equation with solution ρ_i(i = 1,2,3, 4,5,6) and a, b, c, d, e, f are define in eqs (48)-(51) Therefore, we can set:

$P(n,d) = \sum_{i=0}^{6} c_i\rho_i^n$ n=0,1,2………………… -1 (66)

where c_i, i = 1, 2, 3, 4 ,5,6 are constants to be determined. By plugging (66) in (65),

$P(n,s_2) = \sum_{i=0}^{6} v_i\rho_i^n$ n=1,2,3………………… n_e -1 (67)

Again, by plugging (67) in (61), we get:

$P(n, s_1) = \sum_{i=0}^{6} w_i\rho_i^n$ n=2,3………………… n_e -2 (68)

Again, by plugging (68) in (58), we get:

$P(n, r) = \sum_{i=0}^{6} u_i\rho_i^n$ n=3, 4,……………… n_e -3 (69)

Where v_i , w_i and u_i are described in eqs (45)-(47)

By putting n = 1 in (61) and using (67) and (68), it is followed that:

$P(0, s_2) = v_1 + v_2 + v_3 + v_4 + v_5 + v_6$

Then we obtain (39) by combining (67) and the above equation.

In a similar manner, putting n = 1, n = 2 and n = n_e - 1 in (58), and from the above equations, we get (38) .

And putting n=1,n=2,n=3 ,n=n_e -2 and n= n_e – 1 in (55) and from above equation ,we get (37)

from eqs (56), (59) ,(62)and (65), we have

$P(n_e, s_1) = \frac{\lambda+\xi}{\lambda+3\xi}[P(n_e-1, s_1)+ P(n_e-1, r)+ P(n_e-1, d)+ \frac{\xi}{\lambda+\xi} P(n_e-1, s_2)]$

Put the value of eqs (66)-(69) then

$P(n_e, s_1) = \frac{\lambda+\xi}{\lambda+3\xi}\left[\sum_{i=1}^{6}\left(w_i + u_i + c_i + \frac{\xi}{\lambda+\xi}\, v_i\right)\rho_i^{n_e-1}\right]$

Similarly we find (41),(43) and (44)

By plugging (37)–(40) in (54), (57), (60) and (61), we have:

$$\sum_{i=1}^{6}\left\{\frac{(\lambda+\theta)[\ (\lambda+2\xi+\mu)\rho_i-\mu\rho_i{}^2-\lambda][\ (\lambda+\xi+\mu)\rho_i-\mu\rho_i{}^2-\lambda)]\ [(\lambda+\alpha)\rho_i-\lambda]}{2\xi^2\theta\rho_i{}^3}-\alpha\right\}c_i = 0 \qquad (70)$$

$$\sum_{i=1}^{6}\left\{\frac{[\lambda-(\mu+\xi)\rho_i][\ (\lambda+\xi+\mu)\rho_i-\mu\rho_i{}^2-\lambda)]\ [(\lambda+\alpha)\rho_i-\lambda]}{2\xi^2\rho_i{}^2}\right\}c_i = 0 \qquad (71)$$

$$\sum_{i=1}^{6}\left\{\frac{(\lambda-\mu\rho_i)\ [(\lambda+\alpha)\rho_i-\lambda]}{\xi\rho_i{}^2}\right\}c_i = 0 \qquad (72)$$

$$\sum_{i=1}^{6}\left\{\frac{\lambda}{\rho_i}\right\}c_i = 0 \qquad (73)$$

with the help of Eqs. (70)–(73), then the value of c_i, i = 1, 2,..,6 can be determined by using normalization condition.

We now proceed to find the expected net reward of a customer that observes n customers ahead of him and decides to enter. We have the following:

Lemma 2. Consider the partially observable M/M/1 queue with redundant server for breakdowns and delayed repairs where the customers enter to the system according to a threshold strategy 'While arriving at time t, observe N(t); enter if N(t) ≤ n_e- 1 and balk otherwise'. The net benefit of a customer that observes n customers and decides to enter is given by

$$S(n) = R - C\left[(n+1)\left(1+\frac{\xi}{\alpha}+\frac{\xi}{\theta}\right)\frac{1}{\mu}+\frac{\frac{1}{\theta}\sum_{i=0}^{6}u_i\rho_i^n+\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\sum_{i=0}^{6}c_i\rho_i^n}{\sum_{i=0}^{6}(u_i+w_i+v_i+c_i)\rho_i^n}\right],\quad n = 0,1,2,\ldots\ldots n_e-1 \qquad (74)$$

$$S(n_e)=R-C\left[(n_e+1)\left(1+\frac{\xi}{\alpha}+\frac{\xi}{\theta}\right)\frac{1}{\mu}+\frac{\frac{1}{\theta}P(n_e,r)+\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)P(n_e,d)}{P(n_e,r)+P(n_e,s_1)+P(n_e,s_2)+P(n_e,d)}\right] \qquad (75)$$

Proof. The expected net reward, if he enters, for a customer that observes n customers is

S(n) = R - CT(n) (76)

where T(n) = E[S|N^- = n] denotes his expected mean sojourn time given that he finds n customers in the system just before his arrival. We let Pr (I^- = i|N^- = n) be the probability that the state of the server is i when he observes n customers in the system upon his arrival.

Conditioning on the state of the server that he finds upon arrival and taking into account (3) and (6) we obtain:

T(n) = T(n, s_1) Pr (I^- = s_1|N^- = n) +T(n,s_2) Pr (I^- = s_2|N^- = n) +T(n,r) Pr (I^- = r|N^- = n) +T(n,d) Pr (I^- = d|N^- = n)

Put the value of eqs (10), (11) and (12) , now we have

T(n) = T(n, s_1) Pr (I^- = s_1|N^- = n) +T(n,s_1) Pr (I^- = s_2|N^- = n) + $\left(\frac{1}{\theta} + T(n,s_1)\right)$ Pr (I^- = r|N^- = n) + $\left(\frac{1}{\theta}+\frac{1}{\alpha}+T(n,s_1)\right)$ Pr (I^- = d|N^- = n)

= T(n,s_1) + $\frac{1}{\theta}$ Pr (I^- = r|N^- = n) + $(\frac{1}{\theta}+\frac{1}{\alpha})$ Pr (I^- = d|N^- = n) (77)

$$\Pr(I^- = r|N^- = n) = \frac{\lambda P(n,r)}{\lambda P(n,r)+\lambda P(n,s_1)+\lambda P(n,s_2)+\lambda P(n,d)}$$

$$\Pr(I^- = d|N^- = n) = \frac{\lambda P(n,d)}{\lambda P(n,r)+\lambda P(n,s_1)+\lambda P(n,s_2)+\lambda P(n,d)}\qquad n = 0,1,2,\ldots\ldots n_e$$

Using the stationary probabilities obtained in Lemma 1 we obtain the probabilities Pr (I^- = r|N^- = n) and Pr (I^- = d|N^- = n) for n = 0,1, . . . , n_e.

So, we have:

$$T(n) = (n+1)\left(1+\frac{\xi}{\alpha}+\frac{\xi}{\theta}\right)\frac{1}{\mu}+\frac{\frac{1}{\theta}\sum_{i=0}^{6}u_i\rho_i^n+\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\sum_{i=0}^{6}c_i\rho_i^n}{\sum_{i=0}^{6}(u_i+w_i+v_i+c_i)\rho_i^n},\quad n = 0,1,2,\ldots\ldots n_e-1 \qquad (78)$$

$$T(n_e) = (n_e+1)\left(1+\frac{\xi}{\alpha}+\frac{\xi}{\theta}\right)\frac{1}{\mu}+\frac{\frac{1}{\theta}P(n_e,r)+\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)P(n_e,d)}{P(n_e,r)+P(n_e,s_1)+P(n_e,s_2)+P(n_e,d)} \qquad (79)$$

where $P(n_e, r)$, $P(n_e, s_1)$, $P(n_e, s_2)$ and $P(n_e, d)$ are given by Eqs. (41)–(44). And then, Eqs. (74) and (75) can be obtained by put value of Eqs.(78) and (79) in (76) for every ne.

It should be noted that a customer does not enter the system even if he finds no customers in front of him if S(0) < 0, otherwise, he enters the queue.
Next, we describe the equilibrium balking pure threshold strategies in the partially observable case by assuming S(0) > 0. We have the following result.

Theorem 2. Define the sequences ($f_1(n)$: n = 0, 1, . . .) and ($f_2(n)$:n = 0,1, . . .) by

$$f_1(n) = R - C\left[(n+1)\left(1+\frac{\xi}{\alpha}+\frac{\xi}{\theta}\right)\frac{1}{\mu}+\frac{\frac{1}{\theta}\sum_{i=0}^{6} u_i\rho_i^n+\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\sum_{i=0}^{6} c_i\rho_i^n}{\sum_{i=0}^{6}(u_i+w_i+v_i+c_i)\rho_i^n}\right] \quad , n = 0,1,2,\ldots \qquad (80)$$

$$f_2(n) = R - C\left[\begin{array}{c}(n_e+1)\left(1+\frac{\xi}{\alpha}+\frac{\xi}{\theta}\right)\frac{1}{\mu}+ \\ \frac{(\lambda+2\xi)\left\{\left(\frac{1}{\theta^2}+\frac{1}{\theta\alpha}+\frac{1}{\alpha^2}\right)\psi_n-\lambda\left(\frac{1}{\theta^3}+\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\frac{1}{\alpha^2}\right)\sum_{i=0}^{6} w_i\rho_i^{n_e-1}-\lambda\left(\frac{1}{\theta}+\frac{1}{\alpha}\right)\frac{1}{\alpha^2}\sum_{i=0}^{6} u_i\rho_i^{n_e-1}\right\}}{\left\{(\lambda+2\xi)\left(\frac{1}{\theta}+\frac{1}{\alpha}+\frac{1}{\xi}\right)+1\right\}\psi_n-\lambda(\lambda+2\xi)\left(\frac{1}{\theta^2}+\frac{1}{\alpha^2}+\frac{1}{\xi^2}\right)\sum_{i=0}^{6} w_i\rho_i^{n_e-1}-\lambda(\lambda+2\xi)\left(\frac{1}{\xi}+\frac{1}{\alpha}\right)\sum_{i=0}^{6} u_i\rho_i^{n_e-1}-\lambda(\lambda+2\xi)\frac{1}{\xi^2}\sum_{i=0}^{6} c_i\rho_i^{n_e-1}}\end{array}\right] \quad n=0,1\ldots.. \qquad (81)$$

Where $\psi_n = \sum_{i=1}^{6}\left(w_i+u_i+c_i+\frac{\xi}{\lambda+\xi}v_i\right)\rho_i^{n_e-1}$

By definition $f_1(n) = S(n)$, n=0,1,2…. $n_e - 1$ and $f_2(n_e) = S(n_e)$

Moreover, $f_1(n) > f_2(n)$, n=0,1,…..

Then there exist finite non-negative integers $n_L \leq n_U$ such that

$f_1(0)$, $f_1(1)$, $f_1(2)$ …….. $f_1(n_U - 1) > 0$, $\qquad f_1(n_U) \leq 0$ (82)

and

$f_2(n_U)$, $f_2(n_U - 1)$, $f_2(n_U - 2)$ …… $f_2(n_L) \leq 0$, $\qquad f_2(n_L - 1) > 0$ (83)

or

$f_2(n_U)$, $f_2(n_U - 1)$, $f_2(n_U - 2)$, …… $f_2(1)$, $f_2(0) \leq 0$ (84)

In the partially observable M/M/1 queue with redundant server with breakdowns and delayed repairs the pure threshold strategies of the form 'While arriving at time t, observe N(t); enter if N(t) ≤ n_e-1 and balk otherwise ' .
For $n_e \epsilon \{n_L, n_L+1, \ldots \ldots. n_U\}$ are equilibrium strategies.

Proof. if we assume that S(0) > 0 then We have $f_1(0) > 0$ and $\lim_{n\to\infty} f_1(n) = -\infty$
so if n_U is the subscript of the first negative term of the sequence ($f_1(n)$), we have that for the finite number n_U the condition (82) holds.
On the other hand, $f_1(n) > f_2(n)$,n = 0,1, ... In particular we conclude that $f_2(n_U) < f_1(n_U) \leq 0$. Now, we begin to go backwards, starting from the subscript n_U, towards 0 and we let $n_L - 1$ be the subscript of the first positive term of the sequence ($f_2(n)$). Then we have (83). If all the terms of ($f_2(n)$) going backwards from n_U to 0 are non-positive we have (84).
Now we can establish the existence of equilibrium threshold policies in the partially observable case.

In this model we consider an arrival customer assume that all other customers follow the same threshold strategy 'while arriving at time t, observe N(t), enter if N(t) ≤ n_e-1 and balk otherwise ' .for some fixed r $n_e \epsilon \{n_L, n_L+1, \ldots \ldots. n_U\}$
If the tagged customer finds n ≤ n_e-1 customers in front of him and decides to enter, his expected benefit is equal to $f_1(n) > 0$ because of (76), (80) and (82). So in this case the customer prefers to enter.
If the tagged customer finds n = n_e customers in front of him and decides to enter, his expected benefit is equal to $F_2(n_e) \leq 0$ because of (76), (81), (83) or (84). So in this case the customer prefers to balk.
Define

$P(n) = P(n, r) + P(n, s_1) + P(n, s_2) + P(n, d)$ $\qquad$ n=1,2,…… n_e

Because the probability of balking is equal to $P(n_e)$, the social benefit per time unit when all customers follow the threshold policies n_e given in Lemma 2 equals:

$$SB_{po} = \lambda\,(1 - P(n_e)) - C\sum_{n=0}^{n_e} nP(n) \qquad (85)$$

Numerical experiments

In this section by some numerical examples, we calculate the effect of different parameters on customer's behaviour in fully observable queueing system with redundant server for server breakdown and delayed repair. We analysis the sensitivity of equilibrium threshold strategy with the help of main indicator of the system. Here we assume $\lambda = 1, \mu = 2, \alpha = 1, \xi = 3, \theta = 2, c = 2, R = 10$

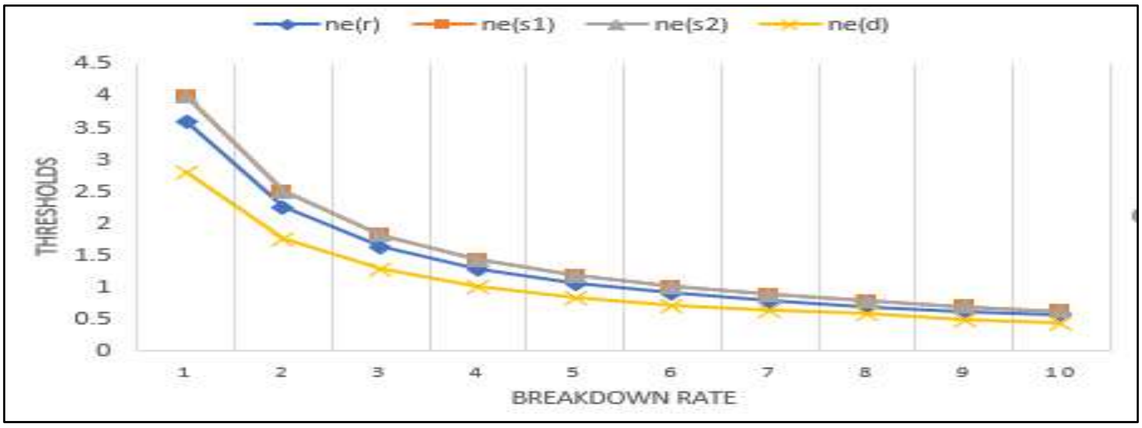

Fig3. Threshold vs. ξ for $\lambda = 1, \mu = 2, \alpha = 1, \theta = 2, c = 2, R = 10$

In the Fig 3, we observe that equilibrium threshold strategy are decreasing function of breakdown rate. This is because Customers waiting time and their payment increases when breakdown of server increases.

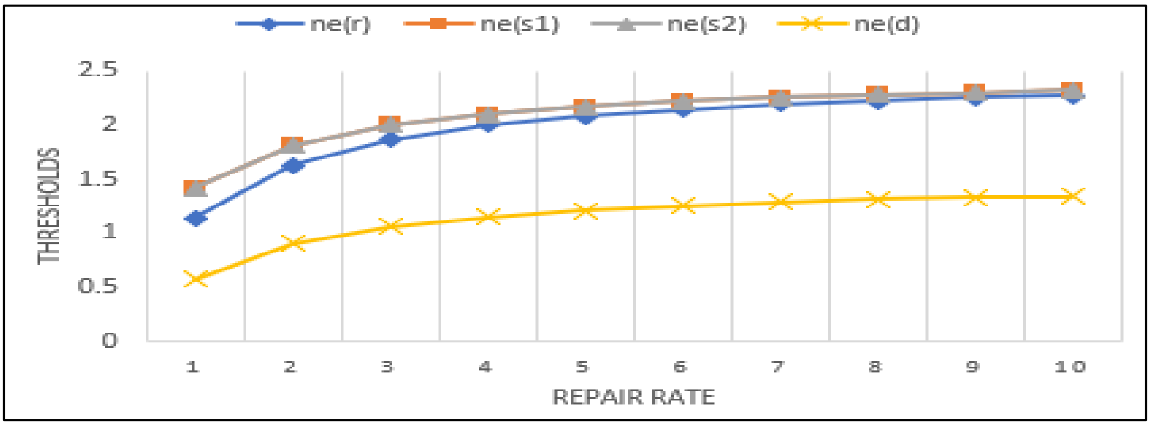

Fig4. Threshold vs. α for $\lambda = 1, \mu = 2, \xi = 3, \theta = 2, c = 2, R = 10$

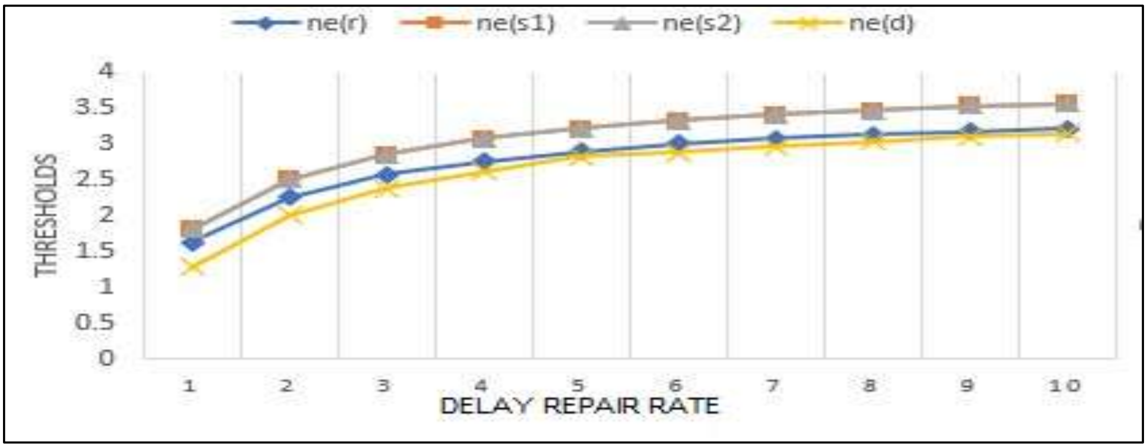

Fig5. Threshold vs. θ for $\lambda = 1, \mu = 2, \xi = 3, \alpha = 1, c = 2, R = 1$

In the Fig 4 and 5, we observe that equilibrium threshold strategy are increasing function of both the repair rate and delayed repair rate. This is because when highly delayed repair rate and highly repair rate are shown in the system upon customers arrival then customers prefer to join the queue.

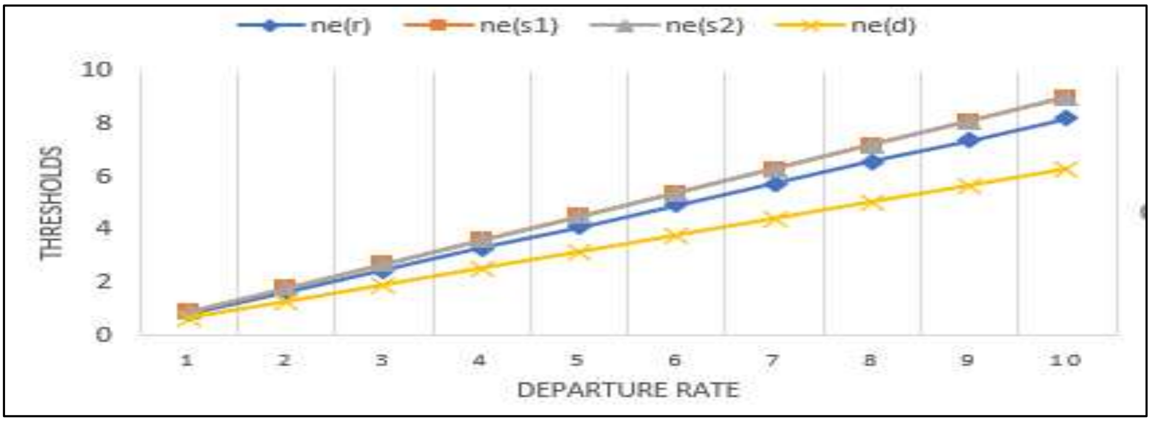

Fig6. Threshold vs. μ for $\lambda = 1, \alpha = 1, \xi = 3, \theta = 2, c = 2, R = 10$

Fig 6 indicats equilibrium threshold strategy versus service rate. we observe that equilibrium threshold strategy are increasingly function of service rate. This is because when server can serve more customer per time in the system then an arriving customer is always prefer to enter the queue.

IV. CONCLUSION

We studied equilibrium thresholds strategy for the observable markovian queue with redundant server for balking and delayed repair. Inspired by Wang and Zhang [9], they describe equilibrium analysis of the observable markovian queue with balking and delayed repair. Here we are derived the equilibrium thresholds balking strategy and equilibrium social benifit for the fully and partially observable markovian queue with redundant server for balking and delayed. Here we use an algorithm Provided to identify for equilibrium strategy for fully observable system or partially observable system with redundant system.

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