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A Mathematical study on Q-Fuzzy Normal Subgroups

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ABSTRACT

This paper on the basis of fuzzy sets introduced by L.A Zadeh , we first gave the definition of α - fuzzy set and then defined α -fuzzy subgroups and α - fuzzy normal subgroups and finally, defined quotient group of the α - fuzzy cosets of an α -fuzzy normal subgroup. This paper, we introduce the concepts of Q-fuzzy normal sub groups and Q-fuzzy cosets and discussed some of its properties.

KEYWORDS: Q-Fuzzy Normal subgroup; Q-Fuzzy Characteristic subgroup; Q-Fuzzy Normalizer; Q-Fuzzy Cosets; Pseudo Q-Fuzzy Cosets.

I. INTRODUCTION

The concept of fuzzy sets is introduced by Zadeh [10]. Then it has become a strong area of research in engineering, medical science, Social science and Graph Theory etc.,. Solairaju [3] gave the idea of Q-Fuzzy Normal sub Groups. Sunderrajan and Senthilkumar[8] introduce and define a L-Fuzzy normal Sub *l*-groups. Sithar Selvam[7] discussed properties of Anti –Q-Fuzzy Normal Subgroups. In this paper we define a new algebraic structure of Q-Fuzzy normal subgroups and Q-Fuzzy cosets and study some of their properties. This paper contains some definitions and results in Q-fuzzy normal subgroup theory and cosets, which are required in the equel. Some theorems are introduced in this paper which have been used by homomorphism and anti-homomorphism of Q-fuzzy normal subgroups.

II. Q-FUZZY NORMAL SUBGROUPS

A. Definition:

Let(G, ·) be a group and Q be a non-empty set. A Q-fuzzy subgroup A of G is said to be a Q-fuzzy normal subgroup(QFNSG) of G if A(xy, q) = A(yx, q), for all x and y in G and q in Q.

B. Definition

Let(G, ·) be a group and Q be a non-empty set. A Q-fuzzy Subgroup A of G is said to be a Q-fuzzy characteristic subgroup (QFCSG) of G if A(x, q) = A(f(x), q), for all x in G and f in AutG and q in Q.

C. Definition

A Q-fuzzy subset A of a set X is said to be normalized if there exists an element x in X such that A(x, q) =1.

D. Definition

Let A be a Q-fuzzy subgroup of a group (G, ·). For any a in G, a A defined by $(aA)(x, q)$ = A($a^{-1} x, q$), for every x in G and q in Q, is called a Q-fuzzy coset of G.

E. Definition

Let A be a Q-fuzzy subgroup of a group (G, ·) and H = { x∈ G / A(x, q) = A(e, q) }, then O(A), the order of A is defined as O(A) =O(H).

F. Definition

Let A be a Q-fuzzy subgroup of a group (G , ·). Then for any a and b in G, a Q-fuzzy middle coset aAb of G is defined by $(aAb)(x, q)$ = A ($a^{-1} x\, b^{-1}, q$), for every x in G and q in Q.

G. Definition

Let A be a Q-fuzzy subgroup of a group (G, ·) and a in G. Then the pseudo Q-fuzzy coset $(aA)^P$ is defined by $((aA)^P)(x, q)$ = p(a) A(x, q), for every x in G and for some p in P and q in Q.

H. Definition

A Q-fuzzy subgroup A of a group G is called a generalized characteristic Q-fuzzy subgroup(GCQFSG) if for all x and y in G, O(x) = O(y) implies A(x, q) = A(y, q), for all q in Q.

III. SOME PROPERTIES OF Q-FUZZY NORMAL SUBGROUPS

A. Theorem

Let (G, ·) be a group and Q be a non- empty set . If A and B are two Q-fuzzy normal subgroups of G, then their intersection A∩B is a Q-fuzzy normal subgroup of G.

Proof:

Let x and y in G and q in Q and A ={⟨(x, q), A(x, q) ⟩} / x in G and q in Q } and B = {⟨(x, q), B(x, q) ⟩} / x in G and q in Q } be any two Q-fuzzy normal subgroups of G.

Let C = A∩B and C = { ⟨(x, q), C(x, q) ⟩} / x in G and q in Q }, where C(x, q) = min{A(x, q), B(x, q)}, Then, Clearly C is a Q-fuzzy subgroup of G, Since A and B are two Q-fuzzy subgroups of G.

And, C (xy, q) = min { A (xy, q), B(xy, q) },
= min { A (yx, q), B(yx, q) }
= C(yx, q).

Therefore, C(xy, q) = C(yx, q) , for all x and y in G and q in Q.

Hence A∩B is a Q-fuzzy normal subgroup of the group G.

B. Theorem

Let (G , ·) be a group and Q be a non-empty set. The intersection of a family of Q-fuzzy normal subgroups of G is a Q-fuzzy normal subgroup of G.

Proof

Let $\{A_i\}_{i\in I}$ be a family of Q-fuzzy normal subgroups of G and A = $\bigcap_{i\in I} A_i$.

Then for x and y in G and q in Q, clearly the intersection of a family of Q-fuzzy subgroups of a group G is a Q-fuzzy subgroup of a group G.

Now, A (xy, q) = $\underset{i\in I}{\inf} A_i(xy, q)$

= $\underset{i\in I}{\inf} A_i(yx, q)$

= A (yx, q).

Therefore, A (xy, q) = A (yx, q), for all x and y in G and q in Q. Hence the intersection of a family of Q-fuzzy normal subgroups of a group G is a Q-fuzzy normal subgroup of G.

C. Theorem

If A is a Q-fuzzy characteristic subgroup of a group G, then A is a Q-fuzzy normal subgroup of the group G.

Proof

Let A be a Q-fuzzy characteristic subgroup of a group G, x and y in G and q in Q. Consider the map f : G→G defined by f(x)=yxy^{-1}. Clearly, f in Aut G.

Now, A (xy, q) = A (f(xy), q)
= A ($y(xy)y^{-1}$,q)
= A (yx, q).

Therefore, A(xy, q) = A(yx, q) , for all x and y in G and q in Q. Hence A is a Q-fuzzy normal subgroup of the group G .

D. Theorem

A Q-fuzzy subgroup A of a group G is a Q-fuzzy normal subgroup of G if and only if A is constant on the conjugate classes of G.

Proof

Suppose that A is a Q-fuzzy normal subgroup of a group G.

Let x and y in G and q in Q.

Now, A ($y^{-1} xy, q$) = A(xyy^{-1} ,q) = A(x, q).

Therefore, A ($y^{-1} xy, q$) = A (x, q) , for all x and y in G and in Q. We note that, (x)={$y^{-1}xy$/ y∈G} is called the conjugate class of x in G. Hence A is constant on the conjugate classes of G.

Conversely, suppose that A is constant on the conjugate classes of G.

Then, A (xy, q) = A (x^{-1} $(xy)x, q$)

$A(\ (x^{-1}\ x)yx, q)$

Therefore, A(xy, q) = A(yx, q) , for all x and y in G and q in Q.

Hence A is a Q-fuzzy normal subgroup of a group G.

E. Theorem

Let A and B be Q-fuzzy subgroups of the groups G and H, respectively. If A and B are Q-fuzzy normal subgroups, then A×B is a Q-fuzzy normal subgroup of G×H.

Proof:

Let A and B be Q-fuzzy normal subgroups of the groups G and H respectively. Clearly A×B is a Q-fuzzy subgroup of G×H, since A and B are Q-fuzzy subgroups G and H.

Let x_1 and x_2 be in G, y_1 and y_2 be in H and q in Q.

Then (x_1,y_1) and (x_2,y_2) are in G×H.

Now,A×B$[(x_1, y_1)(x_2, y_2), q]$ = A×B$((x_1x_2, y_1y_2), q)$
= min {A (x_1x_2, q), B (y_1y_2, q)

$= \min \{ A\,(x_2x_1,q)\ B(y_2y_1,q) \}$,
$= AxB\,((x_2x_1,y_2y_1),q)$
$= AxB.\,[(x_2,y_2)(x_1,y_1),q]$

Therefore, AxB $[(x_1,y_1)(x_2,y_2),q]$ = AxB $[(x_2,y_2)(x_1,y_1),q]$.

Hence AxB is a Q-fuzzy normal subgroup of GxH.

IV. PROPERTIES OF Q-FUZZY COSETS

In P.Pandiammal, R. Natarajan and N.Palaniappan have proved some results on Properties of Anti L-Fuzzy M-cosets of M-groups. This motivated us to examine the results for Q-fuzzy cosets. We have found out that the results perfectly fit with Q-fuzzy cosets. Here we prove the analogue of the Lagrange's theorem.

A. Theorem

Let A be a Q-fuzzy subgroup of a finite group G, then O(A) /O(G).

Proof

Let A be a Q-fuzzy subgroup of a finite group G with e as its identity element. Clearly H ={x∈ G / A (x,q)= A(e,q) } is a subgroup of G for H is a Q-level subset of G. By Lagrange's theorem O(H) / O(G). Hence by the definition of the order of the Q-fuzzy subgroup of G, we have O(A) / O(G).

B. Theorem

Let A and B be two Q-fuzzy subsets of an abelian group G. Then A and B are conjugate Q-fuzzy subsets of the abelian group G if and only if A =B.

Proof

Let A and B be conjugate Q-fuzzy subsets of abelian group G, then for some y in G, we have A(x,q) = B$(y^{-1}xy,q)$, for every x in G and q in Q

= B $(yy^{-1}x,q)$, since G is an abelian group
= B(ex,q)
= B(x,q).

Therefore, A(x,q) = B(x,q), for every x in G an q in Q.

Hence A = B. Conversely, if A = B, then for the identity element e of G, we have, A(x,q) = B$(e^{-1}xe,q)$, for every x in G and q in Q. Hence A and B are conjugate Q-fuzzy subsets of G.

C. Theorem

If A and B are conjugate Q-fuzzy subgroups of the normal group G, then O(A)= O(B).

Proof

Let A and B be conjugate Q-fuzzy subgroups of G.

Now, O(A) = order of { x∈ G / A(x,q) = A(e,q) }
= order of { x∈ G / B$(y^{-1}xy,q)$ = B$(y^{-1}ey,q)$
= order of { x∈ G / B(x,q) = B(e,q)}
= O (B).

Hence, O(A) = O(B).

D. Theorem

If A is a Q-fuzzy subgroup of a group G, then for any a in G the Q-fuzzy middle coset $aAa^{-1})$ of G is also a Q-fuzzy subgroup of G.

Proof

Let A be a Q-fuzzy subgroup of G and a in G. To prove aAa^{-1} is a Q-fuzzy subgroup of G. Let x and y in G and q in Q.

Then, $(aAa^{-1})\,(xy^{-1},q) = A(a^{-1}xy^{-1}a,q)$,
$= A(a^{-1}xaa^{-1}y^{-1}a,q)$,
$= A\ ((a^{-1}xa)(a^{-1}ya)^{-1},q)$
$\geq \min\{A(a^{-1}xa,q), A(a^{-1}ya)^{-1},q)$
$\geq \min\{A(a^{-1}xa,q), A(a^{-1}ya,q)$

Since A is a Q-fuzzy subgroup of G

$= \min\{(aAa^{-1})(x,q),(aAa^{-1})(y,q)\}$

Therefore, $(aAa^{-1}(xy^{-1},q)$

$\geq \min\{(aAa^{-1})(x,q),(aAa^{-1})(y,q)\}$

Hence aAa^{-1} is a Q-fuzzy subgroup of the group G.

V. CONCLUSION

In this paper we have discussed Q-Fuzzy Normal Subgroups, Q-Fuzzy Normaliser and Q-fuzzy subgroups under homomorphism, Interestingly, It has been observed that Q-Fuzzy concepts add an another dimension to the defined fuzzy normal subgroups. This concept can further be extended for new results.

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