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Modelling of the Scheme of the Solution of the Flat Task Theory of Elasticity in Polar Coordinates in the Systems of Computer Mathematics

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ABSTRACT

In this paper the algorithm of an output of the solution of the general two-dimensional task of the theory of elasticity in polar coordinates according to the initial's functions method of V.Z. Vlasov and his son is considered V.V Vlasov. The algorithm is a reference point for automation of an output of the solution of an asymmetric task of the theory of elasticity in cylindrical coordinates and supplements the tool system developed by the author. Behind the scheme offered in the article, it is possible to automate the complete process of an output of the solution of two-dimensional tasks of the theory of elasticity in polar coordinates on a computer. What, in turn, increases the accuracy and reliability of the selected settlement scheme in each specific objective which uses the common analytical decision received in the systems of computer mathematics?

KEYWORDS: a system of computer mathematics (SCM), automation, symbolical solution, polar coordinates.

I. INTRODUCTION

With the intensive development of a computer in researches, analytical methods that allow receiving more generalized and reliable results in comparison with numerical methods are more often applied. However, the use of analytical methods puts a new class of poorly studied problems before researchers. The main problem of analytical methods - complexity of their algorithmizing and programming that in certain cases results in the impossibility of calculations on a computer. The second problem of analytical methods consists in the absence of a community. Each analytical method solves often one specific objective and does not extend to a class of tasks. But despite shortcomings, the relevance of their use increases, especially in mathematical modeling. The systems of computer mathematics which still develop the accelerated rates were developed, trying to solve programming problems of analytical methods. The special contribution to the development of SCM was made by scientists: Victor Aladyev, Vladimir Dyakonov, Bill Schelter, Alexander Matrosov. The offered work is devoted to a question of mathematical modeling of solutions of tasks of the elasticity's theory in the systems of computer mathematics. One of the authors A.G. Ovsky developed on the basis of SCM Maple and Maxima a tool system in which he implemented a method of initial functions of V.Z. Vlasov, V.V. Vlasov for tasks of the theory of elasticity in different coordinate systems. In the article, the algorithm for a polar coordinate system is stated and the example of modeling of a simple task in a system is given. It is a continuation of works [5, 6, 7].

II. PROBLEM STATEMENT AND SOLUTION

The general flat task of the theory of elasticity in polar coordinates of r and φ is considered. It is necessary to find movements of $u(r, \varphi)$, $\upsilon(r, \varphi)$ and tension $\sigma_r(r,\varphi)$, $\sigma_\varphi(r,\varphi)$, $\tau_{r\varphi}(r,\varphi)$ bodies at set initial ($\varphi = 0$) movements of $u(r, 0)$, $\upsilon(r, 0)$ and tension $\sigma_r(r,0)$, $\sigma_\varphi(r,0)$, $\tau_{r\varphi}(r,0)$.

The main equations of the theory of elasticity for polar coordinates of r and φ have an appearance [3]:

$$\begin{cases} \dfrac{\partial\,(r\sigma_r)}{\partial r} - \sigma_\varphi + \dfrac{\partial \tau_{r\varphi}}{\partial \varphi} = 0, \\ \dfrac{\partial\,(r\tau_{r\varphi})}{\partial r} + \tau_{r\varphi} + \dfrac{\partial \sigma_\varphi}{\partial \varphi} = 0, \end{cases} \qquad (1)$$

$$\begin{cases} r\sigma_r = \frac{2G}{1-\nu}\left[r\dfrac{\partial u}{\partial r} + \nu\left(\dfrac{\partial \upsilon}{\partial \varphi} + u\right)\right], \\ r\sigma_\varphi = \frac{2G}{1-\nu}\left[\dfrac{\partial \upsilon}{\partial \varphi} + u + \nu r\,\dfrac{\partial u}{\partial r}\right], \\ r\tau_{r\varphi} = G\left(\dfrac{\partial u}{\partial \varphi} + r\dfrac{\partial \upsilon}{\partial r} - \upsilon\right), \end{cases} \qquad (2)$$

where σ_r, σ_φ and $\tau_{r\varphi}$ - tension;
u, υ - movements;
r - radius; φ - a rotation angle.

Formulas (1) - equilibrium equations in polar coordinates. Expressions (2) - Guk's dependences for flat tension of a body. Dependences (1) - (2) is the system of five differential equations in private derivatives with float factors. It will be transformed into a system of equations convenient for work on SCM by means of replacement:

$$R = r\sigma_r,\ T = r\tau_{r\varphi},\ S = r\sigma_\varphi,\ U = G\,u,\ V = G\,\upsilon. \qquad (3)$$

The simplifying symbolics for $r = e^p$ allowing to receive from (1), (2) systems with constant coefficients:

$$\begin{cases} \dfrac{\partial R}{\partial p} - \Phi + \dfrac{\partial T}{\partial \varphi} = 0, \\ \dfrac{\partial T}{\partial p} + T + \dfrac{\partial \Phi}{\partial \varphi} = 0, \\ R = \frac{2}{1-\nu}\left[\dfrac{\partial U}{\partial p} + \nu\left(\dfrac{\partial V}{\partial \varphi} + U\right)\right], \\ S = \frac{2}{1-\nu}\left[\dfrac{\partial V}{\partial \varphi} + U + \nu\dfrac{\partial U}{\partial p}\right], \\ T = \dfrac{\partial U}{\partial \varphi} + \dfrac{\partial V}{\partial p} - V, \end{cases} \qquad (4)$$

here ratios are considered:

$$\frac{\partial}{\partial r} = \frac{1}{e^p}\cdot\frac{\partial}{\partial p},\ r\frac{\partial}{\partial r} = \frac{\partial}{\partial p}. \qquad (5)$$

Substitution of tension's values of R, S, T with reduction of derivatives for U, V will transform these two equations to a view to the first two equations of a system (4):

$$\begin{cases} \left[2\left(\dfrac{\partial^2}{\partial p^2} - 1\right) + (1-\nu)\dfrac{\partial^2}{\partial \varphi^2}\right]U + \left[(1+\nu)\dfrac{\partial^2}{\partial p\,\partial \varphi} - (3-\nu)\,\dfrac{\partial}{\partial \varphi}\right]V = 0, \\ \left[(1+\nu)\dfrac{\partial^2}{\partial p\,\partial \varphi} + (3-\nu)\,\dfrac{\partial}{\partial \varphi}\right]U + \left[(1-\nu)\left(\dfrac{\partial^2}{\partial p^2} - 1\right) + 2\dfrac{\partial^2}{\partial \varphi^2}\right]V = 0. \end{cases} \qquad (6)$$

For the solution of a system (6) rather identically to satisfy to the first equation of a system having entered function of movements of $F(p, \varphi)$, and to set movements in the form of derivative of this function [2]:

$$U = \left[(1+\nu)\frac{\partial^2}{\partial p\,\partial \varphi} - (3-\nu)\,\frac{\partial}{\partial \varphi}\right]F,\ V = -\left[2\left(\frac{\partial^2}{\partial p^2} - 1\right) + (1-\nu)\frac{\partial^2}{\partial \varphi^2}\right]F. \qquad (7)$$

Substitution (7) in the second equation of a system (6) after conversion gives differential equation of a view:

$$\left(\frac{\partial^2}{\partial p^2}+\frac{\partial^2}{\partial \varphi^2}\right)^2 F-2\frac{\partial^2 F}{\partial p^2}+2\frac{\partial^2 F}{\partial \varphi^2}+F=0. \tag{8}$$

The equations (7) are substituted in expressions for tension of *R*, *S*, *T* from (4). Expressions of tension take a form:

$$\begin{cases} R=2\frac{\partial}{\partial \varphi}\left[\frac{\partial^2}{\partial p^2}-\nu\frac{\partial^2}{\partial \varphi^2}-(3+\nu)\frac{\partial}{\partial p}-\nu\right]F, \\ S=2\frac{\partial}{\partial \varphi}\left[-\frac{\partial^2}{\partial \varphi^2}-(2+\nu)\frac{\partial^2}{\partial p^2}+(1-\nu)\frac{\partial}{\partial p}-1\right]F, \\ T=2\left[\nu\frac{\partial^3}{\partial \varphi^2 \partial p}-\frac{\partial^2}{\partial \varphi^2}-\left(\frac{\partial^3}{\partial p^3}-\frac{\partial^2}{\partial p^2}-\frac{\partial}{\partial p}+1\right)\right]F. \end{cases} \tag{9}$$

Transition to a symbolical form is carried out and algebraic conversions on a tool system are made. Designations by analogy with work [5], but for a derivative are used $\propto=\frac{\partial}{\partial p}$. From a course of differential equations, it is known that if $\frac{\partial}{\partial p}$ formally to consider the constant number, then the solution of the equation (8) will register in the form of [4]:

$$F=\sin(1+\propto)\,\varphi\, f_0+\cos(1+\alpha)\,\varphi\, f_1+\sin(1-\alpha)\,\varphi\, f_2+\cos(1-\alpha)\,\varphi\, f_3. \tag{10}$$

The solution (10) is substituted in ratios (7) and (9), coefficients f_0, f_1, f_2, f_3 are as the solution of a system of the algebraic equations in a symbolical view at $\varphi=0$. At the same time movements (7) and (9) become initial, register:

$$U(p,0)=u_0(p), V(p,0)=\ v_0(p), S(p,0)=s_0(p), R(p,0)=r_0(p).$$

The found coefficients are substituted in (10), the solution (10) is again substituted in (7) and (9). Result the solution of a task of the theory of elasticity through initial functions [6]. Solutions are rewritten in an operator form:

$$\begin{cases} U(p,\varphi)=L_{uu}(\alpha,\varphi)\,u_0(p)+L_{uv}(\alpha,\varphi)\,v_0(p)+L_{us}(\alpha,\varphi)\,s_0(p)+L_{ur}(\alpha,\varphi)r_0(p), \\ V(p,\varphi)=L_{vu}(\alpha,\varphi)\,u_0(p)+L_{vv}(\alpha,\varphi)\,v_0(p)+L_{vs}(\alpha,\varphi)\,s_0(p)+L_{vr}(\alpha,\varphi)\,r_0(p), \\ S(p,\varphi)=L_{\mathrm{su}}(\alpha,\varphi)\,u_0(p)+L_{sv}(\alpha,\varphi)\,v_0(p)+L_{ss}(\alpha,\varphi)\,s_0(p)+L_{sr}(\alpha,\varphi)\,r_0(p), \\ R(p,\varphi)=L_{ru}(\alpha,\varphi)\,u_0(p)+L_{rv}(\alpha,\varphi)\,v_0(p)+L_{rs}(\alpha,\varphi)\,s_0(p)+L_{rr}(\alpha,\varphi)\,r_0(p), \\ T(p,\varphi)=A_u(\alpha,\varphi)\,u_0(p)+A_v(\alpha,\varphi)\,v_0(p)+A_s(\alpha,\varphi)\,\mathrm{s}_0(p)+A_r(\alpha,\varphi)r_0(p). \end{cases} \tag{11}$$

Operators turn out in a tool system automatically, by a call of the relevant library [7]. Example of a call:

```
load(vlas);
operators (D2, polar);
```

Their view for SCM Maxima of fig. 1:

$$Luu = \frac{((\alpha+1)\nu+\alpha+1)\cos((\alpha+1)\varphi) + ((-\alpha-1)\nu-\alpha+3)\cos((\alpha-1)\varphi)}{4}$$

$$Luv = -\frac{((\alpha-1)\nu+\alpha-1)\sin((\alpha+1)\varphi) + ((-\alpha-1)\nu-\alpha+3)\sin((\alpha-1)\varphi)}{4}$$

$$Lus = \frac{((\alpha+1)\nu+\alpha-3)\cos((\alpha+1)\varphi) + ((-\alpha-1)\nu-\alpha+3)\cos((\alpha-1)\varphi)}{8\alpha}$$

$$Lur = \frac{((\alpha-1)\nu+\alpha+3)\sin((\alpha+1)\varphi) + ((-\alpha-1)\nu-\alpha+3)\sin((\alpha-1)\varphi)}{8\alpha}$$

$$Au = \frac{((\alpha^2+\alpha)\nu+\alpha^2+\alpha)\cos((\alpha+1)\varphi) + ((3\alpha-\alpha^2)\nu-\alpha^2+3\alpha)\cos((\alpha-1)\varphi)}{2}$$

$$Lvu = -\frac{((\alpha+1)\nu+\alpha+1)\sin((\alpha+1)\varphi) + ((1-\alpha)\nu-\alpha-3)\sin((\alpha-1)\varphi)}{4}$$

$$Lvv = -\frac{((\alpha-1)\nu+\alpha-1)\cos((\alpha+1)\varphi) + ((1-\alpha)\nu-\alpha-3)\cos((\alpha-1)\varphi)}{4}$$

$$Lvs = -\frac{((\alpha+1)\nu+\alpha-3)\sin((\alpha+1)\varphi) + ((1-\alpha)\nu-\alpha-3)\sin((\alpha-1)\varphi)}{8\alpha}$$

$$Lvr = \frac{((\alpha-1)\nu+\alpha+3)\cos((\alpha+1)\varphi) + ((1-\alpha)\nu-\alpha-3)\cos((\alpha-1)\varphi)}{8\alpha}$$

$$Av = -\frac{((\alpha^2-\alpha)\nu+\alpha^2-\alpha)\sin((\alpha+1)\varphi) + ((3\alpha-\alpha^2)\nu-\alpha^2+3\alpha)\sin((\alpha-1)\varphi)}{2}$$

$$Lsu = -\frac{((\alpha^2+\alpha)\nu+\alpha^2+\alpha)\cos((\alpha+1)\varphi) + ((-\alpha^2-\alpha)\nu-\alpha^2-\alpha)\cos((\alpha-1)\varphi)}{2}$$

$$Lsv = \frac{((\alpha^2-\alpha)\nu+\alpha^2-\alpha)\sin((\alpha+1)\varphi) + ((-\alpha^2-\alpha)\nu-\alpha^2-\alpha)\sin((\alpha-1)\varphi)}{2}$$

$$Lss = -\frac{((\alpha+1)\nu+\alpha-3)\cos((\alpha+1)\varphi) + ((-\alpha-1)\nu-\alpha-1)\cos((\alpha-1)\varphi)}{4}$$

$$Lsr = -\frac{((\alpha-1)\nu+\alpha+3)\sin((\alpha+1)\varphi) + ((-\alpha-1)\nu-\alpha-1)\sin((\alpha-1)\varphi)}{4}$$

$$As = \frac{((\alpha+1)\nu+\alpha-3)\cos((\alpha+1)\varphi) + ((3-\alpha)\nu-\alpha+3)\cos((\alpha-1)\varphi)}{4}$$

$$Lru = -\frac{((\alpha^2+\alpha)\nu+\alpha^2+\alpha)\sin((\alpha+1)\varphi) + ((\alpha-\alpha^2)\nu-\alpha^2+\alpha)\sin((\alpha-1)\varphi)}{2}$$

$$Lrv = -\frac{((\alpha^2-\alpha)\nu+\alpha^2-\alpha)\cos((\alpha+1)\varphi) + ((\alpha-\alpha^2)\nu-\alpha^2+\alpha)\cos((\alpha-1)\varphi)}{2}$$

$$Lrs = -\frac{((\alpha+1)\nu+\alpha-3)\sin((\alpha+1)\varphi) + ((1-\alpha)\nu-\alpha+1)\sin((\alpha-1)\varphi)}{4}$$

$$Lrr = \frac{((\alpha-1)\nu+\alpha+3)\cos((\alpha+1)\varphi) + ((1-\alpha)\nu-\alpha+1)\cos((\alpha-1)\varphi)}{4}$$

$$Ar = \frac{((\alpha-1)\nu+\alpha+3)\sin((\alpha+1)\varphi) + ((3-\alpha)\nu-\alpha+3)\sin((\alpha-1)\varphi)}{4}$$

Figure 1: Differential operators of the solution of the general task of the theory of elasticity in polar coordinates

Operators of the solution of fig. 1 are written in a symbolical form. For receiving the valid submission of the solution, it is necessary to spread out trigonometric functions in ranks, to replace a character α with a derivative $\frac{\partial}{\partial p}$, after to execute operations of differentiation over initial functions [1].

IV EXAMPLE

For the supported wedge under a distributed load of q the common decision (11) will register in a view:

$$\begin{cases} (L_{ru}(\varphi_a) - 2(1+\nu)a\alpha^2 L_{uu}(\varphi_a))u_0 + \\ +(L_{rs}(\varphi_a) - 2(1+\nu)a\alpha^2 L_{us}(\varphi_a))s_0 = 0, \\ (L_{ru}(\varphi_a)u_0 + L_{ss}(\varphi_a)s_0 = \frac{q}{2\delta}e^p, \end{cases} \quad (12)$$

$$a = \frac{E_1\, c}{E\delta}. \quad (13)$$

where φ_a - edge of a wedge;

E_1 - an elastic modulus on edge material stretching;

c - proportionality coefficient on stretching (compression);

E - Jung's module;

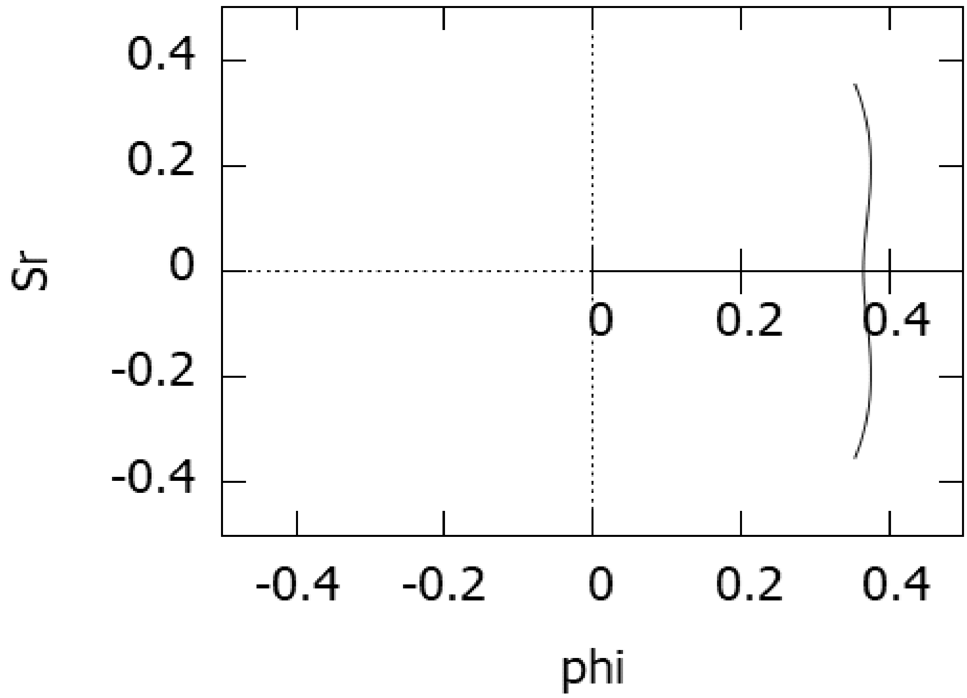
δ - Dirac's function;

$q = const$ - loading.

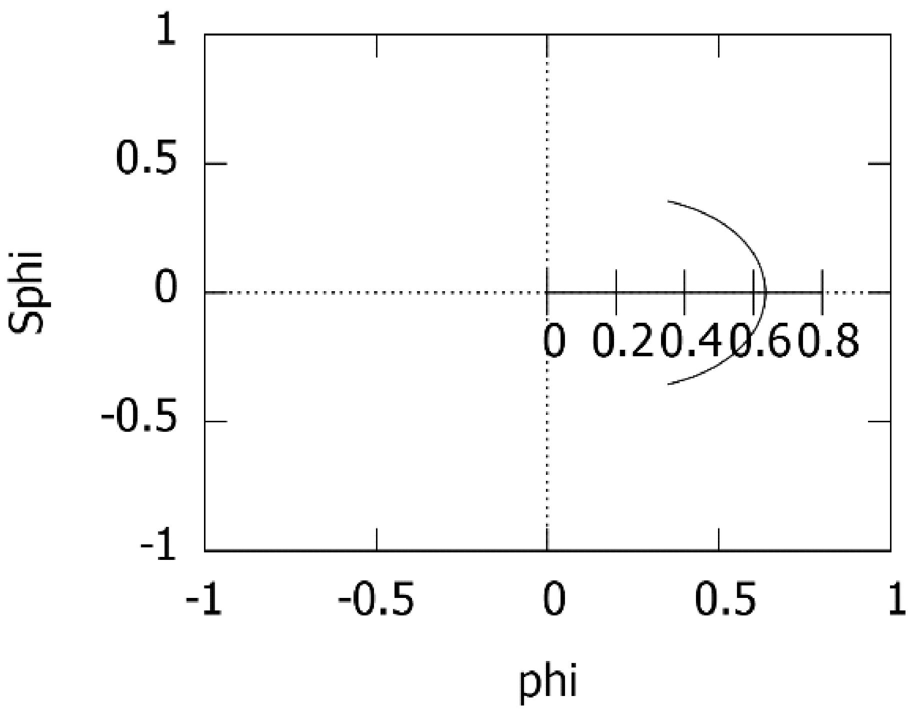
The solution of a system (12) has the following appearance for tension:

$$\begin{cases} \sigma_r = \frac{q}{2(\sin 2\varphi_a + 2a\cos 2\varphi_a)}(\sin 2\varphi_a + a((1+\nu)\cos 2\varphi_a - (1-\nu)cos\ 2\varphi), \\ \sigma_\varphi = \frac{q}{2(\sin 2\varphi_a + 2a\cos 2\varphi_a)}(\sin 2\varphi_a + a((1+\nu)\cos 2\varphi_a + (1-\nu)cos\ 2\varphi), \\ \tau_{r\varphi} = \frac{q}{2(\sin 2\varphi_a + 2a\cos 2\varphi_a)}(1-\nu)a\ sin\ 2\varphi. \end{cases} \quad (14)$$

In a tool system for dimensionless q=1, a=0.3, ν=0.1, $\varphi_a = \frac{\pi}{4}$ diagrams of the solution (14), their view were constructed by fig. 2.



σ_φ



$\tau_{r\varphi}$

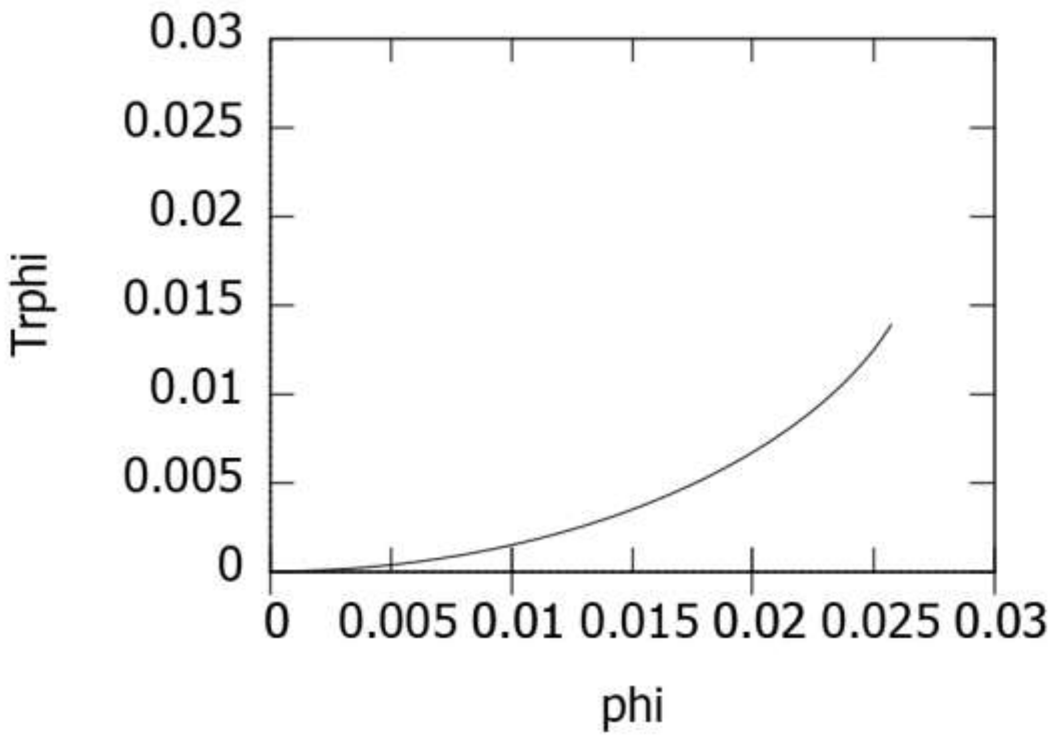


Figure 2: Diagrams of tension for dimensionless parameters

The solution (14) will be agreed with the solution of the elementary theory [3]. It is presented on diagrams visually with different extent of detailing for change of a corner φ from 0 to $\frac{\pi}{4}$.

V. CONSLUSIONS

In this paper the algorithm of an output of the common decision of a two-dimensional task of the theory of elasticity in polar coordinates was provided. The solution is submitted in the uniform of differential operators and is the exact solution of the general task of the theory of elasticity in the mixed setting. For each separate and specifically set task, it is necessary to solve the system of differential equations constructed from boundary conditions, a part of initial functions is selected to satisfy initial conditions of a task, the others are from the system of differential equations. All these processes are easily implemented in a tool system which operates with the received common decision for the coordinate system set by the user.

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