[Volume 5 issue 7 July 2017]
Page No.1783-1789 ISSN :2320-7167

INTERNATIONAL JOURNAL OF MATHEMATICS AND COMPUTER RESEARCH

A Model for Integration Maintenance Planning & Quality Control

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Abstract
The performance of a production system depends on the breakdown-free operation of equipment and processes. Maintenance and quality control play an important role in achieving this goal. In addition to deteriorating with time, equipment may experience a quality shift (i.e. process moves to out-of-control state), which is characterized by a higher rejection rate and a higher tendency to fail. This paper develops an integrated model for joint optimization of preventive maintenance interval and control parameters incorporating the Taguchi loss function. We consider two types of maintenance policies: minimal corrective maintenance that maintains the state of the equipment without affecting the age and imperfect preventive maintenance that upgrades the equipment in between 'as good as new' and 'as bad as old' condition. The proposed model enables the determination of the optimal value of each of the four decision variables, i.e. sample size (n), sample frequency (h), control limit coefficient (k), and preventive maintenance interval (t_{PM}) that minimizes the expected total cost of the integration per unit time. A numerical example is presented to demonstrate the effect of the cost parameters on the joint economic design of preventive maintenance and process quality control policy. The sensitivity of the various parameters is also examined.

Keywords: preventive maintenance; process quality policy; integrated model; cost minimization

Introduction

The performance of a production system strongly depends on the breakdown-free operation of equipment and processes. The performance can be improved if these breakdowns can be minimized in a cost-effective manner. Maintenance and quality control play important roles in achieving this goal. An appropriate Preventive Maintenance (PM) policy not only reduces the probability of machine failure but also improves the performance of the machine in terms of lower production costs and higher product quality. Similarly, an appropriately designed quality control chart may help in identifying any abnormal behaviour of the process, thereby helping to initiate a restoration action. However, both PM and quality control add costs in terms of down time, repair/replacement, sampling, inspection, etc. Traditionally, these two activities have been optimized independently. However, researchers have shown that a relationship exists between equipment maintenance and process quality [1]. and joint consideration of these two shop-floor policies may be more cost-effective in improving the performance of the production system. Recent literature indicates that such joint consideration has started receiving attention from the research community

Notation

$ARL2_E$	average run length during an out-of-control period owing to external reasons
$ARL2_{M/C}$	average run length during an out-of-control period owing to machine failure
ARL1	average run length during an in-control period
K	control limit coefficient

C_{lp} cost of lost production(Rs/job)
C_{Frej} cost of rejection while the process moves out-of –control
$C_{resetting}$ cost of resetting
prd_E evaluation period
$[C_{CM}]_{FM_1}$ expected cost of corrective maintenance (CM) owing to failure model
C_{PM} expected cost of preventive maintenance (PM)
$E[T_{Cycle}]$ expected cycle length
T_1 expected time to determine occurrence of assignable cause
$E[T_{restore}]$ expected time to restore the process which may be moved out-of-control owing to machine degradation
$[TCQ]_{process-failure}$ expected total cost of quality owing to process failure
$([TCT]_{Maintenance*Quality})$ expected total cost per unit time of the system
C_{FCCM} fixed cost per CM(Rs/component)
C_{FCPM} fixed cost per PM(Rs/preventing component)
LC maintenance personnel cost(Rs/h of preventing machine)
MT_{CM} mean time required for corrective repair (h)
MT_{PM} mean time required for preventive repair (h)
N_f number of failure
t_{PM} preventive maintenance interval
β_E probability of nonconforming items produced owing to external cause
$\beta_{M/C}$ type II error probability owing to machine failure mode 2
P_{FM_1} probability of occurrence of failure owing to failure mode 1
P_{FM_2} probability of occurrence of failure owing to failure mode 2
T_s time to sample and chart one item
λ process failure rate
λ_1 failure owing to external causes
λ_2 failure owing to machine degradation
PR production rate(job/h)
n sample size
h sample frequency
α type I error probability machine failures are divided into two failure modes[2]
FM_1(failure mode1): leads to immediate breakdown of the machine.
FM_2(failure mode2): leads to reduction in process quality owing to shifting the process mean.
The FM_1 can be considered as partial failure and are defined by black and mejabi(1995).[3]

3. Model description

If FM_1 occurs, it immediately stops the machine. Corrective actions are taken to repair the machine to its operating condition. Thus, the expected cost of corrective maintenance ($[C_{CM}]_{FM_1}$) includes the cost of down time, and the cost of repair/restore action. FM_2 affects the functionality of the machine and in turn increases the rejection level. In other words, FM_2 affects the process rejection rate. It is assumed that whenever FM_2 is detected, the process is stopped immediately, and corrective actions are taken to repair the process back to the normal condition. Apart from failures owing to FM_2, the process may also deteriorate owing to external causes (E) such as environmental effects, operators' mistakes, use of wrong tool, etc. The process is reset to the in-control state if an external event 'E' is detected. [4].

The detection of FM_2 or an external cause is achieved by monitoring the process. In this paper a control chart mechanism is considered for process monitoring. Let the design parameters of the control chart be sample size (n), length of the sampling interval (h), and coefficient (k) that determines the distance between the center line and the control limits. Thus, the expected total cost of process failure owing to FM_2 and external events($[TCQ]_{process-failure}$) considering the cost of down time, cost of rejections owing to process shifts, cost of repair/resetting, cost of sampling/inspection, cost of investigation of false alarm, and cost of deviation from the target value of the CTQ. Apart from the above corrective actions, the machine can undergo preventive maintenance (PM) to minimize the unplanned downtime losses. In this paper, imperfect preventive maintenance has been considered. This means that the PM upgrades the equipment to a state between the as-good-as-new and as-bad-as-old conditions. The frequency of failures can be significantly decreased through PM, i.e. it reduces the occurrence of both FM_1 and FM_2. Reduction in FM_2 reduces the quality costs related to the out-of-control operation. However, PM also consumes some resources and productive machine time that could otherwise be used for production. The expected cost of PM (C_{PM}) comprises the cost of downtime and cost of performing preventive maintenance actions.

4. Optimization model

The problem is to determine the optimal values of the decision variables (n, h, k, and t_{PM}) that minimize the expected total cost per unit time of the system ($[TCT]_{Maintenance*Quality}$). Recall that the age of the equipment after a PM is reduced according to the restoration factor. The expected total cost per unit time of preventive maintenance and control chart policy ($[TCT]_{Maintenance*Quality}$) is the ratio of the sum of the expected total cost of the process quality control ($[TCQ]_{process-failure}$), expected total costs of the preventive maintenance (C_{PM}) and expected total cost of machine failure ($[C_{CM}]_{FM_1}$) to the evaluation time. the expected total cost per unit time for the integrated model is given as:

$$[TCT]_{Maintenance*Quality} = f(n,h,k,t_{PM}) = \frac{1}{prd_E}\left([C_{CM}]_{FM_1} + C_{PM} + [TCQ]_{process-failure}\right) \qquad 1$$

And we have:

Minimize $[TCT]_{Maintenance*Quality}$

Subject to

$$a_1 \le n \le b_1$$
$$a_2 \le h \le b_2$$
$$a_3 \le k \le b_3$$
$$a_4 \le t_{PM} \le b_4$$

n,h,k, $t_{PM} \ge 0$

that a_i and b_i is the value of down and up band for decision variables

4.1. Expected cost model for corrective maintenance owing to FM_1 and preventive maintenance ($[C_{CM}]_{FM_1}$)

To estimate the expected cost of corrective maintenance owing to FM_1 and preventive maintenance, the analyst must have the following information:

- The amount of time that the equipment is expected to be down each time CM/PM is required. This can include the time to perform the maintenance as well as any logistical delays (i.e. waiting for labor and/or materials required).

The cost of CM/PM including the downtime, labor, materials, and other costs.

The degree to which the equipment will be restored by CM/PM (e.g. 'as good as new,' 'as bad as old,' or 'Imperfect'). This is quantified in terms of a restoration factor. The restoration factor can be determined empirically or based on expert judgment as calculated in and Lad and Kulkarni (2010) respectively.[1],[5]

The probability that the equipment will fail owing to a particular failure mode
The expected cost of minimal corrective maintenance owing to FM_1 is given as:

$$[C_{CM}]_{FM_1} = \{MT_{CM} \,.\, [PR \,.\, C_{lp} + LC] + C_{FCCM}\} \times P_{FM_1} \times N_f \qquad 2$$

where $\{MT_{CM} \,.\, [PR \,.\, C_{lp} + LC] + C_{FCCM}\}$ is the down time cost owing to corrective maintenance and $N_f$is depends to t_{PM}.

4.2 The expected total cost of preventive maintenance action of component will be:

$$C_{PM} = \{MT_{PM} \,.\, [PR \,.\, C_{lp} + LC] + C_{FCPM}\} \times \frac{prd_E}{t_{PM}} \qquad 3$$

Where $\{MT_{PM} \,.\, [PR \,.\, C_{lp} + LC] + C_{FCPM}\}$ is the down time cost owing to preventive maintenance and $\frac{prd_E}{t_{PM}} = N_{PM}$ the number of preventive maintenances[1]

4.2. Expected cost model for quality loss owing to process failure ($[\mathbf{TCQ}]_{\mathbf{process-failure}}$):

In this section we first derive the expression for the expected cycle length $E[T_{Cycle}]$ and then for the expected total cost of process failure ($[\mathbf{TCQ}]_{process-failure}$**),** The expected cycle length has been defined as the expected time between the start of successive in-control periods. There are costs incurred during the in-control period owing to sampling the process, defectives produced, and false alarms. When the process goes out of control, we assume that it cannot return to the in-control state without intervention. Again, there are costs incurred owing to sampling and increased level of defectives produced, as well as cost of searching for the cause, restoring the system, and downtime. Upon restoring the system, one quality cycle is completed and the next cycle begins.

4.3. ($[\mathbf{TCQ}]_{\mathbf{process-failure}}$):

the expected cost of detecting the assignable cause;

(2) the cost of sampling
(3) the expected cost of operating while in out-of-control state
(4) the proportion of cost of restoring in the case of process shifts owing to machine degradation and cost of resetting in the case of process shifts owing to external reasons.
Let C_F be the fixed cost per sample of sampling and C_V be the variable cost per unit sampled. Thus, the expected cost per cycle for sampling is the sum of the fixed cost per sample and variable cost per unit sampled, and is given as:[5]bai and lee

$$E[C_{sampling}] = (C_F + C_V . n) \times \left(ARL2_{M/C} \times {}^{\lambda_2}/_{\lambda} + ARL2_E \times {}^{\lambda_2}/_{\lambda} \right) \qquad 4$$

the proportion of cost of restoring in the case of process shifts owing to machine degradation:

$$E[c_o]_{M/C} = (PR \times P_{M/C} \times C_{Rej}) \times \{ (h+n.T_s) \times (ARL2_{M/c} \times {}^{\lambda_2}/_{\lambda} + ARL2_E \times {}^{\lambda_1}/_{\lambda}) - \tau + T_1)\} \times \left({}^{\lambda_2}/_{\lambda} \right) \qquad 5$$

cost of resetting in the case of process shifts owing to external reasons:

$$E[c_o]_E=(PR\times P_E \times C_{Rej})\times \{ (h+ n.T_s) \times (ARL2_{M/c} \times {}^{\lambda_2}/_{\lambda} +ARL2_E \times {}^{\lambda_1}/_{\lambda}) -\tau+T_1)\}\times ({}^{\lambda_1}/_{\lambda}) \quad 6$$

$P_{M/C}$ and P_E is the probability of non-conforming products owing to machine degradation and external reason:

$$P_{M/C}=1- Pr(LSL\leq X\leq USL)=1-Pr(\frac{LSL-(\mu+\delta_{M/C})}{\sigma} \leq N(0,1) \leq \frac{USL-(\mu+\delta_{M/C})}{\sigma}) \quad 7$$

$$P_E=1- Pr(LSL\leq X\leq USL)=1-Pr(\frac{LSL-(\mu+\delta_E)}{\sigma} \leq N(0,1) \leq \frac{USL-(\mu+\delta_E)}{\sigma}) \quad 8$$

LSL and USL are the lower and upper process specification limits.

Let $C_{resetting}$ be the cost for finding and resetting the assignable cause owing to external reasons, downtime if process ceases functioning, and for finding and resetting the process. The expected value of $C_{resetting}$ can be calculated as:

$$E[C_{resetting}]=[C_{resetting} \times T_{resetting}]\times({}^{\lambda_1}/_{\lambda}) \quad 9$$

The expected cost of corrective maintenance action owing to failure mode FM_2 of the component and for finding and repairing the assignable cause owing to machine failure is given by:

$$E[c_{Repair}]_{FM_2}=\{(MT_{CM}).[PR.C_{lp}+LC]+C_{FCCM}\}\times({}^{\lambda_2}/_{\lambda}) \quad 10$$

The expected cost of corrective maintenance owing to FM_2 includes the cost of lost production, labor cost, and fixed cost per corrective maintenance during the time to repair.

Adding Equations (4), (5), (6), (9) and (10) gives the expected cost of process failure per cycle as:

$$E[C_{process}]= E[C_{sampling}]+ E[c_o]_{M/C}+ E[c_o]_E+ E[C_{resetting}]+ E[c_{Repair}]_{FM_2} \quad 11$$

We assume that process failure is repetitive in nature, i.e. every time when the process moves out-of-control from the in-control state and is again restored, it will take the same expected time (having fixed expected cycle length). If there are M process failure cycles in a given evaluation period, the expected process quality control cost for the evaluation period will be:

$$[TCQ]_{process-failure}=[E(C_{process})]\times M \quad 12$$

Where M is:

$$M=\frac{prd_E}{E[T_{cycle}]} \quad 13$$

4.4. Calculation of process-cycle length

The expected cycle time is the sum of the following: (1) the time , (2) the time to analyze a sample and chart the result, (3) the time until the chart gives an out-of-control signal, (4) the time to discover and analyze the assignable cause, and (5) the time to reset the process if failure is due to external causes or to repair the process if failure is due to FM_2. It is assumed that the in-control time follows a negative exponential distribution with mean=${}^1/_{\lambda}$. In fact The expected cycle time is the in-control time and out-of-control time. It is from the in-control state until the next in-control time.

ARL2 is the average run length when the process has shifted to an out-of-control state, and if the sampled statistics are independent:

$$ARL2_{M/C}={}^{1}/_{1-\beta_{M/C}} \quad 14$$

$$ARL2_{E}={}^{1}/_{1-\beta_{E}} \quad 15$$

β= Pr(in control signal | process is out of control)

$$\beta_{M/C}=Pr\left(LCL \leq \bar{x} \leq UCL \middle| \mu=\mu_1=\mu_0+\delta_{M/C}\sigma_p\right) \quad 16$$

$$\beta_{E}=Pr\left(LCL \leq \bar{x} \leq UCL \middle| \mu=\mu_1=\mu_0+\delta_{E}\sigma_p\right) \quad 17$$

We have $\bar{X}\sim N(\mu,\sigma_P^2/n)$, then LCL and UCL are calculated:

$$UCL=\mu_0+k\sigma_p/\sqrt{n} \quad 18$$

$$LCL=\mu_0-k\sigma_p/\sqrt{n} \quad 19$$

Then we will have:

$$\beta_{M/C}=F\left(\frac{UCL-(\mu_0+\delta_{M/C}\sigma_p)}{\sigma_p/\sqrt{n}}\right) - F\left(\frac{LCL-(\mu_0+\delta_{M/C}\sigma_p)}{\sigma_p/\sqrt{n}}\right) \quad 20$$

$$\beta_{E}=F\left(\frac{UCL-(\mu_0+\delta_{E}\sigma_p)}{\sigma_p/\sqrt{n}}\right) - F\left(\frac{LCL-(\mu_0+\delta_{E}\sigma_p)}{\sigma_p/\sqrt{n}}\right) \quad 21$$

Where F denotes the standard normal cumulative distribution function. This is reduced to

$$\beta_{M/C}=F(k-\delta_{M/C}\sqrt{n})-F(-k-\delta_{M/C}\sqrt{n}).$$

For a sample of n units, the time to analyze the sample and chart the result is given by = $n.T_s$

Then the out-of-control time is:

$$(h+n.T_s)\times(ARL2_{M/c}\times{}^{\lambda_2}/_{\lambda}+ARL2_E\times{}^{\lambda_1}/_{\lambda}) \quad 22$$

Let T_1 be the expected time to investigate the assignable cause and $E[T_{restore}]$ be the expected time to restore the process which may machine degradation or owing to external causes. Accordingly, the expected time to repair or reset the process ($E[T_{restore}]$) is considered and can be calculated as:

$$E[T_{restore}]=(T_{resetting}\times{}^{\lambda_1}/_{\lambda}+MT_{CM}\times{}^{\lambda_2}/_{\lambda}) \quad 23$$

The expected cycle time is[4]

$$E[T_{Cycle}]={}^{1}/_{\lambda}+\{(h+n.T_s)\times(ARL2_{M/c}\times{}^{\lambda_2}/_{\lambda}+ARL2_E\times{}^{\lambda_1}/_{\lambda})\}+T_1+E[T_{restore}] \quad 24$$

5. Failure rate of machine and process

In this paper, we consider machine failures in terms of a machine operating with a degraded functionality and the sudden breakdown which ceases the machine operation. The probability of occurrence of machine failures is captured from past failure data. Similarly, the process may fail because of machine degradation or some external causes as mentioned above. Let the rate of failure owing to machine degradation be λ_2 and that owing to external causes be λ_1. Thus, the process failure rate (λ) is the sum of failure rates owing to machine degradation and owing to assignable causes. It can be written as:

The process failure rate owing to machine degradation can be calculated as[7]

$$\lambda_2=\frac{1}{prd_E}(N_f) \quad 25$$

and the process failure rate owing to assignable causes is calculated as:

$$\lambda_1 = \frac{1}{MTTF} \qquad 26$$

Where MTTF is the Mean Time between process failure.

Model for number of failures ($\mathbf{N_f}$) of a machine for a given evaluation period as a function of preventive maintenance interval ($\mathbf{t_{PM}}$). we have $N_f = a\ (t_{PM})^b$ [6]. The value of a,b are estimated by regression between N_f , t_{PM}.

Statistical process control (SPC) model without maintenance

This model has been investigated a lot in the literature. The expected cycle length and the expected cost of control chart are given as[7].

$$E[T_{cycle}]_{SPC} = {}^{1}/_{\lambda_E} + \{(h+nT_S) \times (ARL2)_E\} - \tau + T_1 + T_{reset} \qquad 27$$

And the cost of quality control function is:

$$C_{SPC} = \frac{(C_F + C_V.n).({}^{1}/_{\lambda_E}) + \{(h+nT_S) \times (ARL2)_E\}}{h} + (\alpha.\ PR\ .C_{Rej})\ .({}^{1}/_{\lambda_E} + (PR \times P_E \times C_{Rej}).\ (h\ +nT_S)\ (ARL2)_E\ +\ (C_{resetting} \times T_{resetting})$$

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Therefore the expected total cost per unit time for the SPC model is given as:

$$CPUT_{SPC} = \frac{C_{SPC}}{E[T_{cycle}]_{SPC}} \qquad 29$$

6. Numerical example

consider a single component operating as a part of a machine. Machine failure is assumed to follow a two-parameter Weibull distribution with $\eta = 1000$ and $\beta = 2.5$ as the characteristic life and shape parameter respectively. The machine considered here is expected to operate for three shifts of seven hours each for six days in a week. Time to execute a preventive maintenance action 7 time units and time to execute corrective maintenance action 12 time. The time to failure for the component was obtained through simulation. The "kijimas" model was used to calculate the virtual age of the component after corrective and preventive action.[8]

Table 1: the value of parameters

parameters	C_v	C_F	$T_{resetting}$	T_1	T_0	T_s	$\delta_{M/C}$	δ_E
value	50	100	2	1	1	$\frac{20}{60}$	0/6	1/5
parameter	PR	C_{reset}	LC	C_{Lp}	C_{FCPM}	C_{FCCM}	$C_{false-Alare}$	C_{Rej}
value	10	5000	500	400	1000	10000	1200	2500

Maple 13 has been used to solve the optimization problem:

$(n^*, k^*, h^*, t_{PM}{}^*) = (12, 1.80, 6, 652)$

$f^*(12,1.80,6,652) = 112$

6.1 the numerical example for Statistical process control (SPC) model without maintenance :

For Statistical process control (SPC) model without maintenance we will have**:**

$(n^*, k^*, h^*) = (11, 3.44, 9)$

$f^*(11, 3.44, 9) = 359.8$

References

1. Pandey, D., Kulkarni, 2010. A model for optimal maintenance interval incorporating the cost of rejections in manufacturing International Journal of Advanced Operations Management
2. Lad, B.K. and Kulkarni, M.S., 2008. Integrated reliability and optimal maintenance schedule design: a life cycle cost based approach. International Journal of Product Life Cycle Management, 3 (1), 78–90.
3. Black, J. and Mejabi, O., 1995. Simulation of complex manufacturing equipment reliability using object oriented methods. Reliability Engineering and System Safety, 48 (1), 11–18.
4. Ben-Daya, M. (1999). Quality and reliability Improvement in Multistage Production Systems. Proceedings of The 5th ISSAT International Conference on Reliability and Quality Design, pp. 229-233.
5. Bai, D.C. and Lee, K.T., 1998. An economic design of variable sampling interval X-control charts. International Journal of Production Economics, 54, 57–64.
6. Yeung, T.G., Cassady, C.R., and Schneider, K., 2008. Simultaneous optimization of X-control chart and age-based preventive maintenance policies under an economic objective. IIE Transactions, 40, 147–159.
7. Lorenzen, T. J., & Vance, L. C. (1986). The economic design of control charts: A unified approach. Techno metrics, 28(1), 3–10.
8. Kijima, M., 1989. Some results for repairable systems with general repair. Journal of Applied Probability, 26, 89–102.