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Graphs in Game Theory

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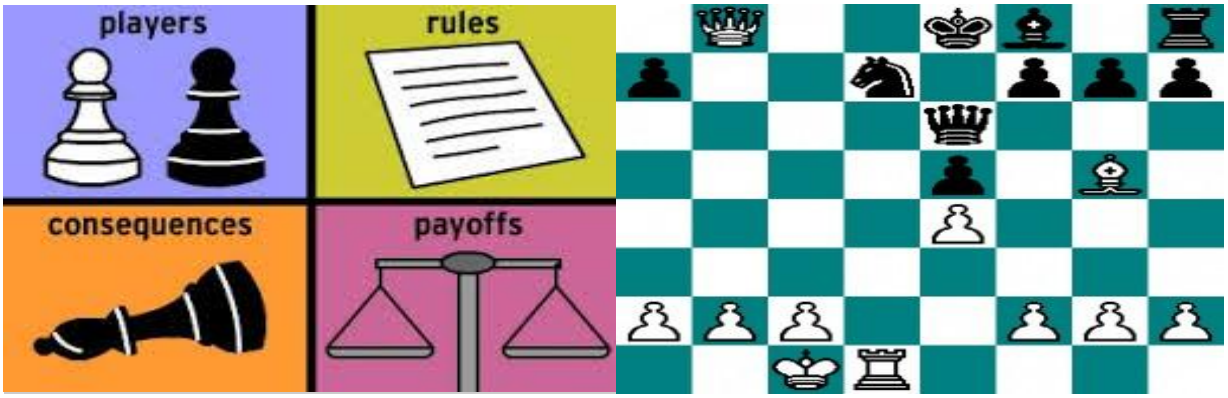
Abstract

This paper describes the application of Graphs in Game Theory. Game Theory can be applied to problems in Engineering ,economics,and war science to find the optimal way of performing certain tasks in competitive environment.This paper describes the applicability of Game Theory in operation Research using Graph Theoriticaltools.They deal with the representation of system structure by means of a connectedgraph and subsequent analysis through appropriate study of digraph.Game theory is the corner stone of combinatorial operation Research.The central topic in Discrete Mathematics is Graph Theory and the language of Graph Theory allows us to visualize combinatorial problems diagrametically .In real problems of operation Research the theory of Games provides an approach rather than complete analysis. Graph Theory is applicable to very special but important classes of Games

Key words: Introduction,Representation of Game Digraph,Characteristicsof game tree

Introduction

The general idea of game theory is same as one as associated with parlor games such as chess ,bridge, and checkers.A.game may be played between two persons known as two person games and a game among more than two persons is known as n-person game.A game is a natural representation of game between two persons with perfect information with out chance moves.The vertices represent the positions also called the states in the game and edges represent moves.If the game can be transformed from vertex u to vertex v is more permissible underrules of game then there is a directed edge from vertex u to v.



Playing carom board with own self interest of winning is a pleasurable game because each player is interested in securing more points than other.A kind of copetativecomflict in which some one is declared as winner and the other as the loser is a game.Theself interest dominated rationality is the driving force for all such struggles.Thus game theory is a special type of decision theory in which one's own choice of action is called strategy can be determined after taking in to account of all possible strategies of his opponents.The final outcome depends upon the strategy and skill of the player concerned

Two person zero and positive sum game:The final out come of actions and reactions of all participants ends with a definite outcome may be positive or negative or zeroBy positive end result means a gain, the negative end means a loss and by zero end means no gain no loss situation.

Representation of Game digraph

The simplified version of a game is called NIM. Two piles of sticks are given and players A and B takes turns, each taking any number of sticks from any pile. The player who takes the last stick wins, since the finite quantity of sticks will be eventually exhausted. For further simplification if the starts with two piles containing two sticks each the complete game can be described by digraph. Each stage of the game is described by ordered pair of labels (x,y) indicating the number of sticks in the first and second pile respectively.

Properties if a digraph:

1. The digraph has unique vertex with zero in degree. This vertex represents the starting position in the game is called starting vertex.
2. There are one or more vertices with zero out degree. These corresponds to the closing positions in the game called closing vertices.
3. A game digraph is a connected acyclic digraph.

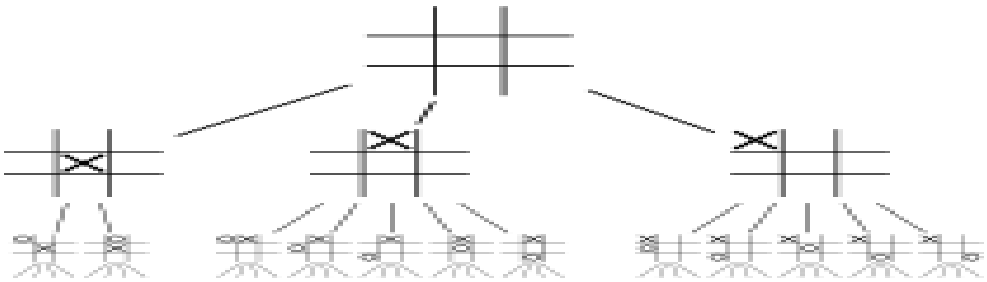
Each directed path from the starting vertex to closing vertex represents the complete play of the game. This path consist of edges representing the moves of two players alternatively.

A game tree (also called the extensive form) is a graphical representation of a sequential game. It provides information about the players, payoffs, strategies, and the order of moves. The game tree consists of nodes (or vertices), which are points at which players can take actions, connected by edges, which represent the actions that may be taken at that node. An initial (or root) node represents the first decision to be made. Every set of edges from the first node through the tree eventually arrives at a terminal node, representing an end to the game. Each terminal node is labeled with the payoffs earned by each player if the game ends at that node.

Game tree representation:

Trees can be used to analyze certain types of games such as tic-tac-toe, nim, checkers and chess ,etc. each of these games two players take turns moving moves ,the vertices of the trees represent the positions of the game, the edges represent the legal moves between these positions. Game trees are usually large hence they can be simplified by representing all symmetric positions of the game by same vertex. The root represents the starting position. The vertices at even levels can be represented by boxes and vertices at odd intervals can be represented by circles. When the game is in the position represented by a vertex at an even level it is the first players move when the game is in the position represented by a vertex at odd level it is the second players move.

The leaves of a game tree represent the final position of the game. We assign a value to each leaf indicating the payoff to the first player if the game terminates in the position represented by the leaf.



For games that are win-lose we labeled a terminal vertex represented by a circle with 1 to indicate a win by the first player and we label a terminal vertex represented by a box with -1 to indicate a win by the second player.

For win-lose game if we assign values to terminal vertices so that the

longer the value the better outcome for the first player. For games where draws are allowed we label the terminal vertex corresponding to draw vertex with O.

Preliminaries:

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Game:A game is a formal description of a strategic situation.
Game theory:Game theory is the formal study of decision-making where several players must makechoices that potentially affect the interests of the other play
Strategy: A strategy means a set of rules that tells a player how to select moves to win the game. Optimal strategy: For the first player is the strategy that maximizing the payoffs to the player and for the second player that minimizes the payoffs
Zero-sum game;A game is said to be zero-sum if for any outcome, the sum of the payoffs to all players iszero. In a two-player zero-sum game, one player's gain is the other player's loss, so theirinterests are diametrically opposed
Value of vertex in game tree:

It Is defined as

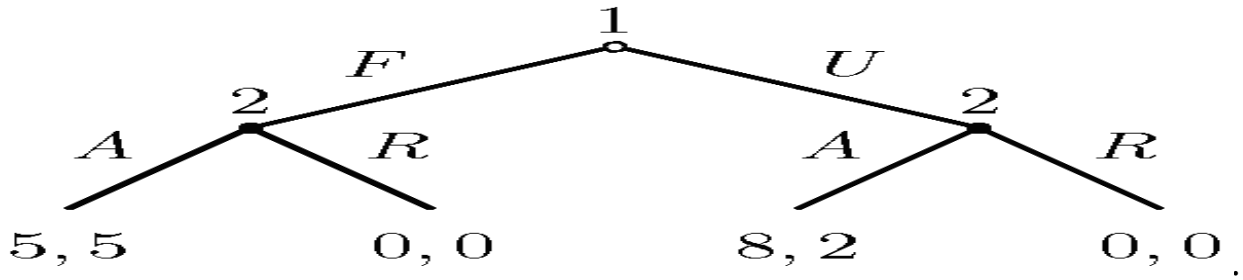
1. The value of a leaf is the payoff to the first player when the game terminates in the position represented by this leaf.
2. The value of vertex at an even level is the maximum of the values of the children and at an odd level is the minimum of values of children.

Techniques for game tree:

- Alfa beta pruning : eliminates much computation by pruning portions of the game tree that cant affect the ancestor vertices.
- Evaluation function: Which estimates the values of internal vertices in game tree when it is not sensible to complete these values exactly.

Tree Diagrams of sequential Games

A sequential game is a game where each player takes turns making decisions instead of making them at the same timeThe diagram below is a sequential game mapped out in a form called a "game tree".



This is a simple model where two actors make a choice between two strategies in a sequential order. Player 1 goes first and chooses between F and U, then Player 2

chooses between A and R. The outcomes of each strategy are the numbers given after player two makes his choice, the first number in the parenthesis is the outcome for Player 1 and the second is the outcome for Player 2. In this case, both players know the options and outcomes of their choices and their opponent's choices as well. Using the looking forward and reasoning back strategy, player 1 can see that player two will never choose R. Five, is a better outcome for player 2 than zero and two is a better outcome than zero. Therefore, player 1 can choose between an outcome of five and an outcome of eight. The outcome of this game will always be (8, 2) because player 1 can move first and knows the outcome of each move he makes. Player 2 is essentially helpless in this game

Characteristics of Game Tree

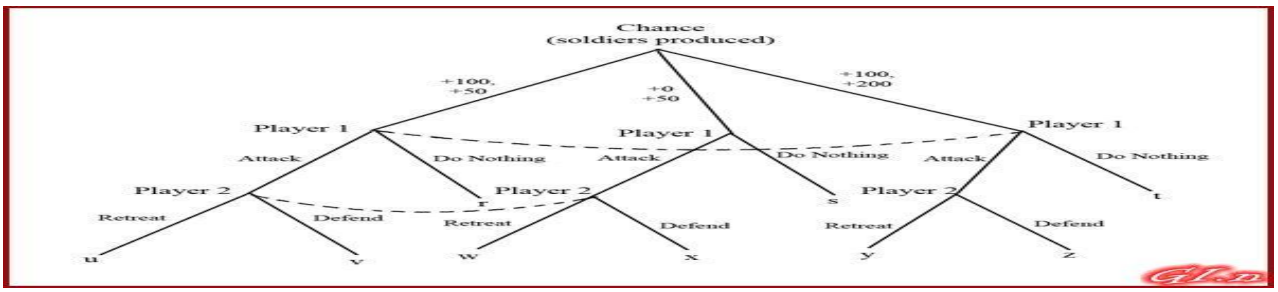
A game tree has the following basic components: Nodes and branches.

A node specifies where we are in the game, and is labeled by the player whose turn it is to move.

A branch takes us from one node to another, and is labeled by the action corresponding to that branch.

A game tree starts with a single initial node, and all pathsfromnode to node (connected by branches) ends with a terminal node. The terminal node specifies payoffs toeach of the players.

If a player whose turn it is to move does not know where he/she is in the game tree, then two or more nodes are connected by a dotted line . This is called an information set.



Conclusion

Game theory is the formal study of conflict and cooperation. Game theoretic conceptsapply whenever the actions of several agents are interdependent.Game theory is the corner stone of combinatorial operation Research. Graph Theory is applicable to very special but important classes of Games

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