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Polya-Aeppli Noncentral Chi-Square Process and its Applications in Risk Analysis

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ABSTRACT

In this paper we introduce Polya-Aeppli non-central chi-square process as a mixed Polya-Aeppli process with mixing random variable having non-central chi-square distribution. We derive expression for PMF and discuss several properties. We consider a risk model with Polya-Aeppli non-central chi- square process as the counting process. The joint distribution of the time to ruin and deficit at the time of ruin is derived. The differential equation of the ruin probability is given. As example, we consider the case of exponentially distributed claims.

KEYWORDS: Polya-Aeppli noncentral chi-square process, Ruin probability.

1 Introduction

Polya-Aeppli process was introduced by Minkova(2004) as a compound Poisson process with geometric compounding distribution. It is a generalization of homogeneous Poisson process and is used to model over-dispersed count data. In order to allow a for lack of homogeneity, some random variation is introduced in the parameter λ (see Minkova(2013))of Polya-Aeppli process. This leads to the notion of a mixed Polya-Aeppli process. It is a modification of Polya-Aeppli process. One reason of the interest on these mixed Polya-Aeppli process lies on the fact that they are over-dispersed relative to Polya-Aeppli process and offer more flexibility than Polya-Aeppli process.

Recently, many researchers used mixed Polya-Aeppli process as a claim counting process in risk modeling. I-Polya process was introduced by Minkova (2011) as a mixed Polya-Aeppli process with gamma mixing distribution. It is a generalization of the classical Polya process. Lazarova and Minkova(2015) studied Polya-Aeppli process with shifted gamma mixing distribution and called it I-Delaporate process. If $\rho = 0$, I-Delaporate process reduces to Delaporate process.

In this study we, introduce a new mixed Polya-Aeppli distribution which is called the Polya-Aeppli non-central chi-square distribution. It is a mixture of Polya-Aeppli distribution by mixing the Polya-Aeppli distribution and non-central chi-square distribution. Then we define a counting process with Polya-Aeppli non-central chi-square distribution and consider the risk model with Polya-Aeppli non-central chi-square counting process . The motivation behind to make a choice of non-central chi-square distribution as the mixing distribution is that it can view as a Poisson mixture of certain gamma distribution and it has various financial applications.

2. Preliminary Results

The Polya-Aeppli distribution with parameters λ and ρ is specified by the PMF:

$$P(N=m) = e^{-\lambda}, \quad m=0$$

$$= e^{-\lambda}\sum_{i=1}^{m}\binom{m-1}{i-1}\frac{[\lambda(1-\rho)]^i}{i!}\rho^{m-i}, \; m=1,2,\ldots$$

The PGF of Polya-Aeppli distribution is given by

$$\Psi_1(s) = e^{-\frac{\lambda(1-s)}{(1-\rho s)}}$$

The factorial moment of order k of Polya-Aeppli distribution is given by

$$\mu_{(k)} = \frac{1}{(1-\rho)^k} \sum_{j=1}^{k} \frac{k!\binom{k-1}{j-1}(\lambda)^j \rho^{k-j}}{j!}. \tag{1}$$

For a thorough discussion of Polya-Aeppli distribution, see Johnson et al.(2005) and Minkova (2002).

The non-central chi-square distribution with r degrees of freedom and non-centrality parameter δ is denoted with PDF:

$$f(x) = \sum_{k=0}^{\infty} \frac{e^{-\frac{(\delta+x)}{2}} (\frac{\delta}{2})^k x^{\frac{r+2k}{2}-1}}{2^{\frac{r+2k}{2}} \Gamma(\frac{r+2k}{2}) k!}, \qquad x > 0$$

The Laplace transform of non-central chi-square distribution(see Johnson and Kotz (2010)) given by

$$\tilde{f}(s) = (1+2t)^{-\frac{r}{2}} e^{\frac{\delta}{2}\left((1+2s)^{-1}-1\right)}. \tag{2}$$

This is the convolution of a gamma and a compound Poisson distribution.
Throughout this study, we will use the Confluent Hypergeometric function defined by,

$$M(a,b,z) = \sum_{n=0}^{\infty} \frac{a^{(n)} z^n}{b^{(n)} n!},$$

where $a^{(n)} = a(a-1)\ldots(a-n+1)$.

3. Polya-Aeppli Non-central Chi-square Distribution

Definition 3.1. 1 *A random variable* N *has a Polya-Aeppli non-central chi-square* (ρ,δ,r) *distribution when the following conditions satisfy:*

$$N \mid \lambda \sim Polya-Aeppli(\lambda,\rho)$$

and

$$\lambda \sim Noncentral\ \ chisquare(\delta,r),$$

We denote unconditional distribution of N by PANC (ρ,δ,r) and its PMF is given by

$$P(N=n) \quad = 3^{-\frac{r}{2}} e^{-\frac{\delta}{3}}, \quad n=0$$

$$= \frac{e^{-\frac{\delta}{2}} \rho^n}{3^{\frac{r}{2}} n} \sum_{j=1}^{n} \frac{\binom{n}{j}\left(\frac{2(1-\rho)}{3\rho}\right)^i M(\frac{r}{2}+j, \frac{r}{2}, \frac{\delta}{6})}{\beta(j,\frac{r}{2})}, \quad n = 1,2,\ldots.$$

The PGF of N is given by

$$\Psi_N(s) \quad = \int_0^{\infty} e^{-\frac{\lambda(1-s)}{(1-\rho s)}} f(\lambda;\delta,r) d\lambda$$

$$= \tilde{f}\left(\frac{1-s}{1-\rho s}\right),$$

where $\tilde{f}(s)$ is the Laplace transform of non-central chi-square (δ,r) distribution, given by (2).
Hence it follows that

$$\Psi_N(s) = \left(1+\frac{2(1-s)}{1-\rho s}\right)^{-\frac{r}{2}} e^{\frac{\delta}{2}\left(\left(1+\frac{2(1-s)}{1-\rho s}\right)^{-1}-1\right)} \tag{3}$$

Theorem 3.1. 2 *If* $N \sim PANC(\rho,\delta,r)$ *,then the* k^{th} *factorial moment of* N *is given by*

$$\mu_{(k)}(N) = \frac{\Gamma(k)e^{-\frac{\delta}{2}}}{(1-\rho)^k}\sum_{j=1}^{k}\frac{\binom{k}{j}(2)^j\rho^{k-j}M(\frac{r}{2}+j,\frac{r}{2},\frac{\delta}{2})}{\beta(j,\frac{r}{2})} \qquad (4)$$

Proof: The k^{th} factorial moment of $PANC(\rho,\delta,r)$ can be obtained as

$$\begin{aligned}\mu_{(k)}(N) &= E_{\lambda}\left[\mu_{(k)}(N\mid\lambda)\right]\\ &= \int_0^{\infty}\mu_{(k)}(N\mid\lambda)\,f(\lambda;\delta,r)d\lambda,\end{aligned}$$

where $\mu_{(k)}(N\mid\lambda)$ is the k^{th} factorial moment of Polya-Aeppli distribution and is given by (1).
By substituting (1) in (5),we get (4).

Proposition 3.1: 3*The PMF of the PANC* (ρ,δ,r) *satisfies the following recursions:*

$$\begin{aligned}P_1 &= \frac{(1-\rho)(3r+\delta)}{9}P_0,\\ P_i &= \left(\frac{(1-\rho)(3r+\delta)+3(4+5\rho)(i-1)}{9i}\right)P_{i-1}-\left(\frac{(1-\rho)(r\rho+\delta\rho+2r)+(\rho+2)(7\rho+2)(i-2)}{9i}\right)\\ &\quad P_{i-2}+\left(\frac{\rho(\rho+2)^2(i-3)}{9i}\right)P_{i-3}, \qquad i=2,3,\ldots\end{aligned} \qquad (6)$$

and $P_{-1}=0$.

Proof: On differentiating (3) with respect to s, we have

$$(1-\rho s)[3-(2+\rho)s]^2\Psi_N'(s) = (1-\rho)[(3r+\delta)(1-\rho s)-2r(1-\rho)s]\Psi_N(s), \qquad (7)$$

where

$$\begin{aligned}\Psi_N(s) &= \sum_{i=0}^{\infty}P_i s^i,\\ &\textit{and}\\ \Psi_N'(s) &= \sum_{i=0}^{\infty}(i+1)P_{i+1}s^i.\end{aligned}$$

Equating the co-efficient of s^i on both sides of (7), we obtain (6).

Proposition 3.2: 4*Recurrence relation for the factorial moments* $\mu_{(k)}$ *of PANC* (ρ,δ,r) *is the following for* $k=1,2,\ldots$ with $\mu_{(0)}=1$

$$\begin{aligned}(1-\rho)^3\mu_{(k+1)} &= (1-\rho)^2[(r+\delta)+k(3\rho+4)]\mu_{(k)}-k(1-\rho)[2r+\rho(r+\delta)+(k-1)(\rho+2)(3\rho+2)]\\ &\quad \mu_{(k-1)}+\rho(\rho+2)^2k(k-1)(k-2)\mu_{(k-2)}.\end{aligned} \qquad (8)$$

Proof: The factorial MGF of PANC (ρ,δ,r) is

$$\begin{aligned}\eta(t) &= \psi(1+t)\\ &= \left(1-\frac{2t}{1-\rho(1+t)}\right)^{-\frac{r}{2}}e^{\frac{\delta}{2}\left(\left(1-\frac{2t}{1-\rho(1+t)}\right)^{-1}-1\right)}\end{aligned} \qquad (9)$$

On differentiating (9) with respect to t, we get

$$[(1-\rho)-\rho t][(1-\rho)-(2+\rho)t]^2\eta'(t)=(1-\rho)[(r+\delta)(1-\rho)-(2r+(r+\delta))t]\eta(t), \qquad (10)$$

where

$$\eta'(t) = \sum_{k=1}^{\infty}\mu_{(k)}\frac{t^{k-1}}{(k-1)!}$$

$$and$$

$$\eta(t) = \sum_{k=0}^{\infty}\mu_{(k)}\frac{t^{k}}{k!}.$$

Equating the co-efficient of $\frac{t^k}{k!}$ on both sides of (10), we obtain (8).

3.1 Polya-Aeppli Non-central Chi-square Process

Let $N(t)$ denotes the number of the events up to time t.Then $\{N(t), t\geq 0\}$ is a Polya-Aeppli non-central chi-square process if

$$N(t)\,|\,\lambda \sim Polya-Aeppli(\lambda t,\rho)$$

and

$$\lambda \sim Noncentral\ \ chisquare(\delta, r),$$

We use the notation $N(t)\sim \text{PANCP}(\rho,\delta,r)$ and its PMF is given by

$$P(N(t)=n) = (1+2t)^{-\frac{r}{2}}e^{-\frac{\delta t}{(1+2t)}}, \quad n=0$$

$$= \frac{e^{-\frac{\delta}{2}}\rho^n}{n(1+2t)^{\frac{r}{2}}}\sum_{j=1}^{n}\frac{\binom{n}{j}\left(\frac{2t(1-\rho)}{(1+2t)\rho}\right)^{i} M(\frac{r}{2}+j,\frac{r}{2},\frac{\delta}{2(1+2t)})}{\beta(j,\frac{r}{2})}, \quad n=1,2,\ldots \qquad (11)$$

The mean and variance of PANCP (ρ,δ,r) are given by

$$EN(t)=\frac{(r+\delta)t}{(1-\rho)}$$

and

$$V(N(t))=\frac{(1+\rho)(r+\delta)t+2(r+2\delta)t^2}{(1-\rho)^2}.$$

The Fisher index of dispersion is given by

$$FI(N(t)) = \frac{\text{var}(N(t))}{EN(t)}$$

$$=1+\frac{2\rho}{(1-\rho)}+\frac{2(r+2\delta)t}{(1-\rho)(r+\delta)}>1.$$

Therefore PANCP (ρ,δ,r) is overdispersed, related to Polya-Aeppli process.

4. PANCP as a pure birth process

In this section we define PANCP as a pure birth process.

Definition 4.1. 5*A counting process* $\{N(t), t\geq 0\}$ *is said to be a PANCP with parameters* ρ, δ and r *if*

(1) $N(0)=0$;

(2) the state transition probabilities are defined as follows

$$P\big(N(t+h)=n/N(t)=m\big)=\begin{cases} 1-\dfrac{(1-\rho)}{(1+2t)^2}\displaystyle\sum_{k=1}^{\infty}\left(\dfrac{2t+\rho}{1+2t}\right)^{k-1} \\ \times\left[r-\delta+\dfrac{2\delta}{1+2t}\left(1+\dfrac{(k-1)(1-\rho)t}{(2t+\rho)}\right)\right]h+o(h), & n=m, \\ \dfrac{(1-\rho)}{(1+2t)^2}\left(\dfrac{2t+\rho}{1+2t}\right)^{i-1} \\ \times\left[r-\delta+\dfrac{2\delta}{1+2t}\left(1+\dfrac{(i-1)(1-\rho)t}{(2t+\rho)}\right)\right]h+o(h), & n=m+i, i=1,2,\ldots \end{cases} \tag{12}$$

for every $m=0,1,\ldots$, where $o(h)\to 0$ as $h\to 0$.

Let $P_n(t)=P(N(t)=n),\quad n=0,1,2,\ \ldots$.

Then the above postulates yield the following Kolmogorov forward equations:

$$P_0'(t)=-\left(\frac{r(1+2t)+\delta}{(1+2t)^2}\right)P_0(t),$$

$$P_n'(t)=-\left(\frac{r(1+2t)+\delta}{c(1+2t)^2}\right)P_n(t)+\frac{(1-\rho)}{(1+2t)^2}\sum_{k=1}^{n}\left(\frac{2t+\rho}{1+2t}\right)^{k-1}\left[r-\delta+\frac{2\delta}{1+2t}\left(1+\frac{(k-1)(1-\rho)t}{(2t+\rho)}\right)\right]P_{n-k}(t),\ \ n\geq 1$$

with initial conditions

$$P_0(0)=1 \text{ and } P_n(0)=0,\quad n=1,2,\ldots$$

As the solutions of above Kolmogorov forward equations, the marginal distributions of the process are obtained , given by (11). Therefore two definitions of the Process are equivalent.

5 Properties of PANCP (ρ,δ,r)

In this section, we discuss some properties of $PANCP(\rho,\delta,r)$.

Theorem 5.1. 6 *Let* $N(t)\sim PAPNCP(\rho,r,\delta)$. *Then:*

1. *The time interval* T_1 *to the first arrival is a non-central chi-square mixture of exponential(NCME) pdf's and inter-arrival times* $T_2,T_3\ldots$ *are non-central chi-square mixture of exponential(NCME) with mass at zero equal to* ρ.
2. *The distribution of the waiting time until the* n^{th} *event is*

$$f_{S_n}(t)=\frac{2e^{-\frac{\delta}{2}}}{(1+2t)^{\frac{r}{2}+1}}\sum_{i=0}^{n-1}\frac{\binom{n-1}{i}\left(\frac{2t(1-\rho)}{1+2t}\right)^{i}\rho^{n-1-i}M(\frac{r}{2}+i+1,\frac{r}{2},\frac{\delta}{2(1+2t)})}{\beta(\frac{r}{2},i+1)} \tag{13}$$

Proof: Let $\{T_k\}_{k\geq 2}$ are the inter-arrival times and $S_n=\sum_{i=1}^{n}T_i$ be the waiting time until the n^{th} event.

For any $t\geq 0$ and $n\geq 0$, the following relation holds.

$$P(N(t)=n)=P(S_n\leq t)-P(S_{n+1}\leq t),\ n=0,1,\ldots. \tag{14}$$

(1). The conditional C.D.F of T_1 given λ (See Minkova(2004)) is

$$P(T_1 \le t \mid \lambda) = 1 - e^{-\lambda t} \quad t > 0,\ \lambda > 0.$$

Then the unconditional C.D.F of T_1 is

$$F_{T_1}(t) = 1 - (1+2t)^{-\frac{r}{2}} e^{-\frac{\delta t}{1+2t}}.$$

Hence the density function of T_1 is

$$f_{T_1}(t) = \left(r(1+2t)+\delta\right)(1+2t)^{-\frac{r}{2}-2} e^{-\frac{\delta t}{1+2t}},\ t \ge 0. \tag{15}$$

i.e, It is a non-central chi-square mixture of exponential(NCME) pdf's and is denoted by $\nu(r,\delta;t)$.

Proceeding similar way we get unconditional distribution of T_2 as

$$f_{T_2}(t) = \rho\delta(0) + (1-\rho)\left(r(1+2t)+\delta\right)(1+2t)^{-\frac{r}{2}-2} e^{-\frac{\delta t}{1+2t}}, t \ge 0.$$

which is the pdf of non-central chi-square mixture of exponential(NCME) with mass at zero equal to ρ.

(2). We will prove the result by using mathematical induction.

From (15), it follows that

$$f_{S_1}(t) = \frac{2e^{-\frac{\delta}{2}}}{(1+2t)^{\frac{r}{2}+1}} \frac{M(\frac{r}{2}+1, \frac{r}{2}, \frac{\delta}{2(1+2t)})}{\beta(\frac{r}{2},1)}.$$

For $n=1$, (14) becomes

$$P(N(t)=1) = P(S_1 \le t) - P(S_2 \le t),\ n = 0,1,\ldots \tag{16}$$

Applying (11) for $m=1$ and then differentiating (16), we get

$$f_{S_2}(t) = \frac{2e^{-\frac{\delta}{2}}}{(1+2t)^{\frac{r}{2}+1}} \left(\frac{\rho M(\frac{r}{2}+1, \frac{r}{2}, \frac{\delta}{2(1+2t)})}{\beta(\frac{r}{2},1)} + \frac{\frac{2t(1-\rho)}{(1+2t)} M(\frac{r}{2}+2, \frac{r}{2}, \frac{\delta}{2(1+2t)})}{\beta(\frac{r}{2},2)} \right)$$

Now, suppose that for $n \ge 1$, the distribution of waiting time given by (13) is true. Applying (11) for $m = n$ and then differentiating (14) and substituting (13), we get

$$f_{S_{n+1}}(t) = \frac{2e^{-\frac{\delta}{2}}}{(1+2t)^{\frac{r}{2}+1}} \sum_{i=0}^{n} \frac{\binom{n}{i}\left(\frac{2t(1-\rho)}{1+2t}\right)^{i} \rho^{n-i} M(\frac{r}{2}+i+1, \frac{r}{2}, \frac{\delta}{2(1+2t)})}{\beta(\frac{r}{2}, i+1)}.$$

6. Application to Risk Theory

We consider here the standard risk model $\{X(t), t \ge 0\}$, defined on the probability space (Ω, F, P)

$$X(t) = ct - \sum_{k=1}^{N(t)} Y_k, \left(\sum_{1}^{0} = 0\right).$$

where c is the rate of insurer's premium income and the claim sizes $\{Y_i, i \in N\}$ independent of the counting process $\{N(t), t \ge 0\}$, are i.i.d positive random variables with common distribution function $F(x)$ such that $F(0) = 0$. In this model we assume that $\{N(t), t \ge 0\}$ is a $PANCP(\lambda, \delta, r)$.

The relative safety loading $\delta > 0$ satisfies the equation $c = \dfrac{(1+\theta)\mu(r+\delta)}{1-\rho}$, where $\mu = E(X_i)$.

The time to ruin is denoted by T and is defined by

$$T = \begin{cases} \inf\{t : u + X(t) < 0\} \\ \infty \quad if \;\; u + X(t) \geq 0 \, for all \, t > 0. \end{cases}$$

The probability of ultimate ruin from initial capital u is denoted by $\Psi(u)$ and is given by

$$\Psi(u) = P(T < \infty).$$

The non ruin probability is defined by $\Phi(u) = 1 - \Psi(u)$.

Let $W(u, z)$ denote the joint CDF of the time to ruin T and deficit at the time of ruin $D = |u + X(T)|$ is given by

$$W(u, z) = P(T < \infty, D \leq z), \quad z \geq 0,$$

It is clear that

$$\lim_{z \to \infty} W(u, z) = \Psi(u). \tag{17}$$

Using the postulates (12), we get

$$\begin{aligned} W(u,z) \quad &= \left[1 - \left(\frac{r(1+2t)+\delta}{(1+2t)^2}\right) h\right] W(u+ch, z) \\ &+ \frac{(1-\rho)}{(1+2t)^2} \sum_{k=1}^{\infty} \left(\frac{2t+\rho}{1+2t}\right)^{k-1} \left[r - \delta + \frac{2\delta}{1+2t}\left(1 + \frac{(k-1)(1-\rho)t}{(2t+\rho)}\right)\right] h \\ &\left[\int_0^{u+ch} W(u+ch-x, z) dF^{*k}(x) + F^{*k}(u+ch+z) - F^{*k}(u+ch)\right] + o(h), \end{aligned}$$

where $F^{*k}(x), k = 1, 2, \ldots$ is the k-fold convolution of claim amount distribution.

or,equivalently

$$\begin{aligned} \frac{W(u+ch, z) - W(u, z)}{ch} \quad &= \left(\frac{r(1+2t)+\delta}{c(1+2t)^2}\right) W(u+ch, z) \\ &- \frac{(1-\rho)}{c(1+2t)^2} \sum_{k=1}^{\infty} \left(\frac{2t+\rho}{1+2t}\right)^{k-1} \left[r - \delta + \frac{2\delta}{1+2t}\left(1 + \frac{(k-1)(1-\rho)t}{(2t+\rho)}\right)\right] \\ &\times \left[\int_0^{u+ch} W(u+ch-x, z) dF^{*k}(x) + F^{*k}(u+ch+z) - F^{*}(u+ch)\right] + o(h). \end{aligned}$$

Taking the limit $h \to 0$, leads to the following differential equation:

$$\frac{\partial}{\partial u} W(u,z) = \left(\frac{r(1+2t)+\delta}{c(1+2t)^2}\right)\left[W(u,y) - \int_0^u W(u-x, z) dG(x) - (G(u+y) - G(u))\right] \tag{18}$$

where $G(x) = \dfrac{(1-\rho)}{(r(1+2t)+\delta)} \sum_{k=1}^{\infty} \left(\dfrac{2t+\rho}{1+2t}\right)^{k-1} \left[r - \delta + \dfrac{2\delta}{1+2t}\left(1 + \dfrac{(k-1)(1-\rho)t}{(2t+\rho)}\right)\right] F^{*k}(x),$ is a non defective distribution function of the claims with $G(0) = 0, G(\infty) = 1$.

The above equation can be expressed in terms of the safety loading as follows:

$$\frac{\partial}{\partial u} W(u,z) \quad = \frac{(1-\rho)(r(1+2t)+\delta)}{\mu(1+\theta)(r+\delta)(1+2t)^2}\left[W(u,z) - \int_0^u W(u-x,z) dG(x) - (G(u+z) - G(u))\right].$$

Using (17) and (18) we obtain the following integro- differential equation for the ruin probability.

$$\frac{\partial}{\partial u} \Psi(u) = \left(\frac{r(1+2t)+\delta}{c(1+2t)^2}\right)\left[\Psi(u) - \int_0^u \Psi(u-x) dG(x) - (1 - G(u))\right], \; u \geq 0. \tag{19}$$

Theorem 6.1. 7*The probability of ruin with zero initial capital satisfies*

$$\Psi(0) = \frac{\mu(r+\delta)}{c(1-\rho)}. \tag{20}$$

Proof: If we integrate (18) from $u = 0$ to $u = \infty$ with $W(\infty, z) = 0$, we get the following equation.

$$-W(0,z) = \left(\frac{r(1+2t)+\delta}{c(1+2t)^2}\right)\left[\int_0^\infty W(u,z)du - \int_0^\infty\int_0^u W(u-x,z)dG(x)du - \int_0^\infty (G(u+z)-G(u))du\right].$$

Changing the order of integration in the II^{nd} integral and then making use of some transformation, we get

$$W(0,z) = \frac{r(1+2t)+\delta}{c(1+2t)^2}\int_0^\infty (G(u+z)-G(u))du.$$

Hence

$$W(0,z) = \frac{r(1+2t)+\delta}{c(1+2t)^2}\int_0^z (1-G(u))du. \tag{21}$$

Using (17) and (21) we can write,

$$\begin{aligned}\Psi(0) &= \frac{r(1+2t)+\delta}{c(1+2t)^2}\int_0^\infty (1-G(u))du \\ &= \frac{r(1+2t)+\delta}{c(1+2t)^2}E(X).\end{aligned}$$

where EX is the mean of the random variable X with distribution function $G(x)$ and is given by

$$\begin{aligned}E(X) &= \frac{\mu(1-\rho)}{(r(1+2t)+\delta)}\sum_{k=1}^{\infty} k\left(\frac{2t+\rho}{1+2t}\right)^{k-1}\left[r-\delta+\frac{2\delta}{1+2t}\left(1+\frac{(k-1)(1-\rho)t}{(2t+\rho)}\right)\right] \\ &= \frac{\mu(r+\delta)(1+2t)^2}{(1-\rho)(r(1+2t)+\delta)}.\end{aligned}$$

Hence the result.

Exponential claims

Consider exponential claim sizes with p.d.f $f(x) = \frac{1}{\mu}e^{-\frac{x}{\mu}}$, $x \geq 0$, $\mu > 0$. The survival function $\overline{G}(x)$ is given by

$$\overline{G}(x) = e^{-\frac{(1-\rho)x}{\mu(1+2t)}}\left(1+\frac{2t\delta(1-\rho)}{\mu(1+2t)(r(1+2t)+\delta)}\right),\ x > 0.$$

In this case we obtain $W(0,z)$ from (21) and is given by

$$W(0,z) = \frac{\mu(r+\delta)}{c\rho}\left(1-e^{-\frac{(1-\rho)z}{\mu(1+2t)}}\right) - \frac{2t\delta}{c(1+2t)^2}ze^{-\frac{(1-\rho)z}{\mu(1+2t)}}.$$

Differentiating (19) w.r.to u twice we get the following differential equation for the ruin probability, in the case of exponentially distributed claims.

$$\frac{\partial^3}{\partial u^3}\Psi(u) + \frac{(1-\rho)}{\mu(1+2t)}\left(2-\frac{\mu(r(1+2t)+\delta)}{c(1-\rho)(1+2t)}\right)\frac{\partial^2}{\partial u^2}\Psi(u) + \frac{(1-\rho)}{\mu(1+2t)}\left(c(1-\rho)-\mu(r+\delta)\right)\frac{\partial^2}{\partial u^2}\Psi(u) = 0.$$

Conclusions

In this study we introduced PANCP as a mixed Polya-Aeppli process with mixing random variable having non-central chi-square distribution. We found that this model is more suitable for handling over-dispersed count data. We have defined the risk model with PANCP as a counting process and have studied probability of ruin for this model.

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