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VARIOUS LABELINGS FOR THE GRAPH $C(t \cdot P_n)$

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Abstract

In this paper we investigate graceful labeling, cordial labeling, product cordial labeling and total edge product cordial labeling for the cycle graph $C(t \cdot P_n)$. The cycle graph $C(t \cdot P_n)$ formed by joining corresponding vertices of t copies of path P_n.

Key words : Graceful labeling, cordial labeling and cycle of graphs.

AMS **subject classification** (2010) **:** 05C78.

1 Introduction :

Various graph labelings have been introduced so far. It explored as well by many researchers. Graph labelings have many applications within mathematics, computer sciences and communication networks. Such applications of graph labeling have been studied by Yegnanaryanan and Vaidhyanathan [1]. An extensive survey on graph labeling and bibliographic references are given in Gallian [2].

The graceful labeling was introduced by A Rosa [3] and he proved that the cycle C_n ($n \equiv 0, 3 \pmod 4$) is graceful. Cahit [4] has introduced the concept of cordial labeling. Some labelings like A-cordial labeling, H-cordial labeling, prime cordial labeling, product cordial labeling, total produact cordial labeling, edge product cordial labeling, total edge product cordial labeling are also introduced with minor variations in cordial theme.

We shall begin with a simple graph $G = (V, E)$, which is a finite, undirected graph with $|V| = p$ vertices and $|E| = q$ edges. For all terminology and notations we follows Harary [5]. First we shall recall some definitions which are used in this paper.

Definition−1.1 **:** A function f is called *graceful labeling* of a graph $G = (V, E)$ if $f : V(G) \longrightarrow \{0, 1, \ldots, q\}$ is injective and the induce function $f^* : E(G) \longrightarrow \{1, 2, \ldots, q\}$ defined as $f^*(e) = |f(u) - f(v)|$ is bijective for every edge $e = (u, v) \in E$. A graph G is called *graceful graph* if it admits a graceful labeling.

Definition−1.2 **:** A function $f : V(G) \longrightarrow \{0, 1\}$ is called *binary vertex labeling* of a graph G and $f(v)$ is called *label of the vertex* v of G under f.

For an edge $e = (u, v)$, the induced function $f^* : E(G) \longrightarrow \{0, 1\}$ defined as $f^*(e) = |f(u) - f(v)|$, for every edge $e = (u, v) \in E$. Let $v_f(0)$, $v_f(1)$ be number of vertices of G having labels 0 and 1 respectively under f and let $e_f(0)$, $e_f(1)$ be number of edges of G having labels 0 and 1 respectively under f^*.

A binary vertex labeling f of a graph G is called *cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph which admits cordial labeling is called *cordial graph*.

For a graph $G = (V(G), E(G))$, a vertex labeling function $f : V(G) \longrightarrow \{0, 1\}$ induces an edge labeling function $f^* : E(G) \longrightarrow \{0, 1\}$ defined as $f^*(e) = f(u) * f(v)$, for every edge $e = (u, v) \in E$.

Then f is called *product cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph which admits product cordial labeling is called *product cordial graph*.

For a graph $G = (V(G), E(G))$, an edge labeling function $f : E(G) \longrightarrow \{0, 1\}$ induces a vertex labeling function $f^* : V(G) \longrightarrow \{0, 1\}$ defined as $f^*(v) = \Pi f(e)$, for every edge $e = (u, v) \in E$.

Then f is called *total edge product cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph which admits total edge product cordial labeling is called *total edge product cordial graph*.

Definition-1.3 **:** For a cycle C_n, each vertices of C_n is replace by connected graphs $G_1, G_2, \ldots, G_n$ is known as *cycle of graphs* and we shall denote it by $C(G_1, G_2, \ldots, G_n)$. If we replace each vertices by a graph G i.e. $G_1 = G$, $G_2 = G$, ..., $G_n = G$, such cycle of a graph G, we shall denote it by $C(n \cdot G)$.

In this paper we have discussed various graph labelings for the cycle graph $C(t \cdot P_n)$.

2 Main Results :

Theorem-2.1 **:** $C(t \cdot P_n)$ is graceful, where $t \equiv 0, 3 \pmod 4$ and $n \in N$.

Proof : Let G be a cycle of graphs formed by t copies of path P_n. Let $u_{i,j}$ $(1 \leq j \leq n)$ be vertices of i^{th} copy of path P_n in $C(t \cdot P_n)$, $\forall\, i = 1, 2, \ldots, t$. We shall join $u_{i,n}$ last vertex of $P_n^{(i)}$ with $u_{i+1,n}$ vertex of $P_n^{(i+1)}$ by an edge, $\forall\, i = 1, 2, \ldots, t-1$ and also join $u_{t,n}$ last vertex of $P_n^{(t)}$ with $u_{1,n}$ to form the cycle of graphs $C(t \cdot P_n)$.

We define the labeling function $f : V(G) \longrightarrow \{0, 1, \ldots, q\}$, where $q = t \cdot n$ as follows.

$f(u_{1,j}) = q - (\frac{j-1}{2}),$ when $j \equiv 1 \pmod 2$

$\quad = (\frac{j-2}{2}),$ when $j \equiv 0 \pmod 2$, $\forall\, j = 1, 2, \ldots, n;$

$f(u_{2,j}) = f(u_{1,j}) + (-1)^j(q + 1 - n),$ $\forall\, j = 1, 2, \ldots, n;$

$f(u_{i,j}) = f(u_{i-2,j}) - (-1)^{i+j}(n),$ $\forall\, j = 1, 2, \ldots, n$, $\forall\, i = 3, 4, \ldots, \lceil \frac{t}{2} \rceil;$

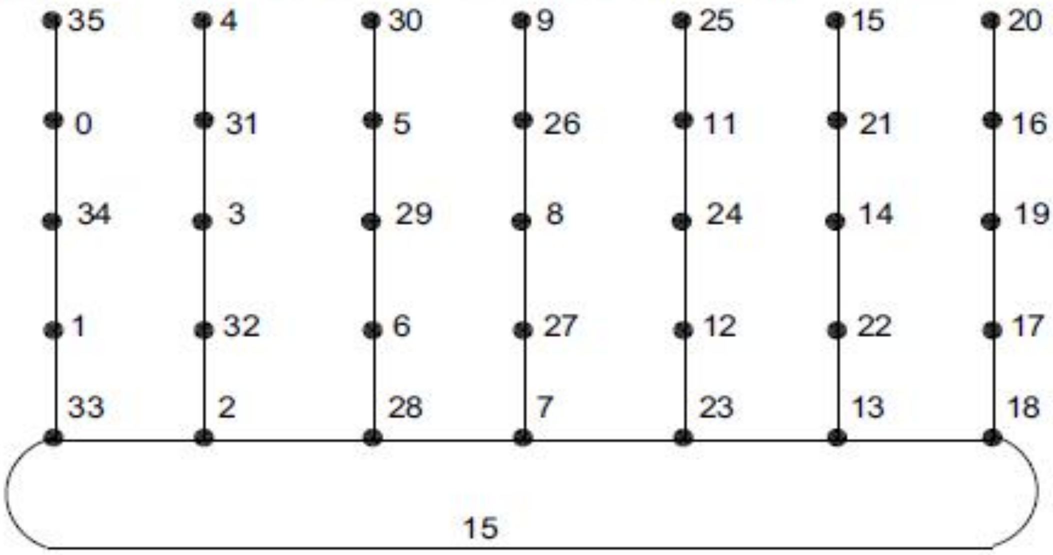
$f(u_{\lceil \frac{t}{2} \rceil + 1,j}) = f(u_{\lceil \frac{t}{2} \rceil - 1,j}) + \frac{1}{2} + (-1)^j(n + \frac{1}{2}),$ $\forall\, j = 1, 2, \ldots, n;$

$f(u_{\lceil \frac{t}{2} \rceil + 2,j}) = f(u_{\lceil \frac{t}{2} \rceil,j}) + \frac{1}{2} - (-1)^j(n + \frac{1}{2}),$ $\forall\, j = 1, 2, \ldots, n;$

$f(u_{i,j}) = f(u_{i-2,j}) - (-1)^{i+j}(n),$ $\forall\, j = 1, 2, \ldots, n$, $\forall\, i = \lceil \frac{t}{2} \rceil + 3, \lceil \frac{t}{2} \rceil + 4, \ldots, t.$

Above labeling pattern give rise a graceful labeling to the given graph G. Thus $G = C(t \cdot P_n)$ is a graceful graph.

Illustration−2.2 **:** $C(7 \cdot P_5)$ and its graceful labeling shown in *figure*−1, where $p = q = 35$.



Figure−1 Cycle graph $C(7 \cdot P_5)$ and its graceful labeling.

Theorem−2.3 **:** $C(t \cdot P_n)$ is cordial, where $t \equiv 0, 1, 3 \pmod 4$ and n is odd or $t \in N$ and n is even.

Proof : Let $u_{i,j}$ $(1 \leq i \leq t,\ 1 \leq j \leq n)$ be vertices of the cycle graph $C(t \cdot P_n)$ like previous *theorem*−2.1.

To define the labeling function $f : V(t \cdot P_n) \longrightarrow \{0, 1\}$, we shall consider following two cases.

Case−**I :** $t \equiv 0, 1, 3 \pmod 4$ and $n \equiv 1 \pmod 2$

$f(u_{1,j}) = 0$ when $j \equiv 0, 1 \pmod 4$

$= 1,$ when $j \equiv 2, 3 \pmod 4$, $\forall\ j = 1, 2, \ldots, n;$

$f(u_{i,j}) = f(u_{1,j})$ when $i \equiv 0, 1 \pmod 4$

$= 1 - f(u_{1,j})$, when $i \equiv 2, 3 \pmod 4$, $\forall\ j = 1, 2, \ldots, n$, $\forall\ i = 1, 2, \ldots, t$.

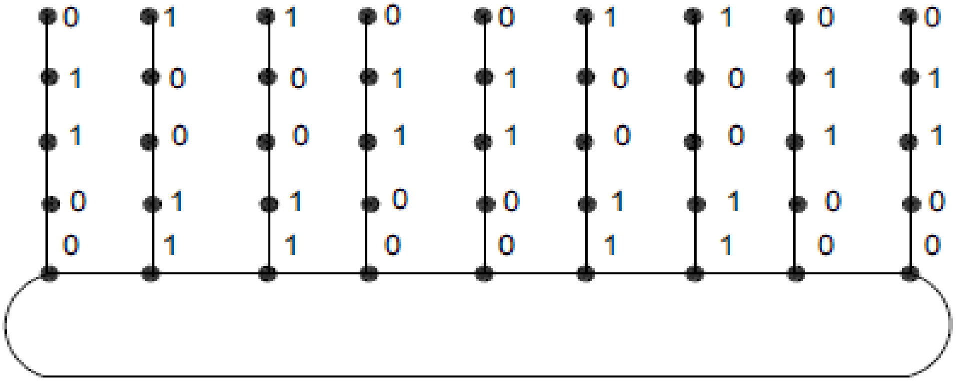
Case−**II :** $t \in N$ and $n \equiv 0 \pmod 2$

$f(u_{i,j}) = 0$ when $j \equiv 0, 1 \pmod 4$

$= 1,$ when $j \equiv 2, 3 \pmod 4$, $\forall\ j = 1, 2, \ldots, n$, $\forall\ i = 1, 2, \ldots, t$.

Above labeling pattern give rise cordial labeling to the graph $C(t \cdot P_n)$ and so it is a cordial graph.

Illustration−2.4 **:** $C(9 \cdot P_5)$ and its cordial labeling shown in *figure*−2, where $p = q = 45$, $v_f(0) = 23$, $v_f(1) = 22$, $e_f(0) = 23$ and $e_f(1) = 22$.



Figure−2 Cycle graph $C(9 \cdot P_5)$ and its cordial labeling.

Theorem−2.5 **:** $C(t \cdot P_n)$ is product cordial graph, where $t, n \in N$ and $n \geq 2$.

Proof : Let G be a cycle of t copies of path P_n. It is obvious that $|V(G)| = p = q = |E(G)| = t \cdot n$. Let $u_{i,j}$ $(1 \leq i \leq t,\ 1 \leq j \leq n)$ be vertices of the cycle graph G like previous *theorem*−2.1. Let us redefine $V(G) = \{u_{i,j} \ / \ i = 1, 2, \ldots, t, j = 1, 2, \ldots, n\}$ by $V(G) = \{v_l \ / \ l = 1, 2, \ldots, p - t\} \cup \{w_s \ / \ s = 1, 2, \ldots, t\}$, where
$u_{i,j} = v_{i(n-1)+1-j}$, $\forall \ i = 1, 2, \ldots, t$, $\forall \ j = 1, 2, \ldots, n-1$ and $u_{i,n} = w_i$, $\forall \ i = 1, 2, \ldots, t$.

We define the labeling function $f : V(G) \longrightarrow \{0, 1\}$ as follows.

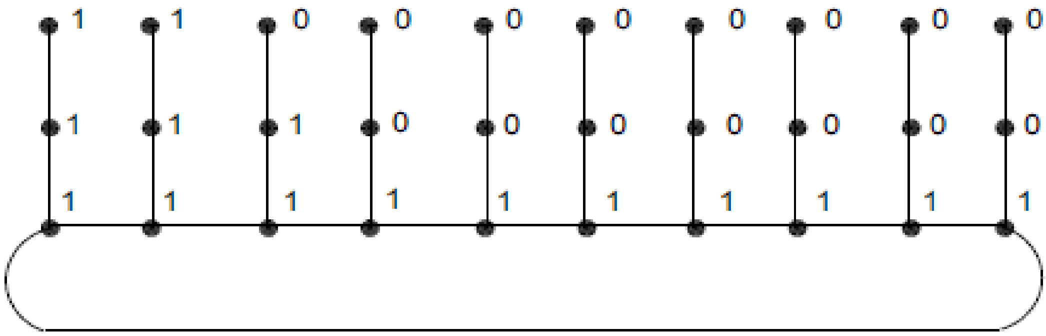
$f(w_s) = 1 \qquad \forall \ s = 1, 2, \ldots, t;$

$f(v_l) = 1 \qquad$ when $l \leq \lceil \frac{q}{2} \rceil - t$

$\quad = 0, \qquad$ when $l > \lceil \frac{q}{2} \rceil - t$, $\forall \ l = 1, 2, \ldots, p - t$.

Above labeling pattern give rise a product cordial labeling to the graph $C(t \cdot P_n)$ and so it is a product cordial graph.

Illustration−2.6: $C(10 \cdot P_3)$ and its product cordial labeling shown in *figure*−3, where $p = q = 30$, $v_f(0) = 15$, $v_f(1) = 15$, $e_f(0) = 15$ and $e_f(1) = 15$.



Figure−3 Cycle graph $C(10 \cdot P_3)$ and its product cordial labeling.

Theorem−2.7 **:** $C(t \cdot P_n)$ is total edge product cordial graph, where $t, n \in N$ and $n \geq 2$.

Proof : Let G be a cycle of t copies of path P_n. It is obvious that $|V(G)| = p = q = |E(G)| = t \cdot n$. Let $u_{i,j}$ $(1 \leq i \leq t,\ 1 \leq j \leq n)$ be vertices of the cycle graph G like previous *theorem*−2.1. Let us redefine $V(G) = \{u_{i,j} \ / \ i = 1, 2, \ldots, t, j = 1, 2, \ldots, n\}$ by $V(G) = \{w_k \ / \ k = 1, 2, \ldots, p\}$, where $u_{i,j} = w_{(j-1)t+i}$, $\forall\ i = 1, 2, \ldots, t$, $\forall\ j = 1, 2, \ldots, n$.

We shall define $E(G) = \{e_k \ / \ k = 1, 2, \ldots, q\}$ by

$e_k = (w_k, w_{k+t})$, $\forall\ k = 1, 2, \ldots, q-t$.

$e_k = (w_k, w_{k+1})$, $\forall\ k = q-t+1, q-t+2, \ldots, q-1$.

$e_q = (w_q, w_{q-t})$.

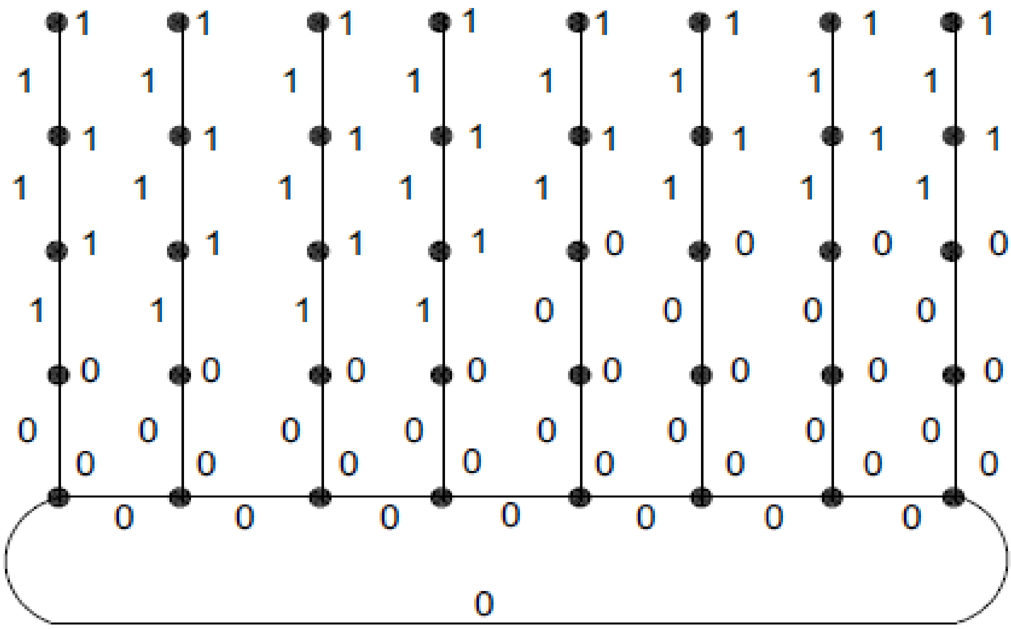
Now we shall define the labeling function $f : E(G) \longrightarrow \{0, 1\}$ as follows.

$f(e_k) = 1$ when $k \leq \lceil \frac{q}{2} \rceil$

$= 0,$ when $k > \lceil \frac{q}{2} \rceil$, $\forall\ k = 1, 2, \ldots, q$.

Above labeling pattern give rise total edge product cordial labeling to the graph G and so G is a total edge product cordial graph.

Illustration−2.8: $C(8 \cdot P_5)$ and its total edge product cordial labeling shown in *figure*−4, where $p = q = 40$, $v_f(0) = 20$, $v_f(1) = 20$, $e_f(0) = 20$ and $e_f(1) = 20$.



Figure−4 Cycle graph $C(8 \cdot P_5)$ and its total edge product cordial labeling.

3 Concluding Remarks :

Kaneria et al. [7] introduced above cycle of graphs. Here we investigate graceful labeling, cordial labeling, product cordial labeling and total edge product cordial labeling for the cycle graph $C(t \cdot P_n)$. In this way we got various graph labelings for the cycle of paths.

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