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g''' - OPEN SETS IN FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we offer a new class of sets called fuzzy g'''-open sets in fuzzy topological spaces. It turns out that this class lies between the class of fuzzy open sets and the class of fuzzy generalized open sets.

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Key words and Phrases: Fuzzy g-open set, fuzzy ω-open set, fuzzy g*s -open set, fuzzy gs-open set, fuzzy sg-open set, fuzzy gsp-open set, fuzzy αg-open set, fuzzy αgs-open set, fuzzy g'''-open set and fuzzy g'''_{α}-open set.

INTRODUCTION

Recently, Jeyaraman et al. [6] have introduced the concept of fuzzy g'''-closed sets and studied its basic fundamental properties in fuzzy topological spaces. In this paper, we introduce a new class of sets namely fuzzy g'''-open sets in fuzzy topological spaces. Also, we investigate the relationships among related fuzzy generalized open sets.

PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) (or X and Y) represent fuzzy topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a fuzzy subset A of a space (X, τ), cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively.

We recall the following definitions which are useful in the sequel.

Definition 2.1:

A subset A of a space (X, τ) is called:

(i) fuzzy semi-open set [1] if A ≤ cl(int(A));

(ii) fuzzy preopen set [4] if A ≤ int(cl(A));

(iii) fuzzy α -open set [4] if A ≤ int(cl(int(A)));

(iv) fuzzy β-open set [13] (= fuzzy semi-preopen [13]) if A ≤ cl(int(cl(A)));

(v) fuzzy regular open set [1] if A = int(cl(A)).

The complements of the above mentioned fuzzy open sets are called their respective fuzzy closed sets.

The fuzzy semi-closure [15] (resp. fuzzy α -closure [7], fuzzy semi-preclosure [13]) of a fuzzy subset A of X, denoted by scl(A) (resp. α cl(A), spcl(A)) is defined to be the intersection of all fuzzy semi-closed (resp. fuzzy α -closed, fuzzy semi-preclosed) sets of (X, τ) containing A. It is known that scl(A) (resp. α cl(A), spcl(A)) is a fuzzy semi-closed (resp. fuzzy α -closed, fuzzy semi-preclosed) set.

Definition 2.2:

A fuzzy subset A of a space (X, τ) is called:

(i) a fuzzy generalized closed (briefly, fuzzy g-closed) set [2] if cl(A) ≤ U whenever A ≤ U and U is fuzzy open in (X, τ). The complement of fuzzy g-closed set is called fuzzy g-open set;

(ii) a fuzzy semi-generalized closed (briefly fsg-closed) set [3] if scl(A) ≤ U whenever A ≤ U and U is fuzzy semi-open in (X, τ). The complement of fsg-closed set is called fsg-open set;

(iii) a fuzzy generalized semi-closed (briefly fgs-closed) set [10] if scl(A) ≤U whenever A ≤ U and U is fuzzy open in (X, τ). The complement of fgs-closed set is called fgs-open set;

(iv) a fuzzy α -generalized closed (briefly f α g-closed) set [11] if α cl(A) ≤ U whenever A ≤ U and U is fuzzy open in (X, τ). The complement of f α g-closed set is called f α g-open set;

(v) a fuzzy generalized semi-preclosed (briefly fgsp-closed) set [9] if spcl(A) ≤ U whenever A ≤ U and U is fuzzy open in (X, τ). The complement of fgsp-closed set is called fgsp-open set;

(vi) a fuzzy ω -closed set (briefly f ω -closed) [12] if cl(A) ≤ U whenever A ≤ U and U is fuzzy semi-open in (X, τ). The complement of f ω -closed set is called f ω -open set;

(vii) a fuzzy αgs -closed set(briefly f αgs -closed)[6] if α cl(A) ≤ U whenever A ≤ U and U is fuzzy semi-open in (X, τ). The complement of f αgs -closed set is called f αgs -open set;

(viii) a fuzzy g*s-closed set(briefly f g*s-closed)[6] if scl(A) ≤ U whenever A ≤ U and U is fgs- open in (X, τ). The complement of f g*s closed set is called f g*s open set;

(ix) a fuzzy g''' -closed set (briefly f g''' -closed)[6] if cl(A) ≤ U whenever A ≤ U and U is fgs-open in (X, τ). The complement of f g''' -closed set is called f g''' -open set;

(x) a fuzzy g_{α}''' -closed set(briefly f g_{α}''' -closed)[6] if α cl(A) ≤ U whenever A ≤ U and U is fgs-open in (X, τ). The complement of f g_{α}''' -closed set is called f g_{α}''' -open set;

Proposition 2.3[6]:

For any fuzzy topological space **(**X, τ),the following assertions hold:

(i) Every fuzzy closed set is fuzzy g''' -closed but not conversely.

(ii) Every fuzzy g''' -closed set is fuzzy g_{α}''' -closed but not conversely.

(iii) Every fuzzy g''' -closed set is fuzzy g*s-closed but not conversely.

(iv) Every fuzzy g''' -closed set is fuzzy ω -closed but not conversely.

(v) Every fuzzy g''' -closed set is fuzzy sg-closed but not conversely.

(vi) Every fuzzy g''' -closed set is fuzzy g-closed but not conversely.

(vii) Every fuzzy g''' -closed set is fuzzy αgs-closed but not conversely.

(viii) Every fuzzy g'''-closed set is fuzzy αg-closed but not conversely.

(ix) Every fuzzy g'''-closed set is fuzzy gs-closed but not conversely.

(x) Every fuzzy g'''-closed set is fuzzy gsp-closed but not conversely.

3. FUZZY g'''-OPEN SETS

In this section, we discuss some relation between fuzzy g'''-open set and fuzzy generalized open sets.

Definition 3.1:

A fuzzy subset A of a space (X, τ) is called fuzzy g'''-open set in X if A^c is fuzzy g'''-closed set in (X, τ).

The collection of all fuzzy g'''-open set in X is denoted by G''' O(X).

Proposition 3.2:

Every fuzzy open set is fuzzy g'''-open set.

Proof:

If A is fuzzy open set in (X, τ), then A^c is fuzzy closed set. Since, by proposition 2.3, every fuzzy closed set is fuzzy g'''-closed. Therefore A^c is fuzzy g'''-closed set. Hence A is fuzzy g'''-open set.

The converse of Proposition 3.2 need not be true as seen from the following example.

Example 3.3:

Let X = {a, b} with τ = {0_x, A, 1_x} where A is fuzzy set in X defined by A(a)=1, A(b)=0. Then (X, τ) is a fuzzy topological space. Clearly B defined by B(a)=0.5, B(b)=0 is fuzzy g'''-open set but not fuzzy open.

Proposition 3.4:

Every fuzzy g'''-open set is fuzzy g'''_α-open set.

Proof:

If A is fuzzy g'''-open set in (X, τ), then A^c is fuzzy g'''-closed set. Since, by proposition 2.3, every fuzzy g'''-closed set is fuzzy g'''_α-closed. Therefore A^c is fuzzy g'''_α-closed set. Hence A is fuzzy g'''_α-open in (X, τ).

The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5:

Let X = {a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5. Then (X, τ) is a fuzzy topological space. Clearly μ defined by μ(a)=0.6, μ(b)=0.6 is fuzzy g'''_α-open set but not fuzzy g'''-open set in (X, τ).

Proposition 3.6:

Every fuzzy g'''-open set is fuzzy g*s-open.

Proof:

If A is fuzzy g'''-open set in (X, τ), then A^c is fuzzy g'''-closed set. Since, by proposition 2.3, every fuzzy g'''-closed set is fuzzy g*s-closed. Therefore A^c is fuzzy g*s -closed set. Hence A is fuzzy g*s -open in (X, τ).

The converse of Proposition 3.6 need not be true as seen from the following example.

Example 3.7:

Let X = {a, b} with τ = {0_x, α, 1_x} where α is fuzzy set in X defined by α(a)=0.4, α(b)=0.5. Then (X, τ) is a fuzzy topological space. Clearly β defined by β(a)=0.6, β(b)=0.5 is fuzzy g*s-open but not fuzzy g'''-open set in (X, τ).

Proposition 3.8:

Every fuzzy g'''-open set is fuzzy ω-open set.

Proof:

If A is fuzzy g'''-open set in (X, τ), then A^c is fuzzy g'''-closed set. Since, by proposition 2.3, e very fuzzy g'''-closed set is fuzzy ω-closed. Therefore A^c is fuzzy ω-closed set. Hence A is fuzzy ω-open in (X, τ).

The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9:

Let X = {a, b}. Consider the fuzzy topology τ as in Example 3.5, where μ is fuzzy set in X defined by μ(a)=0.6, μ(b)=0.6. Clearly μ is fuzzy ω-open but not fuzzy g'''-open set in (X, τ).

Proposition 3.10:

Every fuzzy g'''-open set is fuzzy sg-open.

Proof:

If A is fuzzy g'''-open set in (X, τ), then A^c is fuzzy g'''-closed set. Since, by proposition 2.3, e very fuzzy g'''-closed set is fuzzy sg-closed. Therefore A^c is fuzzy sg-closed set. Hence A is fuzzy sg-open in (X, τ).

The converse of Proposition 3.10 need not be true as seen from the following example.

Example 3.11:

Let X = {a, b}. Consider the fuzzy topology τ as in Example 3.7, where β is fuzzy set in X defined by β(a)=0.6, β(b)=0.6. Clearly β is fuzzy sg-open but not fuzzy g'''-open set in (X, τ).

Proposition 3.12:

Every fuzzy g'''-open set is fuzzy g-open.

Proof:

If A is fuzzy g'''-open set in (X, τ), then A^c is fuzzy g'''-closed set. Since, by proposition 2.3, every fuzzy g'''-closed set is fuzzy g-closed. Therefore A^c is fuzzy g-closed set. Hence A is fuzzy g-open in (X, τ).

The converse of Proposition 3.12 need not be true as seen from the following example.

Example 3.13:

Let X = {a, b}. Consider the fuzzy topology τ as in Example 3.7, where γ is fuzzy set in X defined by γ(a)=0.5, γ(b)=0.5. Clearly γ is fuzzy g-open but not fuzzy g'''-open set in (X, τ).

Proposition 3.14:

Every fuzzy g'''-open set is fαgs-open.

Proof:

If A is fuzzy g'''-open set in (X, τ), then A^c is fuzzy g'''-closed set. Since, by proposition 2.3, every fuzzy g'''-closed set is f αgs-closed. Therefore A^c is f αgs-closed set. Hence A is f αgs-open in (X, τ).

The converse of Proposition 3.14 need not be true as seen from the following example.

Example 3.15:

Let X = {a, b}. Consider the fuzzy topology τ as in Example 3.3, where C is fuzzy set in X defined by C(a)=1, C(b)=0.5. Clearly C is fuzzy αgs-open but not fuzzy g'''-open set in (X, τ).

Proposition 3.16:

Every fuzzy g'''-open set is fuzzy α g-open.

Proof:

If A is fuzzy g'''-open set in (X, τ), then A^c is fuzzy g'''-closed set. Since, by proposition 2.3, every fuzzy g'''-closed set is fuzzy α g-closed. Therefore A^c is fuzzy α g-closed set. Hence A is fuzzy α g-open in (X, τ).

The converse of Proposition 3.16 need not be true as seen from the following example.

Example 3.17:

Let X = {a, b}. Consider the fuzzy topology τ as in Example 3.3, where C is fuzzy set in X defined by C(a)=1, C(b)=0.5. Clearly C is fuzzy α g-open but not fuzzy g'''-open set in (X, τ).

Proposition 3.18:

Every fuzzy g'''-open set is fgs-open.

Proof:

If A is fuzzy g'''-open set in (X, τ), then A^c is fuzzy g'''-closed set. Since, by proposition 2.3, every fuzzy g'''-closed set is fgs-closed. Therefore A^c is fgs-closed set. Hence A is fgs-open in (X, τ).

The converse of Proposition 3.18 need not be true as seen from the following example.

Example 3.19:

Let X = {a, b}. Consider the fuzzy topology τ as in Example 3.7, where γ is fuzzy set in X defined by γ(a)=0.5, γ(b)=0.5. Clearly γ is fgs-open but not fuzzy g'''-open set in (X, τ).

Proposition 3.20:

Every fuzzy g'''-open set is fuzzy gsp-open.

Proof:

If A is fuzzy g'''-open set in (X, τ), then A^c is fuzzy g'''-closed set. Since, by proposition 2.3, every fuzzy g'''-closed set is fuzzy gsp-closed. Therefore A^c is fuzzy gsp-closed set. Hence A is fuzzy gsp-open in (X, τ).

The converse of Proposition 3.20 need not be true as seen from the following example.

Example 3.21:

Let X = {a, b}. Consider the fuzzy topology τ as in Example 3.7, where γ is fuzzy set in X defined by γ(a)=0.5, γ(b)=0.5. Clearly γ is fuzzy gsp-open but not fuzzy g'''-open set in (X, τ).

Lemma 3.22:

A fuzzy subset A of (X, τ) is fuzzy g'''-open if and only if F ≤ int(A) whenever F is fuzzy gs-closed and F ≤ A.

Proof:

Suppose that F≤ int(A) such that F is fgs-closed set and F ≤ A. Let A^c ≤ U where U is fsg-open . Then U^c ≤ A and U^c is fsg-closed . Therefore U^c ≤ int(A) by hypothesis. Since U^c ≤ int(A), we have $(\text{int}(A))^c$ ≤ U. i.e.,cl(A^c) ≤ U, since cl(A^c) = $(\text{int}(A))^c$. Thus A^c is f g'''-closed set. i.e., A is f g'''-open.

Conversely, suppose that A is f g'''-open such that F ≤ A and F is fgs-closed. Then F^c is fsg-open and $A^c \leq F^c$. Therefore, cl(A^c) ≤ F^c by definition of f g'''-closedness and so F ≤ int(A) , cl(A^c) = $(\text{int}(A))^c$.

4. PROPERTIES OF FUZZY g'''-OPEN SETS

In this section, we discuss some basic properties of fuzzy g'''-open sets.

Theorem 4.1:

If A and B are fuzzy g'''-open sets in (X,τ), then A∧B is fuzzy g'''-open set in (X, τ).

Proof:

If $G \leq A \wedge B$ and G is fgs-closed set, then $G \leq A$ and $G \leq B$. Since A and B are fuzzy g''' -open sets, $G \leq int(A)$ and $G \leq int(B)$ and hence $G \leq int(A) \wedge int(B) = int(A \wedge B)$. Thus $A \wedge B$ is fuzzy g''' -open set in (X, τ).

Theorem 4.2:

If A is fuzzy g''' -open set in (X, τ) and $int(A) \leq B \leq A$, then B is fuzzy g''' -open set in (X, τ).

Proof:

Let $G \leq B$ where G is fgs-closed set. Since $B \leq A$, $G \leq A$. Since A is fuzzy g''' -open set, $G \leq int(A)$. Since $int(A) \leq B$, $G \leq int(A) \leq int(B)$. Therefore B is fuzzy g''' -open

set in X.

Theorem 4.3:

If A is a fgs-closed set and fuzzy g''' -open set in (X, τ), then A is fuzzy open set in (X, τ).

Proof:

Since A is fgs-closed set and fuzzy g''' -open set, $A \leq int(A)$ and hence A is fuzzy open set in (X, τ).

Theorem 4.4:

Let A be a fuzzy g''' -open set of a topological space (X, τ). If A is fuzzy regular closed set, then sint(A) is also fuzzy g''' -open sets.

Proof:

Since A is fuzzy regular closed in X, $A = cl(int(A))$. Then $sint(A)=A \wedge cl(int(A)) = A$. Thus, sint(A) is fuzzy g''' -open sets in (X, τ).

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