Some Results on Prime Cordial Labeling Of Graphs

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Abstract

A prime cordial labeling of a graph G with the vertex set V(G) is a bijection f: V(G)→{1, 2, 3, ... $\lfloor$V(G)$\rfloor$} such that each edge uv is assigned the label 1 if gcd(f(u), f(v)) = 1 and 0 if gcd(f(u), f(v)) > 1 then the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. A graph which admits a prime cordial labeling is called a prime cordial graph. In this paper we prove that Y-tree and X-tree are prime cordial.

Keywords : labeling, prime cordial labeling, X-tree, Y-tree.

1. Introduction

We consider a finite, connected undirected graph G=(V(G), E(G)). For standard terminology and notations we follow Bondy J.A and Murthy USR [4]. In this section we provide brief summary of definition and the required information for our investigation.

The *graph labeling* is an assignment of numbers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) then the labeling is called a *vertex labeling* (*edge labeling*)

Many types of labeling schemes have been introduced so far and explored as well by many researchers. Graph labelings have enromous applications within mathematics as well

as to several areas of computer science and communication networks. According to Beineke and Hegde [3] graph labeling serves as a frontier between number theory and structure of graphs. Various applications of graph labeling have been studied by Yegnanaryanan and Vaidhyanathan [16]. For a dynamic survey on various graph labeling problems along with an extensive bibliography we refer to Gallian [5].

The study of prime numbers is of great importance as prime numbers are scattered and there are arbitrarily large gaps in the sequence of prime numbers. The notion of prime labeling was originated by Entringer and was introduced by Tout *et al.* [7].

After this many researchers have explored the notion of prime labeling for various graphs. Vaidya and Prajapati [13,14] have investigated many results on prime labeling. Same authors [15] have discussed prime labeling in the context of duplication of graph elements. Motivated through the concepts of prime labeling and cordial labeling a new concept termed as a prime cordial labeling was introduced by Sundaram *et al.* [6] which contains blend of both the labelings.

Definition: 1.1

A prime cordial labeling of a graph G with vertex set V(G) is a bijection f: V(G) → {1, 2, ... $\lvert$V$\rvert$} and the induced function f* : E (G) → {0,1} is defined by

$$f*(e:uv) \ = \ 1, \quad \text{if gcd } (f(u), f(v))=1$$

$$= \ 0, \quad \text{otherwise}$$

the number of edges having label 0, and the number of edges having label 1, differ by atmost 1. A graph which admits prime cordial labeling is called a prime cordial graph.

Sundaram et. al [6] have investigated several results on prime cordial labeling. Prime cordial labeling for some cycle related graphs have been discussed by Vaidya and Vihol [8]. Prime cordial labeling in the context of some graph operations have been discussed by Vaidya and Vihol [9] and Vaidya and Shah [10,11]. Vaidya and Shah [11] have proved that the wheel graph Wn admits prime cordial labeling for n ≥ 8 while same authors in [12] have discussed prime cordial labeling for some wheel related graphs. Babitha and Babuji [2] have

exhibited prime cordial labeling for some cycle related graphs and discussed the duality of prime cordial labeling. The same authors in [1] have investigated some characterization of prime cordial graphs and derived various methods to construct large prime cordial graph using existing prime cordial graphs.

Definition: 1.2

Y-tree is the tree obtained by taking three paths of same length and identifying one end point of each path. In otherwords Y – tree is got by sub division of $K_{1,3}$ n times.

Definition: 1.3

X-tree is the graph obtained by taking five paths P_1, P_2, P_3, P_4 and P_5 of same length and identifying the end vertices of P_1, P_2, P_3 and then identifying the other end vertex of P_3 with the end vertices of P_4 and P_5.

In this paper we prove that star $K_{1,n}$ for n even, X-tree and Y-tree are prime cordial.

2. Main Results

Theorem: 2.1

$K_{1,n}$ is prime cordial if n is even and $n \not\equiv 0 \pmod 4$, $n \geq 4$

Proof:

Let $V(K_{1,n}) = \{c, v_1, v_2, ... v_n\}$

$E(K_{1,n}) = \{c\, v_i \,/\, 1 \leq i \leq n\}$

Define a labeling f: $V(K_{1,n}) \rightarrow \{1, 2, ... n+1\}$ by

$f(c) = n,$

$f(v_i) = i$ for $1 \leq n - 1$

$f(v_n) = n+1$

Then   gcd $(f(c), f(v_i)) = (n, i) = 1$ if i is odd.

$$\neq 1 \quad \text{if i is even}$$

$$\gcd(f(c), f(v_n)) = (n, n+1) = 1$$

Thus $K_{1,n}$ is prime cordial.

Theorem: 2.2

Y-tree is prime cordial for all $n \geq 4$

Proof:

Let $V(G) = \{u_1, u_2, ... u_{n-1},$

$v_1, v_2, ... v_{n-1},$

$u_n = v_n = w_1, w_2, ...w_n\}$

and $E(G) = \{ u_i u_{i+1} \mid 1 \leq i \leq n-1 \}$

$\cup \{ v_i v_{i+1} \mid 1 \leq i \leq n-1 \}$

$\cup \{w_i w_{i+1} \mid 1 \leq i \leq n-1 \}$

Case (i): n is even

Define a labeling $f: v(G) \rightarrow \{1, 2, ... 3n-2\}$ by

$f(u_i) = 2i-1$ for $1 \leq i \leq n-1$

$f(v_i) = 2i$ for $1 \leq i \leq n-1$

$f(w_i) = 2n + (2i-2)$ for $1 \leq i \leq \lfloor n/2 \rfloor$

$f(w_n) = 3n-3,$

$f(w_{n-i}) = f(w_n) - 2i$ for $1 \leq i \leq \lfloor n/2 \rfloor$

Case (ii): n is odd and $n+1 \equiv 0 \pmod 3$

Define a labeling $f: V(G) \rightarrow \{1, 2, ... 3n-2\}$ by

$f(u_i) = 2i-1$ for $1 \leq i \leq n-1$

$f(v_i) = 2i$ for $1 \leq i \leq in-1$

$f(w_i) = 2n + (2i-2)$ for $1 \leq i \leq \lfloor n/2 \rfloor - 1$

$f(w_n) = 3n-2,$

$f(w_{n-i}) = f(w_n) - 2i$ for $1 \leq i \leq \lfloor n/2 \rfloor$

and then interchange the labels of w_{n-2} and $w_{\lfloor n/2 \rfloor +1}$

Case (iii): n is odd and $n+1 \equiv 0 \pmod 3$

Define a labeling $f: V(G) \rightarrow \{1, 2, ... 3n-2\}$ by

$f(u_i) = 2i-1$ for $1 \le i \le n-1$

$f(v_i) = 2i$ for $1 \le i \le n-1$

$f(w_i) = 2n + (2i - 2)$ for $1 \le i \le \lfloor n/2 \rfloor - 1$

$f(w_n) = 3n - 2,$

$f(w_{n-i}) = f(w_n) - 2i$ for $1 \le i \le \lfloor n/2 \rfloor$

In all the above cases $\left| e_{f(0)} - e_{f(1)} \right| \le 1$.

Hence Y tree is prime cordial.

Theorem: 2.3

X-tree is prime cordial for all $n \ge 9$ except $n = 3^p$ where p is odd.

Proof:

Let $V(G) = \{u_1, u_2, ... u_{n-1}, u_{n+1}, u_{n+2}, ... u_{2n-1},$

         $v_1, v_2, ... v_{n-1}, v_{n+1}, v_{n+2}, ... v_{2n-1},$

         $u_n = v_n = w_1, w_2, ... w_n = u_{2n} = v_{2n}\}$

and $E(G) = \{u_i u_{i+1} \mid 1 \le i \le n-1$ and $n+1 \le i \le 2n\}$
                $\cup \{v_i v_{i+1} \mid 1 \le i \le n-1$ and $n+1 \le i \le 2n\}$
                $\cup \{w_i w_{i+1} \mid 1 \le i \le n-1\}$

Case (i): n is even

Define a labeling $f: V(G) \rightarrow \{1, 2, ... 5n - 4\}$ by

$f(u_i) = 2i-1$ for $1 \le i \le n-1$

$f(v_i) = 2i$ for $1 \le i \le n-1$

$f(w_i) = 2n + (2i - 2)$ for $1 \le i \le \lfloor n/2 \rfloor$

$f(w_n) = 3n - 3,$

$f(w_{n-i}) = f(w_n) - 2i$ for $1 \le i \le \lfloor n/2 \rfloor - 1$

$f(v_{n+1}) = 3n,$

$$f(v_i) = f(v_{i-1}) + 2 \qquad \text{for } n+2 \leq i \leq 2n - 1$$

$$f(u_{n+1}) = 3n - 1,$$

$$f(u_i) = f(u_{i-1}) + 2 \qquad \text{for } n+2 \leq i \leq 2n - 1$$

Case (ii): If n is odd, $n \geq 9$, $n+1 \not\equiv 0 \pmod 3$ and $n+1 \equiv 0 \pmod{10}$

Define a labeling f: $V(G) \rightarrow \{1, 2, \dots\dots\dots 5n - 4\}$

$$f(u_i) = 2i-1 \qquad \text{for } 1 \leq i \leq n - 1$$

$$f(v_i) = 2i \qquad \text{for } 1 \leq i \leq n - 1$$

$$f(w_i) = 2n + (2i - 2) \qquad \text{for } 1 \leq i \leq \lfloor n/2 \rfloor$$

$$f(w_n) = 3n - 2,$$

$$f(w_{n-i}) = f(w_n) - 2i \qquad \text{for } 1 \leq i \leq \lfloor n/2 \rfloor$$

$$f(v_{n+1}) = 3n - 1,$$

$$f(v_i) = f(v_{i-1}) + 2 \qquad \text{for } n + 2 \leq i \leq 2n-1$$

$$f(u_{n+1}) = 3n,$$

$$f(u_i) = f(u_{i-1}) + 2 \qquad \text{for } n + 2 \leq i \leq 2n - 1$$

Case (iii)   : If n is odd, $n > 9$, $n+1 \equiv 0 \pmod 3$ and $n+1 \not\equiv 0 \pmod{10}$

In the labeling defined in case (ii) interchange the labels of w_{n-2} and $w_{\lfloor n/2 \rfloor+2}$

Case (iv)   : n is odd and $n \equiv 0 \pmod 3$

Define a labeling f: $V(G) \rightarrow \{1, 2, \dots\dots\dots 5n - 4\}$

$$f(u_i) = 2i - 1 \qquad \text{for } 1 \leq i \leq n - 1$$

$$f(v_i) = 2i \qquad \text{for } 1 \leq i \leq n - 1$$

$$f(w_i) = 2n + (2i - 2) \qquad \text{for } 1 \leq i \leq \lfloor n/2 \rfloor$$

$$f(w_n) = 3n - 2,$$

$$f(w_{n-i}) = f(w_n) - 2i \qquad \text{for } 1 \leq i \leq \lfloor n/2 \rfloor$$

$$f(v_{n+1}) = 3n - 1,$$

$$f(v_i) = f(v_{i-1}) + 2 \qquad \text{for } n+2 \leq i \leq 2n - 1$$

$$f(u_{n+1}) = 3n,$$

$$f(u_i) = f(u_{i-1}) + 2 \qquad \text{for } n + 2 \leq i \leq 2n - 1$$

and then interchange the labels of w_{n-2} and $w_{[n/2]+1}$

In all the above cases $\left| e_f(0) - e_f(1) \right| \leq 1.$

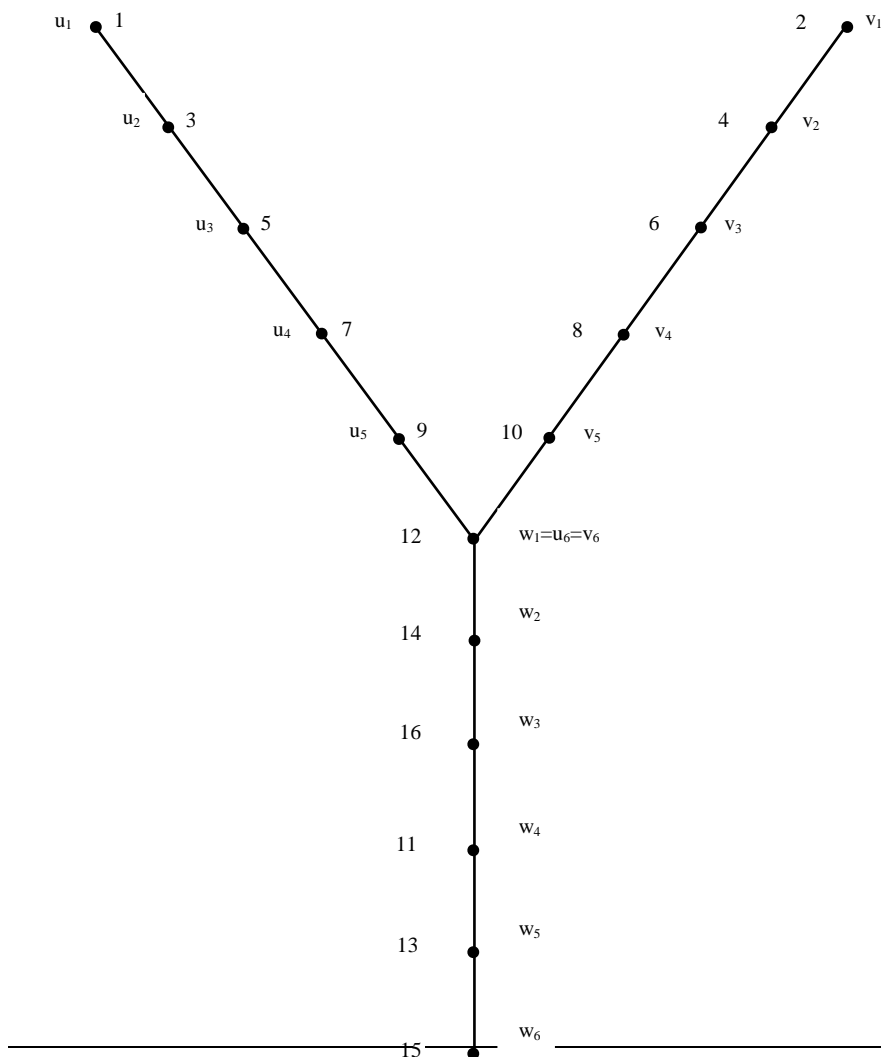Hence X-tree is prime cordial

Remark: 2.4

    X-tree is not a prime cordial if $n = 3^p$ where p is odd or if $n \leq 8$.

Conclusion:

    Here we investigate new results on prime cordial labeling for star $K_{1,n}$ (n is even) X-tree and Y-tree. It is possible to investigate similar results for other graph families. There is a scope to obtain similar results corresponding to
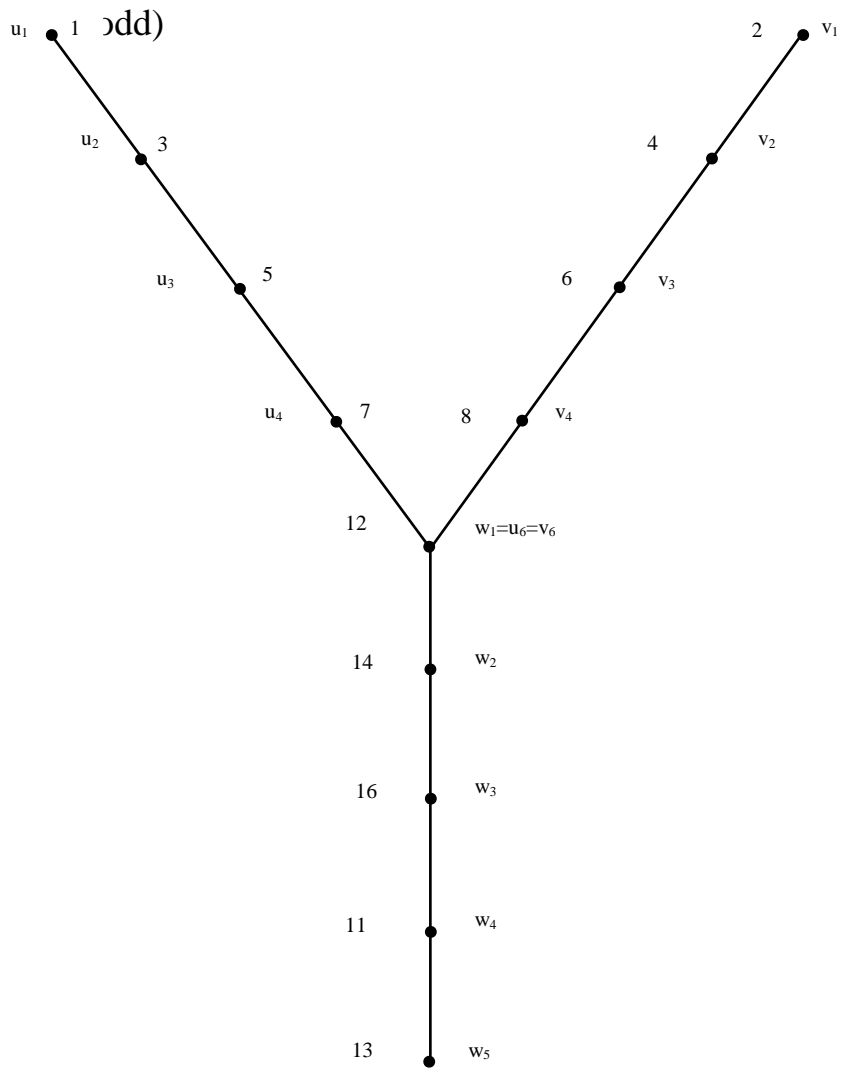
Example: 1

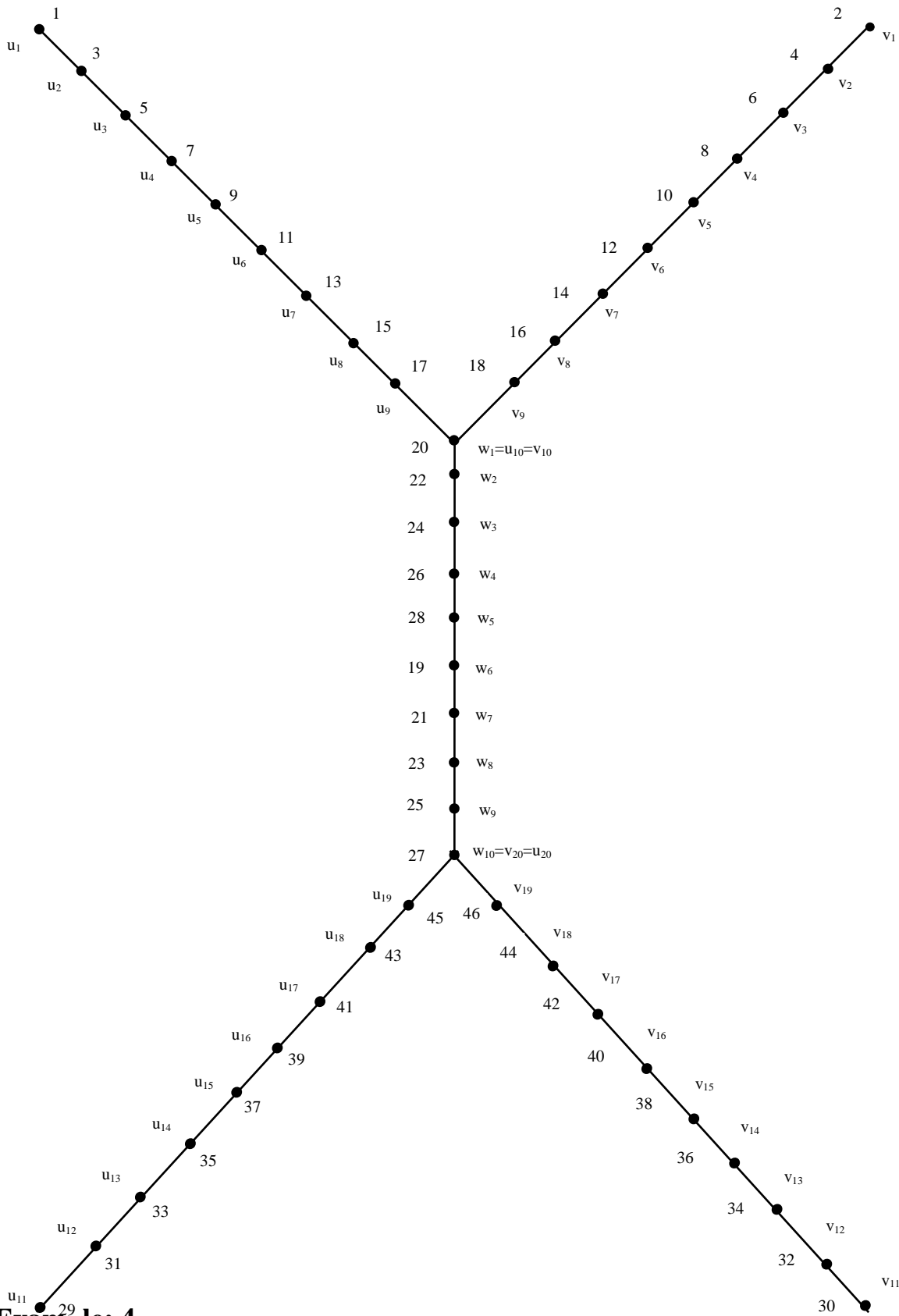   Y-tree is prime cordial when $n = 6$ (even)
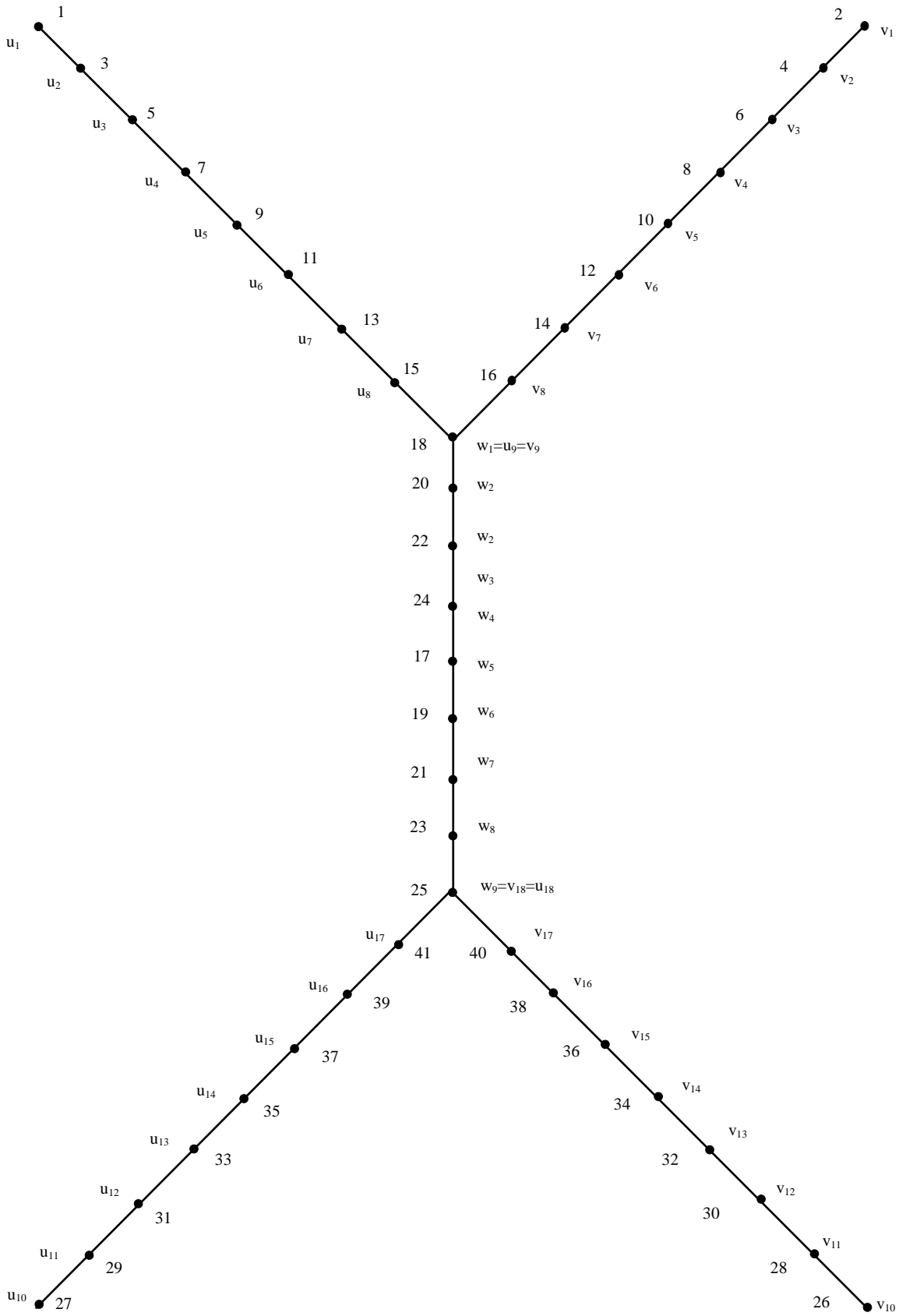
Example : 2

Y-tree is prime cordial when n = 5

(odd)

Example: 3

X-tree is prime cordial, n = 10 (even)

Example: 4

X-tree is prime cordial, n = 9 (odd)

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