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Altered Fibonacci Sequences

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Abstract : Fibonacci numbers are non-negative integers defined by a definite rule have both prime numbers and composite numbers. Here we established the results on altered Fibonacci sequence and the greatest common divisors of its terms. Some identities on the same are also derived.

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Keywords: Fibonacci sequence, Lucas Sequence.

1. INTRODUCTION

Altered Lucas Sequences was defined by S. Harne [1] and Dudley and Tucker [2] defined altered Fibonacci Sequences. They also derived greatest common divisors on the same. Dudley and Tucker [2] gives some results on altered Fibonacci sequence using the identities given by Hoggatt[3]. In a similar manner, here we define the sequence by altering the terms of Fibonacci sequence and derived some results and identities of it.

Let the Fibonacci and Lucas sequence be defined as usual:

$$F_{n+1} = F_n + F_{n-1}, \qquad L_{n+1} = L_n + L_{n-1}, \qquad n = 1, 2, 3, \ldots ,$$

$$F_0 = 0,\ F_1 = 1,\ L_0 = 2,\ L_1 = 1.$$

2. ALTERED FIBONACCI SEQUENCE

If we alter the sequence slightly by letting

$$P_n = F_n + (-1)^n, \qquad n = 1, 2, \ldots \qquad \ldots(2.1)$$

then the nature of the sequence can be studied from the following table:

n	1	2	3	4	5	6	7	8	9	10	11	12	13
(P_n, P_{n+1})	1	1	4	1	3	2	11	1	8	1	29		

n	14	15	16	17	18	19	20
(P_n, P_{n+1})	2	21	1	76	1	55	2

An observation of the table shows that the entries of the line (P_n, P_{n+1}) are either Lucas numbers or Fibonacci numbers.

3. RESULTS

Based on the values of P_n some identities are discussed below:

Theorem 3.1: For all natural number n,

$$P_n + P_{-n} = \begin{cases} F_3 & , n \text{ even} \\ F_3(F_n - F_1) & , n \text{ odd} \end{cases}$$

Proof: Using equation (2.1),

$$P_n + P_{-n} = F_n + (-1)^n + F_{-n} + (-1)^{-n}$$

$$= F_n\left(1 + (-1)^{n+1}\right) + 2(-1)^n$$

$$= \begin{cases} F_3 & , n \text{ even} \\ F_3(F_n - F_1) & , n \text{ odd} \end{cases}$$

Theorem 3.2: For all natural number n,

$$P_n^2 = (F_{-n} - F_1)^2$$

Proof: Using equation (2.1),

$$P_n^2 = \left[F_n + (-1)^n\right]^2$$

$$= F_{-n}^2 - 2F_{-n} + F_1^2$$

$$= (F_{-n} - F_1)^2$$

Theorem 3.3: For all natural number n,

$$P_n^2 = (F_1 \pm F_n)^2$$

Proof:

$$P_n^2 = \left[F_n + (-1)^n\right]^2$$

$$= F_n^2 + 1 + 2(-1)^n F_n$$

$$= (F_1 \pm F_n)^2$$

where sign depends on the value of n.

Theorem 3.4: For all natural number n,

$$P_{-n} = \left(F_{n+1}F_{n-1} - F_n^2\right)(1 - F_n)$$

Proof: Using equation (2.1),

$$P_{-n} = F_{-n} + (-1)^{-n}$$

$$= (-1)^n (1 - F_n)$$

$$= \left(F_{n+1}F_{n-1} - F_n^2\right)(1 - F_n) \qquad \text{\{By Cassini's Identity\}}$$

Theorem 3.5: For all natural number n,

$$P_n + P_{n+1} = P_{n+2} \pm 1$$

Proof: Using equation (2.1),

$$P_n + P_{n+1} = F_n + (-1)^n + F_{n+1} + (-1)^{n+1}$$

$$= F_n + F_{n+1}$$

$$= F_{n+2}$$

$$= P_{n+2} \pm 1$$

where sign depends on the value of n.

Theorem 3.6: For all $m, n \geq 1,$ prove that

$$P_m + P_n = \begin{cases} F_m + F_n + 2 & ,both\ m\ \&\ n\ are\ even \\ F_m + F_n - 2 & ,both\ m\ \&\ n\ are\ odd \\ F_m + F_n & ,one\ is\ even\ \&\ other\ is\ odd \end{cases}$$

Proof: Using equation (2.1) , it can be proved.

Theorem 3.7: For an even natural number n,

$$\sum_{r=1}^{m} P_{rn} = \sum_{r=1}^{m} F_{rn} + m$$

For an odd natural number n,

$$\sum_{r=1}^{m} P_{rn} = \begin{cases} \sum_{r=1}^{m} F_{rn} & ,m\ even \\ \sum_{r=1}^{m} F_{rn} - F_1 & ,m\ odd \end{cases}$$

Proof: Using the identity (2.1) for different values of m, the theorem can be proved.

Theorem 3.8: For $m \geq 1,\ n > 2,$

$$(F_n,\ F_{n+m}) = F_n$$

after every (n-1) jumps and m is multiple of n

and $(F_2,\ F_{2+m}) = F_2 \quad \forall\ m \geq 1$

Proof: The result can be derived by inspection on some values of Fibonacci sequence.

Some more results for the sequence P_n are written below:

Result 3.1: Observing the sequences P_n and F_n we arrive at the result that $P_n + F_n$ is always an odd number for all natural numbers.

Result 3.2: For all natural numbers n and r,

$$P_{rn} = F_{rn} \pm F_1$$

the sign depends on the value of r and n both.

Result 3.3: For all natural numbers,

$$(P_{2n},\ P_{2n+1}) = \begin{cases} F_{n+1} & ,n\, is\, odd\, natural\, number \\ L_{n+1} & ,n\, is\, even natural\, number \end{cases}$$

Result 3.4: For all natural numbers,

$$(P_{2n+1},\ P_{2n+2}) = \begin{cases} F_3 & ,n\, is\, multiple of\ 3 \\ F_2 & ,otherwise \end{cases}$$

which is analogous to the result 4 in paper [1].

Result 3.5: For all even natural numbers,

$$(P_{2n+1},\ P_{2n+2}) = F_3$$

which is again analogous to the result for K_n in paper [1].

Result 3.6: For all natural number n,

$$P_{2n} - 1 = F_{2n}$$

Result 3.3 to 3.5 follows by observing the table and result 3.2 and 3.6 proves by using identity (2.1).

Special Case: If we modify the equation (2.1) by letting

$$Q_n = F_n - (-1)^n \qquad n = 1, 2, 3, \ldots$$

then we have following results:

1) $(Q_{2n+1}, Q_{2n+2}) = L_1$ for all natural number n except when n is multiple of 3.

2) $(Q_{3n+1}, Q_{3n+2}) = L_0$ for all even natural number n

3) $(Q_{2n+2}, Q_{2n+3}) = \begin{cases} F_{n+2} & , for odd\ natural\ number\ n \\ L_{n+2} & for even natural\ number\ n \end{cases}$ which can be proved in similar manner.

4. CONCLUSION

There are many known identities for Fibonacci sequence. This paper describes identities of altered Fibonacci sequence and the greatest common divisors of its consecutive terms. Also we have developed its connection with Lucas and Fibonacci sequence. We have also described greatest common divisors of different Fibonacci numbers. New identities can be discovered by using different concepts.

5. REFERENCES

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