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Numerical Solution of Third Order Homogenous Differential Equations Using Exponentially Fitted Collocation Method (EFCM)

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ABSTRACT

The Exponentially Fitted Method (EFM) is an analytical and numerical method for solving various homogeneous equations. In this study,we proposed a reliable numerical method named Exponentially Fitted Collocation Method (EFCM) which will increase the rate of convergence of the initial value problems. The new method is applied to third order homogenous initial value problems. The comparative study between exact solution and Exponentially Fitted Collocation Method (EFCM) is presented. The results demonstrate liability and efficiency of the new method developed.

Keywords: Exponentially Fitted Collocation Method (EFCM), homogeneous equations, Initial value problems (IVP), rate of convergence, analytical and numerical solutions

INTRODUCTION

Third order differential equations are frequently encountered in applied sciences like engineering, physics and geology. These require the determination of a function of a single independent variable satisfying a given differential equation and subject to different values at the independent variable at the initial domain. Many problems in engineering and science can be formulated as linear and non-linear differential equations e.g., Growth rate equations, Vibrational Motion, Quantum Mechanics, Steady State Condition, Heat transfer in a thin heated wire electric potential inside a thin conductor, deflection of a thin elastic thread under load and many others[1-3].

In this paper, we presented and applied a newly proposed method called Exponentially Fitted Collocation Method (EFCM) to obtain the numerical solution of the third order homogenous initial value problems of the form:

$$\frac{d^3y}{dt^3}+\alpha_1\frac{d^2y}{dt^2}+\alpha_2\frac{dy}{dt}+\alpha_3 y(t)+f(t,y)=g(t)$$

$$0\le t\le 1 \qquad (1.0)$$

Subject to the initial conditions

$$y(0)=A_1,\ y^{/}(0)=A_2,\ y^{//}(0)=A_3 \qquad (1.1)$$

Where and $\alpha_1,\alpha_2,\alpha_3$ are coefficients of the function y(t),

A_1, A_2, A_3 are constants and f(t,y) = 0

g(t)=0

The theory of collocation method has been extensively used to construct numerical solution of Ordinary Differential Equation, Taiwo O.A [4] proposed exponential fitting for the solution of two points boundary value problems with the cubic spline collocation and Tau method, Adejide and Adinira [5] proposed a five-step ninth order hybrid linear multi-step method for the solution of the first order initial value problems via the interpolation and collocation procedure and to mention just few [see 6-10].

This paper is arranged as follows. In section 1 we briefly discussed the concept of linear equations initial value problems and the recurrence formula of the first ten shifted Chebyshev polynomials and its collocated point table is presented. In section 2, the method of the Exponentially Fitted Collocation Method (EFCM) is presented in details. In section 3, the proposed method is applied to solve some third order initial value problems and exact solutions are obtained. Finally we give a brief conclusion in the last section

SHIFTED CHEBYSHEV POLYNOMIALS SERIES

The shifted Chebyshev polynomial is defined on the interval [0, 1] and can be determined with the aid of the following recurrence formula.

$$T_{n+1}(t)=\frac{2(2t-b-a)}{b-a}T_n(t)+T_{n-1}(t)$$

$$T_0(t)=1 \text{ and } T_1(t)=2t-1$$

$a\le t\le b,$

Where $a=0,\ b=1$

The first ten shifted Chebyshev are given as:

$$T_0(t)=1$$
$$T_1(t) = 2t-1$$
$$T_2(t) = 8t^2-8t+1$$
$$T_3(t) = 32t^3-48t^2+18t - 1$$
$$T_4(t) = 12\,8t^4-256t^3+160t^2-32t+1$$
$$T_5(t) = 512\,t^5-128t^4+1120t^3-400t^2+50t-1$$
$$T_6(t) = 2048t^6-6144t^5+6912t^4-3584t^3+840t^2-72t+1$$
$$T_7(t) = 8192t^7-28672t^6+39424t^5-26880t^4+9408t^3-1568t^2+98t-1$$
$$T_8(t) = 32768\,t^8-131072t^7+212992t^6-180224t^5+84480t^4-21304t^3+2688t^2-128t+1$$
$$T_9(t) = 131072\,t^9-589824t^8+1105920t^7-1118208t^6+658944t^5-228096t^4+44352t^3-4320t^2+162t-1$$
$$T_{10}(t) = 524288t^{10}-2621440t^9+5570560t^8-655360t^7+4659200t^6-2050048t^5+549120t^4-84480t^3+6600t^2-200t+1$$

Collocate the above shifted Chebyshev polynomials at a point $t=t_i$ and leads to Table 1

$$\text{Where } t_i = a+\frac{(b-a)i}{N+2},\ i=1,2,3......N+1 \quad a=0 \ \& b=1$$

TABLE 1: **Numerical values of Collocated First Ten Shifted Chebyshev Polynomials**

t_i	$N=0$ 1	$N=1$ $2t_i-1$	$N=2$ $8t_i^2-8t_i+1$	$N=3$ $32t^3-48t^2+18t+1$	$N=4$ $128t_i^4-256t_i^3+$ $160t_i^2-32t_i+1$	$N=5$ $512t_i^5-1280t_i^4+$ $1120t_i^3-400t_i^2+50t_i-1$
1	1	-0.33333333	-0.5000000	0.93600000	-0.97530864	0.74225025
2	--------	0.33333333	-1.0000000	0.56800000	0.30987654	-0.79984530
3	---------	-------	-0.5000000	-0.56800000	1.00000000	-0.65692866
4	--------	-------	-------	-0.9360000	0.20987654	0.65692866
5	--------	-------	-------	-------	-0.97530864	0.79984580
6	--------	-------	-------	-------	-------	-0.74225025

t_i	$N=6$	$N=7$	$N=8$	$N=9$	$N=10$
1	-0.3671875	-0.045312232	0.4219724800	-0.7177765374	0.9104895934
2	1.0000000	-0.8314009562	0.4219724800	0.0751391453	-0.5283916747
3	-0.0546875	0.6909007773	-0.9886899200	0.8935377842	-0.5000000000
4	-1.000000	0.702843568	-0.0400563200	-0.6095976872	0.9672136700
5	-0.0546875	- 0.702843568	1.0000000000	-0.7306769728	0.1034987891
6	1.0000000	-0.6909007773	-0.040056320	0.7306769728	-1.000000000
7	0.3671875	0.8314009562	-0.988689920	0.6095976872	0.1034987891
8	---------	0.045312232	0.4219724800	-0.8935377842	0.9672136700
9	---------	---------	0.4219724800	-0.0751391453	-0.500000000
10	---------	---------	---------	0.7177765374	-0.5283916747
11	---------	---------	---------	------------	0.9104895934

2.0 METHODOLOGY

DISCUSSION OF EXPONENTIALLY FITTED COLLOCATION METHOD (EFCM)

In this section, we derived a new method using power series and substituted into a slightly perturbed general third order homogenous initial value problems of the form:

$$\frac{d^3y}{dt^3}+\alpha_1\frac{d^2y}{dt^2}+\alpha_2\frac{dy}{dt}+\alpha_3 y(t)=0 \qquad (2.1)$$

Subject to initial conditions:

$$y(t_0)=A_1 \qquad (2.2)$$

$$y^{/}(t_0)=A_2 \qquad (2.3)$$

$$y^{//}(t_0)=A_3 \qquad (2.4)$$

Where $\alpha_1,\alpha_2,\alpha_3$ are coefficient of the function y(t)

$A_{1,}A_2, A_3 \in \square$

$t_0 = 0$

Given an approximate solution of the form:

$$y(t) = \sum_{k=0}^{N} Y(t)t^k \tag{2.5}$$

Taking N as arbitrary order of the computation and obtain the third derivative of equation (2.5), we have,

$$y^{/}(t) = \sum_{k=0}^{N} kY(k)t^{k-1} \tag{2.6}$$

$$y^{//}(t) = \sum_{k=0}^{N} k(k-1)Y(k)t^{k-2} \tag{2.7}$$

$$y^{///}(t) = \sum_{k=0}^{N} k(k-1)(k-2)Y(k)t^{k-3} \tag{2.8}$$

Substituting equations (2.5),(2.6),(2.7) and (2.8) into equation (2.1),we obtained

$$\sum_{k=0}^{N} k(k-1)(k-2)Y(k)t^{k-3} + \alpha_1 \sum_{k=0}^{N} k(k-1)Y(k)t^{k-2} + \alpha_2 \sum_{k=0}^{N} kY(k)t^{k-1} +$$
$$\alpha_3 \sum_{k=0}^{N} Y(k)t^k = 0 \tag{2.9}$$

The expansion of equation (2.9), leads to

$$6Y(3) + ... + N(N-2)(N-1)Y(N)t^{N-3} + 2\alpha_1 Y(2) + 6t\alpha_1 Y(3) + ...\alpha_1 N(N-1)Y(N)t^{N-2} +$$
$$\alpha_2 Y(1) + 2t\alpha_2 Y(2) + 3t^2\alpha_2 Y(3) + ... + \alpha_2 NY(N)t^{N-1} + \alpha_3 Y(0) + \alpha_3 tY(1) + \alpha_3 t^2 Y(2) +$$
$$\alpha_3 t^3 Y(3) + ... + \alpha_3 Y(N)t^N = 0 \tag{2.10}$$

$$\alpha_3 Y(0) + \left(\alpha_2 + \alpha_3 t\right)Y(1) + \left(2\alpha_1 + 2\alpha_2 t + \alpha_3 t^2\right)Y(2) + \left(6 + 6t\alpha_1 + 3t^2\alpha_2 + t^3\alpha_3\right)Y(3) +$$
$$....... + \left[N(N-1)(N-2)t^{N-3} + \alpha_1 N(N-1)t^{N-2} + \alpha_2 Nt^{N-1} + \alpha_3 t^N\right]Y(N) = 0 \tag{2.11}$$

Thus, collecting the like terms in equation (2.11) and slightly perturbed to give

$$\alpha_3 Y(0) + \left(\alpha_2 + \alpha_3 t\right)Y(1) + \left(2\alpha_1 + 2\alpha_2 t + \alpha_3 t^2\right)Y(2) + \left(6 + 6\alpha_1 t + 3\alpha_2 t^2 + \alpha_3 t^3\right)Y(3) + ...$$
$$... + \left(N(N-2)(N-1)t^{N-3} + \alpha_1 N(N-1)t^{N-2} + \alpha_2 Nt^{N-1} + \alpha_3 t^N\right)Y(N) = \tau_1 T_N + \tau_2 T_{N-1} \tag{2.12}$$

Here, τ_1 and τ_2 are free tau parameters to be determined and $T_N(t)$ and

$T_{N-1}(t)$ are the Chebyshev polynomials (**see Table 1**)

$$\alpha_3 Y(0)+\left(\alpha_2+\alpha_3 t\right)Y(1)+\left(2\alpha_1+2\alpha_2 t+\alpha_3 t^2\right)Y(2)+\left(6+6\alpha_1 t+3\alpha_2 t^2+\alpha_3 t^3\right)Y(3)+....$$
$$....+\left[N(N-2)(N-1)t^{N-3}+\alpha_1 N(N-1)t^{N-2}+\alpha_2 N t^{N-1}+\alpha_3 t^N\right]Y(N)-\tau_1 T_N-\tau_2 T_{N-1}=0 \qquad (2.13)$$

Hence, equation (2.13) gives rise to (N + 1) algebraic linear system of equations in (N+4) unknown constants. Three extra equations are obtained from the initial condition given as

$$y(t_0)=\sum_{k=0}^{N} Y(k)t^k+\tau_3\ell^{t_0}=A_1 \qquad (2.14)$$

$$y^{/}(t_0)=\sum_{k=0}^{N} kY(k)t^{k-1}+\tau_3\ell^{t_0}=A_2 \qquad (2.15)$$

$$y^{//}(t_0)=\sum_{k=0}^{N} k(K-1)Y(k)t^{k-2}+\tau_3\ell^{t_0}=A_3 \qquad (2.16)$$

Altogether, we obtained (N+4) algebraic linear equations in (N+4) unknown constants. Thus, we put the (N+4) algebraic equations in Matrix form as

$$AX=b \qquad (2.17)$$

Where

$$A=\begin{pmatrix} {}^*b_{11} & {}^*b_{12} & {}^*b_{13} & . \; . \; . {}^*b_{1N} & -T_N(t_1) & -T_{N-1}(t_1) & 0 \\ {}^*b_{21} & {}^*b_{22} & {}^*b_{23} & . \; . \; . {}^*b_{2N} & -T_N(t_2) & -T_{N-1}(t_2) & 0 \\ . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Here

$$^{*}b_{11} = \left[k(k-2)(k-1)t_1^{\,k-3} + \alpha_1 k(k-1)t_1^{\,k-2} + \alpha_2 k t_1^{\,k-1} + \alpha_3 t_1^{\,k} \right] \qquad for \quad k=0$$
$$^{*}b_{12} = \left[k(k-2)(k-1)t_1^{\,k-3} + \alpha_1 k(k-1)t_1^{\,k-2} + \alpha_2 k t_1^{\,k-1} + \alpha_3 t_1^{\,k} \right] \qquad for \quad k=1$$
$$^{*}b_{13} = \left[k(k-2)(k-1)t_1^{\,k-3} + \alpha_1 k(k-1)t_1^{\,k-2} + \alpha_2 k t_1^{\,k-1} + \alpha_3 t_1^{\,k} \right] \qquad for \quad k=0$$
$$\cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot$$
$$\cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot$$
$$^{*}b_{1N} = \left[k(k-2)(k-1)t_1^{\,k-3} + \alpha_1 k(k-1)t_1^{\,k-2} + \alpha_2 k t_1^{\,k-1} + \alpha_3 t_1^{\,k} \right] \qquad for \quad k=N$$

$$X = \left[Y(0), Y(1), Y(2)...., Y(N), \tau_1, \tau_2, \tau_3 \right]^T$$

$$b = \left[0, 0, 0, ...0, A_1, A_2, A_3 \right]^T$$

MAPLE 13 software is used to obtain the unknown constant $Y(0), Y(1), Y(2)...., Y(N), \tau_1, \tau_2$ *and* τ_3

Which are then substituted into the approximate solution

$$y(t) = \sum_{k=0}^{N} Y(k)t^k + \tau_3 \ell^t \qquad (2.18)$$

3.0 NUMERICAL APPLICATION

EXAMPLE 1 Consider the third-order Homogenous initial value problem

$$\frac{d^3y}{dt^3}(t) + 2\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y(t) = 0 \qquad (3.1)$$
$$0 \le t \le 1$$

Subject to initial conditions

$$, \qquad y(0) = 1, y^{/}(0) = 0, y^{//}(0) = 0 \qquad (3.2),$$

Analytical solution is give as

$$y(t)=\left[\frac{-\ell^{-2t}+3\ell^{-t}+\ell^{t}}{3}\right] \qquad (3.3)$$

Consider equation (3.1), we have:

$\alpha_1=2, \alpha_2=-1 \,\&\, \alpha_3=-2$

$a=0, b=1$

Taking arbitrary N = 4 and collocate at a point $t=t_i$

$\Rightarrow T_4(t) \; = 128t_i^4 - 256t_i^3 + 160t_i^2 - 32t_i + 1$ (See table 1)

Substituting the above values into equation (2.17) leads to the matrix below.

$$\begin{bmatrix} -2.0000 & -1.3333 & 3.6111 & 7.9074 & 4.6466 & 0.9753 & -0.9360 & 0 \\ -2.0000 & -1.6667 & 3.1111 & 9.5926 & 10.4938 & -0.2099 & -0.5680 & 0 \\ -2.0000 & -2.0000 & 2.5000 & 11.0000 & 17.3750 & -1.0000 & 0.5680 & 0 \\ -2.0000 & -2.3333 & 1.7778 & 12.0747 & 25.0864 & -0.2099 & 0.9360 & 0 \\ -2.0000 & -2.6667 & 0.9444 & 12.7593 & 33.3873 & 0.9753 & 0.0000 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Maple 13 software is used to obtain the unknown constants:

$\mathrm{Y}(0)=-1.656140560$, $\mathrm{Y}(1) = -2.656140560$, $\mathrm{Y}(2) = -1.328070280$,
$\mathrm{Y}(3) = -0.1233568960$, $\mathrm{Y}(4) = -0.2274466546$,
$\tau_1 = 0.02726930918$, $\tau_2 = 0.05579973579$, $\tau_3 = 2.656140560$

These values are substituted into approximate solution given in equation (2.18):

$$y_4(t) = -1.656140560t^0 - 2.656140560t^1 - 1.328070280t^2 - 0.1233568960t^3$$
$$-0.2274466546t^4 + 2.656140560\ell^t \qquad (3.4)(see \quad table\, 3)$$

EXAMPLE 2 Consider the third-order Homogenous initial value problem

$$\frac{d^3y}{dt^3}(t)+4\frac{d^2y}{dt^2}-\frac{dy}{dt}-4y(t)=0 \qquad (3.5)$$
$$0\le t\le 1$$

Subject to initial conditions

, $y(0)=1, y^{/}(0)=\sqrt{\frac{1}{3}}, y^{//}(0)=1$ (3.6) ,

Analytical solution is give as

$$y(t)=\left(\frac{3-\sqrt{3}}{6}\right)\ell^{-t}+\left(\frac{\sqrt{3}+3}{6}\right)\ell^{t} \quad (3.7)$$

Consider equation (3.5), we have:

$\alpha_1=4, \alpha_2=-1 \,\&\, \alpha_3=-4$

$a=0, b=1$

Taking arbitrary N = 4 and collocate at a point $t=t_i$

$\Rightarrow T_4(t) = 128t_i^4-256t_i^3+160t_i^2-32t_i+1$ (See table 1)

Substituting the above values into equation (2.17) leads to the matrix below.

$$\begin{bmatrix} -4.0000 & -1.6667 & 7.5555 & 9.8981 & 5.3117 & 0.9753 & -0.9360 & 0 \\ -4.0000 & -2.3333 & 6.8889 & 13.5185 & 13.1358 & -0.2099 & -0.5680 & 0 \\ -4.0000 & -3.0000 & 6.0000 & 16.7500 & 23.2500 & -1.0000 & 0.5680 & 0 \\ -4.0000 & -3.6667 & 4.8889 & 19.4815 & 35.3580 & -0.2099 & 0.9360 & 0 \\ -4.0000 & -4.3333 & 3.5556 & 21.6019 & 49.0895 & 0.9753 & 0.0000 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \sqrt{\frac{1}{3}} \\ 1 \end{bmatrix}$$

Maple 13 software is used to obtain the unknown constants:

$Y(0)=0.2937492488$, $Y(1) = -0.1289004820$, $Y(2) = -0.1468746244$,
$Y(3) = -0.21192634701$, $Y(4) = 0.1126331034$, $\tau_1=-0.0009268641178$
$\tau_2=-0.001363368374$, $\tau_3=0.7062507512$

These values are substituted into approximate solution given in equation (2.18):

$$y_4(t)=0.2937492488t^0-0.1289004820t^1-0.1468746244t^2-0.2119263470t^3 1+ 0.1126331034t^4+0.7062507512\ell^t \quad (3.8)(see \quad table\, 4)$$

EXAMPLE 3 Consider the third-order Homogenous initial value problem

$$\frac{d^3y}{dt^3}(t)-2\frac{d^2y}{dt^2}+y(t)=0 \tag{3.9}$$

$$0\le t\le 1$$

Subject to initial conditions

, $y(0)=1, y^{/}(0)=0, y^{//}(0)=0$ (3.10),

Analytical solution is give as

$$\text{y(t)}=\left[\frac{\ell^{t}-\left(\sqrt{5}\right)\ell^{\left(\frac{\sqrt{5}+1}{2}\right)t}+\left(\sqrt{5}\right)\ell^{\left(\frac{-\sqrt{5}-1}{2}\right)t}}{5}\right] \tag{3.11}$$

Consider equation (3.5), we have:

$$\alpha_1=-2,\alpha_2=0\ \&\ \alpha_3=1$$

$$a=0,b=1$$

Taking arbitrary N = 6 and collocate at a point $t=t_i$

$\Rightarrow T_6(t_i)\ =2048t_i^6-6144t_i^5+6912t_i^4-3584t_i^3+840t_i^2-72t_i+1$ (See table 1)

Substituting the above values into equation (2.17) leads to the matrix below.

$$\begin{bmatrix} 1.0000 & 0.1250 & -3.9844 & 4.5020 & 2.6252 & 0.8594 & 0.2197 & 0.3672 & -0.7423 & 0 \\ 1.0000 & 0.2500 & -3.9375 & 3.0156 & 4.5039 & 3.1360 & 1.6409 & -1.000 & 0.7998 & 0 \\ 1.0000 & 0.3750 & -3.8594 & 1.5527 & 5.6448 & 6.3355 & 5.1444 & 0.0547 & 0.6569 & 0 \\ 1.0000 & 0.5000 & -3.7500 & 0.1250 & 6.0625 & 10.0313 & 11.2656 & 1.0000 & -0.6569 & 0 \\ 1.0000 & 0.6250 & -3.6094 & -1.2559 & 5.7776 & 13.7672 & 20.2012 & 0.0547 & 0.7998 & 0 \\ 1.0000 & 0.7500 & -3.4375 & -2.5781 & 4.8164 & 17.1123 & 31.8187 & -1.0000 & 0.7998 & 0 \\ 1.0000 & 0.8750 & -3.2344 & -3.8301 & 3.2112 & 19.6535 & 45.6685 & 0.3672 & 0.0000 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \\ Y(6) \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Maple 13 software is used to obtain the unknown constants:

$Y(0) = 12.17855697$, $Y(1) = 11.17855697$, $Y(2) = 5.589278487$,

$Y(3) = 1.698113467$, $Y(4) = 0.3780143885$, $Y(5) = 0.6590840020$

$Y(6) = 0.02481151893$,

$\tau_1 = -0.01112829119$

$\tau_2 = 0.00005051496109$

$\tau_3 = -11.17855697$

These values are substituted into approximate solution given in equation (2.18):

$$y_4(t) = 12.17855697t^0 + 11.17855697t^1 + 5.589278487t^2 + 0.698113467t^3 + 0.3780143885t^4 + 0.6590840020t^5 + 0.02481151893t^6 - 11.17855697\ell^t \quad (3.11)(see \quad table 5)$$

3.1.0 PROBLEMS SUMMARY TABLE

Table 2.

parameter	**Example 1**	**Example 2**	**Example 3**
α_1	2.00000	4.00000	-2.00000
α_2	-1.00000	-1.00000	0.00000
α_3	-2.00000	-4.00000	1.00000
$y(0)$	1.00000	1.00000	1.000000
$y^{/}(0)$	0.00000	$\sqrt{\frac{1}{3}}$	0.00000
$y^{//}(0)$	0.00000	1.00000	0.000000
$g(t)$	0.00000	0.000000	0.000000
Nature of the function	Homogenous	Homogenous	Homogenous

4.0 NUMERICAL SOLUTION OF EXAMPLE 1 (EXACT AND EFCM SOLUTIONS)

Table 3.

t	EXACT $((y_{exact}(t))$	EFCM (N=4) $(y_{Num}(t))$	$ERROR\|y_{exact}(t)-y_{Num}(t)\|$
0.0	0.9048374180	1.0000000000	0.0951625820
0.1	1.0003174730	1.0003078800	0.0000959300
0.2	1.0024249900	1.0023751530	0.0000498370
0.3	1.0078339450	1.0077327230	0.0001012220
0.4	1.0178186240	1.0176905850	0.0001280390
0.5	1.0334779360	1.0333720020	0.0001059340
0.6	1.0557864990	1.0557512770	0.0000352220
0.7	1.0856372180	1.0856954970	0.0000582790
0.8	1.1238771010	1.1240106590	0.0001335800
0.9	1.1713377340	1.1714926670	0.0001549330
1.0	1.2288616230	1.2289836660	0.0001220430

Table 4. EXAMPLE 2 (EXACT AND EFCM SOLUTIONS)

t	EXACT $(y_{exact}(t))$	EFCM (N=4) $(y_{Num}(t))$	$ERROR\|y_{exact}(t)-y_{Num}(t)\|$
0.0	1.0000000000	1.0000000000	0.000000000
0.1	1.0628354680	1.0628356720	0.000000204
0.2	1.1363081510	1.1363092330	0.000001082
0.3	1.2211533880	1.2211556490	0.000002261
0.4	1.3182203380	1.3182233220	0.000002984
0.5	1.4284804800	1.4284831780	0.000002698
0.6	1.5530373350	1.5530387110	0.000001376
0.7	1.6931375100	1.6931370860	0.000000424
0.8	1.8501831740	1.8501813980	0.000001776
0.9	2.0257460940	2.0257442330	0.000001861
1.0	2.2215833600	2.2215826500	0.000000710

Table 5. EXAMPLE 3 (EXACT AND EFCM SOLUTIONS)

t	EXACT $(y_{exact}(t))$	EFCM (N=6) $(y_{Num}(t))$	$ERROR\lvert y_{exact}(t)-y_{Num}(t)\rvert$
0.0	1.000000000	1.000000000	0.000000000
0.1	0.9998246564	0.9998259500	0.000001294
0.2	0.9985220126	0.9985301700	0.000008157
0.3	0.9947363576	0.9947584800	0.000221220
0.4	0.9868145934	0.9868584300	0.000043840
0.5	0.9727440250	0.9728189100	0.000074885
0.6	0.9500784022	0.9501965600	0.000118160
0.7	0.9158004220	0.9160272500	0.000226830
0.8	0.8664654864	0.8667208000	0.000255310
0.9	0.7975816772	0.7979371100	0.000355430
1.0	0.7039590968	0.7044415700	0.000482400

REMARKS: EFCM: Exponentially Fitted Collocation Method , **EXACT:** Analytical Solutions

N: Computational length

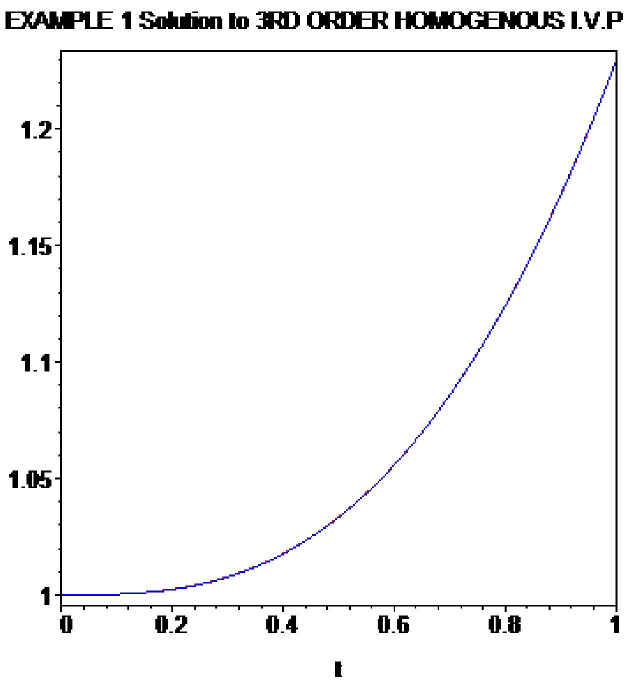

Figure 1. **(EXACT SOLUTION & EXPONENTIALLY FITTED COLLOCATION METHOD)**

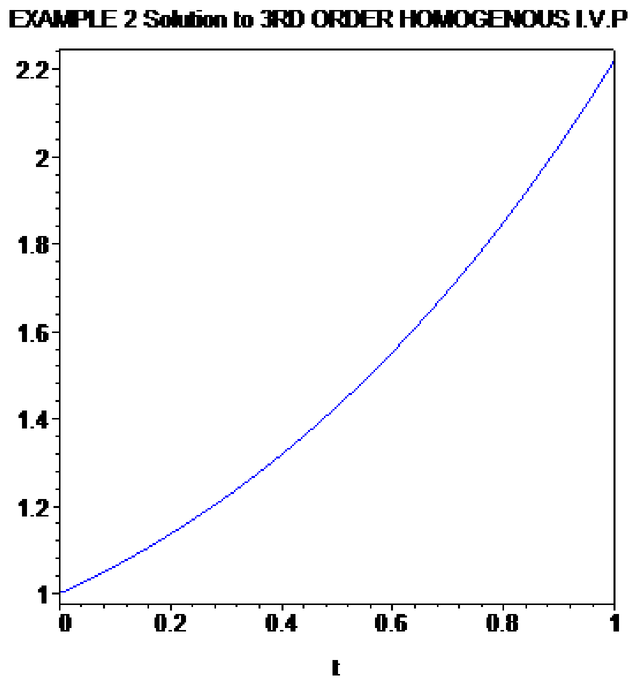

Figure 2. **(EXACT SOLUTION & EXPONENTIALLY FITTED COLLOCATION METHOD)**

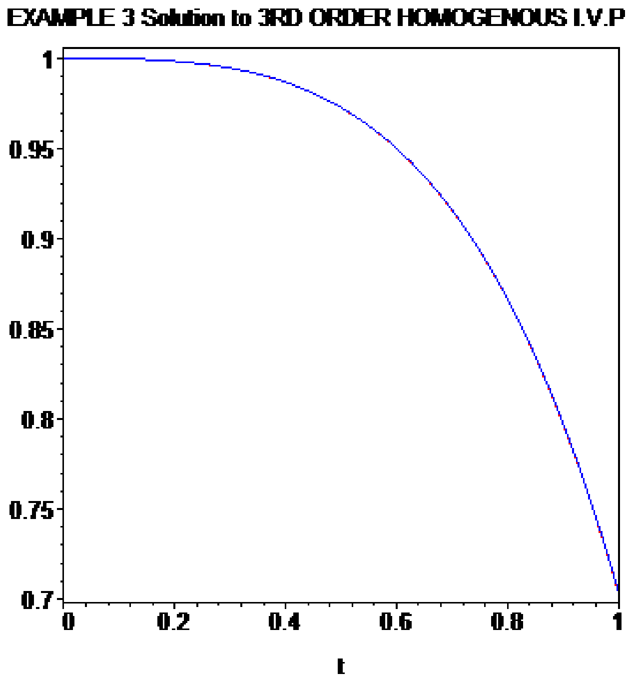

Figure 3. **(EXACT SOLUTION & EXPONENTIALLY FITTED COLLOCATION METHOD)**

CONCLUSION

In this paper we proposed and implemented Exponentially Fitted Collocation Method (EFCM) for solving third order linear homogenous initial value problems. The method is straightforward and very simple to apply. The numerical results by the proposed method (EFCM) show a good agreement with the exact solution. The comparisons are made between approximate solution and exact solution to illustrate the validity and the great potential of the technique. All computation in this paper is done using Maple 13 software.

REFERENCES

1. Morton M. and Joseph W. "**Introduction to General Physics"**second edition pp229-245 (1938)

2.Wilson B. "**College Physics"** Fouth edition prentice Hall inc.Upper saddle river new Jersey pp142-147 (2000)

3.Jeffrey R. "**Introduction to Differential Equations"** Lecture note for MTH2351/2352 pp31-37 (2009)

4. Taiwo O. A. "**Exponential Fitting for the Solution of Two Point Boundary Value Problems (BVP) with Cubic Spline Collocated Tau Method**". International Journal Computer Mathematics pp 1 – 8 (2002)

5.Adejide and Adenira "**A five-step ninth order hybrid linear multi-step method for the solution of the first order initial value problem via the interpolation and collocation procedure**"Internation.Journal of Applied Mathematics .Vol.12 pp314-334 (2002) .

6. Yang C. and Guo.B "**A collocation Method for Initial Value Problems of Second-Order ODEs by using Languarre Functions".** Numerical Mathematics Theory Method Application Vol. 4 No 2 pp 283 – 395. (2011).

7. Fox, L. "**Chebyshev methods for Ordinary Differential equations",** Computer J. 4, pp318 – 331 (1962):

8. Lanczos, C. "**Table of Chebyshev Polynomials"** National Bureau of standards, Appl. Maths Series, 9 Washington: Gov Printing office. (1952)

9. Miller, J.C.P. "**Two numerical approximation of Chebyshev Polynomials**", Proc. Rey. Soc. Edinb. Lxii, pp. 204 – 210 (1946).

10. Onumanyi, P, Oladele, J.O, Adeniyi, R.B and Awoniyi, D.O,"**Derivation of finite differences methods by Collocation",** Abacus Vol. 23.No.2 pp.76-83 , (1993):.