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Numerical solution of fuzzy differential equations by Adams predictor-corrector method and the dependency problem

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Abstract

In this paper, we study the numerical solution of Fuzzy Differential Equations by using predictor-corrector method which is obtained by combining Adams-Bashforth three-step method and Adams-Moulton two-step method. This method is adopted to solve the dependency problem in fuzzy computation. In addition, these methods are illustrated by solving two fuzzy cauchy problems.

Keywords : Fuzzy initial value problem, Dependency problem in fuzzy computation, Adams-Bashforth, Adams-Moulton, predictor-corrector methods.

1 Introduction

Fuzzy Differential Equations(FDE) are used for modelling problems in science and engineering. Most of the problems in science and engineering require the solutions of FDE which are satisfied by fuzzy initial conditions, therefore Fuzzy Initial Value Problems(FIVP) occur and should be solved. Fuzzy set was first introduced by Zadeh[22]. Since then, the theory has been developed and it is now emerged as an independent branch of Applied Mathematics. The elementary fuzzy calculus based on the extension principle was studied by Dubois and Prade[14]. Seikkala[21] and Kaleva[16] have discussed FIVP. Buckley and Feuring[13] compared the solutions of FIVP which where obtained using different derivatives. The numerical solutions of FIVP by Euler's method was studied by Ma et al.[18]. Abbasbandy and Allviranloo[1, 2] proposed the Taylor method and the fourth order Runge-Kutta method for solving FIVP. Palligkinis et al.[20] applied the Runge-Kutta method for more general problems and proved the convergence for n-stage Runge-Kutta method. Allahviranloo et al.[8] and Barnabas Bede[10] studied the numerical solution of fuzzy differential equations by predictor-corrector method. The dependency problem in fuzzy computation was discussed by Ahmad and Hasan[4] and they used Euler's method based on Zadeh's extension principle for finding the numerical solution of FIVP. Later Omar and Hasan[7], adopted the same computation method to derive the fourth order Runge-Kutta method for FIVP. Recently Ahmad and Hasan[4] investigated the dependency problem in fuzzy computation based on Zadeh's extension principle. In this paper we study the dependency problem in fuzzy computations by using predictor-corrector three-step method.

2 Preliminary concepts

In this section, we give some basic definitions and notations.

Definition 2.1. An m-step method for solving the initial-value problem is one whose difference equation for finding the approximation $x(t_{i+1})$ at the mesh point t_{i+1} can be represented by the following equation :

$$\begin{aligned} x(t_{i+1}) &= a_{m-1}x(t_i) + a_{m-2}x(t_{i-1}) + \cdots + a_0 x(t_{i+1-m}) \\ & h\{b_m f(t_{i+1}, x_{i+1}) + b_{m-1} f(t_i, x_i) + \cdots + b_0 f(t_{i+1-m}, x_{i+1-m})\}, \end{aligned} \tag{1}$$

for $i = m-1, m, \ldots, N-1,$ such that $a = t_0 \le t_1 \le \cdots \le t_N = b,\ h = (b-a)/N = t_{i+1} - t_i$ and $a_0, \ldots, a_{m-1}, b_0, \ldots, b_m$ are constants with the starting values

$$x_0 = \alpha_0, x_1 = \alpha_1, x_2 = \alpha_2, \ldots x_{m-1} = \alpha_{m-1}.$$

When $b_m = 0$, the method is known as explicit, since equation (1) gives x_{i+1} explicit in terms of previously determined values. When $b_m \neq 0$, the method is known as implicit, since x_{i+1} occurs on both sides of Equation (1) and is specified only implicitly.

With consideration of Definition 2.1, several multi-step methods are as follow:

Adams-Bashforth two-step method:

$$x_0 = \alpha_0, \quad x_1 = \alpha_1,$$

$$x_{i+1} = x_i + \frac{h}{3}[3f(t_i, x_i) - f(t_{i-1}, x_{i-1})], \qquad where\ i = 1, 2, \ldots, N-1.$$

Adams-Moulton two-step method:

$$x_0 = \alpha_0, \quad x_1 = \alpha_1,$$

$$x_{i+1} = x_i + \frac{h}{12}[5f(t_{i+1}, x_{i+1}) + 8f(t_i, x_i) - f(t_{i-1}, x_{i-1})], \qquad where\ i = 1, 2, \ldots, N-1.$$

Adams-Bashforth three-step method:

$$x_0 = \alpha_0, \quad x_1 = \alpha_1, \quad x_2 = \alpha_2,$$

$$x_{i+1} = x_i + \frac{h}{12}[23f(t_i, x_i) - 16f(t_{i-1}, x_{i-1}) + 5f(t_{i-2}, x_{i-2})], \qquad where\ i = 1, 2, \ldots, N-1.$$

*Adams-Moulton three-step method***:**

$$x_0 = \alpha_0, \quad x_1 = \alpha_1, \quad x_2 = \alpha_2,$$

$$x_{i+1} = x_i + \frac{h}{24}[9f(t_{i+1}, x_{i+1}) + 19f(t_i, x_i) - 5f(t_{i-1}, x_{i-1}) + f(t_{i-2}, x_{i-2})], \qquad where\ i = 1, 2, \ldots, N-1.$$

Definition 2.2. Associated with the difference equation

$$\begin{aligned} & x_{i+1} = a_{m-1}x_i + a_{m-2}x_{i-1} + \cdots + a_0 x_{i+1-m} + F(t_i, h, x_{i+1}, x_i, \ldots, x_{i+1-m}), \\ & x_0 = \alpha_0, x_1 = \alpha_1, \ldots, x_{m-1} = \alpha_{m-1}, \end{aligned} \tag{2}$$

The following, called the characteristic polynomial of the method is

$$p(\lambda) = \lambda^m - a_{m-1}\lambda^{m-1} - a_{m-2}\lambda^{m-2} - \ldots - a_1\lambda - a_0.$$

If $|\lambda_i| \le 1$ for each $i = 1, 2, \ldots, m,$ and all roots with absolute value 1 is simple roots, then the difference method is said to satisfy the root condition.

Proof : See [15].

Definition 2.3. Subset Ã of a universal set X is said to be a fuzzy set if a membership function $\mu_{\tilde{A}}(x)$ takes each object in X onto the interval [0,1]. The function $\mu_{\tilde{A}}(x)$ is the possibility degrees to which each object is compatible with the properties that characterized the group.

A fuzzy set $\tilde{A} \subseteq X$ can also be presented as a set of ordered pairs

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}. \tag{3}$$

The support, the core and the height of A are respectively

$$supp(\tilde{A}) = \{x \in X: x > \mu_{\tilde{A}}(x)\}, \quad (4)$$
$$core(\tilde{A}) = \{x \in X: \mu_{\tilde{A}}(x) = 1\}, \quad (5)$$
$$hgt(\tilde{A}) = \sup_{x \in X} \mu_{\tilde{A}}(x). \quad (6)$$

Definition 2.4. A fuzzy number is a convex fuzzy subset A of R, for which the following conditions are satisfied:

(i) $\tilde{A}$ is normalized. i.e. $hgt(\tilde{A}) = 1$;
(ii) $\mu_{\tilde{A}}(x)$ are upper semicontinuous;
(iii) $\{x \in R: \mu_{\tilde{A}}(x) = \alpha\}$ are compact sets for $0 < \alpha \leq 1$, and
(iv) $\overline{\{x \in R: \mu_{\tilde{A}}(x) = \alpha\}}$ are also compact sets for $0 < \alpha \leq 1$.

Definition 2.5. If $F(R)$ is the set of all fuzzy numbers, and $\tilde{A} \in F(R)$, we can characterize $\tilde{A}$ by its α-levels by the following closed-bounded intervals:

$$[\tilde{A}]^{\alpha} = \{x \in R: \mu_{\tilde{A}}(x) \geq \alpha\} = [a_1^{\alpha}, a_2^{\alpha}], \qquad 0 < \alpha \leq 1, \quad (7)$$
$$[\tilde{A}]^{\alpha} = \overline{\{x \in R: \mu_{\tilde{A}}(x) \geq \alpha\}} = \overline{[a_1^{\alpha}, a_2^{\alpha}]}, \qquad 0 < \alpha \leq 1. \quad (8)$$

Operations on fuzzy numbers can be described as follows: If $\tilde{A}, \tilde{B} \in F(R)$, then for $0 < \alpha \leq 1$

1. $[\tilde{A} + \tilde{B}]^{\alpha} = [a_1^{\alpha} + b_1^{\alpha}, a_2^{\alpha} + b_2^{\alpha}]$;
2. $[\tilde{A} - \tilde{B}]^{\alpha} = [a_1^{\alpha} - b_1^{\alpha}, a_2^{\alpha} - b_2^{\alpha}]$;
3. $[\tilde{A} \cdot \tilde{B}]^{\alpha} = [min\{a_1^{\alpha} \cdot b_1^{\alpha}, a_1^{\alpha} \cdot b_2^{\alpha}, a_2^{\alpha} \cdot b_1^{\alpha}, a_2^{\alpha} \cdot b_2^{\alpha}\}, max\{a_1^{\alpha} \cdot b_1^{\alpha}, a_1^{\alpha} \cdot b_2^{\alpha}, a_2^{\alpha} \cdot b_1^{\alpha}, a_2^{\alpha} \cdot b_2^{\alpha}\}]$;
4. $\left[\frac{A}{B}\right]^{\alpha} = \left[min\left\{\frac{a_1^{\alpha}}{b_1^{\alpha}}, \frac{a_1^{\alpha}}{b_2^{\alpha}}, \frac{a_2^{\alpha}}{b_1^{\alpha}}, \frac{a_2^{\alpha}}{b_2^{\alpha}}\right\}, max\left\{\frac{a_1^{\alpha}}{b_1^{\alpha}}, \frac{a_1^{\alpha}}{b_2^{\alpha}}, \frac{a_2^{\alpha}}{b_1^{\alpha}}, \frac{a_2^{\alpha}}{b_2^{\alpha}}\right\}\right]$ here $0 \notin [\tilde{B}]^{\alpha}$;
5. $[s\tilde{A}]^{\alpha} = s[\tilde{A}]^{\alpha}$ where s is scalar and
6. $[a_1^{\alpha_i}, a_2^{\alpha_i}] = [a_1^{\alpha_j}, a_2^{\alpha_j}]$ for $0 < \alpha_i \leq \alpha_j$.

Definition 2.6. A fuzzy process is a mapping $\tilde{x}: I \to F(R)$, where I is a real interval [17]. This process can be denoted as:

$$[\tilde{x}(t)]^{\alpha} = [x_1^{\alpha}(t), x_2^{\alpha}(t)], \qquad t \in I \quad and \quad 0 < \alpha \leq 1. \quad (9)$$

The fuzzy derivative of a fuzzy process *x(t)* is defined by

$$[\tilde{x}(t)]^{\alpha} = [x_1'^{\alpha}(t), x_2'^{\alpha}(t)], \qquad t \in I \quad and \quad 0 < \alpha \leq 1. \quad (10)$$

Definition 2.7. Triangular fuzzy numbers are those fuzzy sets in $f(R)$ which are characterized by an ordered triple $(x^l, x^c, x^r) \in R^3$ with $x^l \leq x^c \leq x^r$ such that $[U]^0 = [x^l, x^r]$ and $[U]^1 = \{x^c\}$ then

$$[U]^{\alpha} = [x^c - (1-\alpha)(x^c - x^l), x^c + (1-\alpha)(x^r - x^c)], \quad (11)$$

for $\alpha \in I$.

Definition 2.8. The supremum metric d_{∞} on $F(R)$ is defined by

$$d_{\infty}(U,V) = \sup\{d_H([U]^{\alpha}, [V]^{\alpha}) : \alpha \in I\},$$

and $(F(R), d_{\infty})$ is a complete metric spaces.

Definition 2.9. A mapping $F: T \to F(R)$ is Hukuhara differentiability at $t_0 \in T \subseteq R$ if for some $h_0 > 0$ the Hukuhara differences

$$F(t_0 + \Delta t) \sim_h F(t_0), \quad F(t_0) \sim_h F(t_0 - \Delta t),$$

exist in $F(R)$ for all $0 < \Delta t < h_0$ and if there exist an $F'(t_0) \in F(R)$ such that

$$\lim_{\Delta t \to 0_+} d_\infty (F(t_0 + \Delta t) \sim_h F(t_0))/\Delta t, \quad F'(t_0) = 0$$

and

$$\lim_{\Delta t \to 0_+} d_\infty (F(t_0) \sim_h F(t_0 - \Delta t))/\Delta t, \quad F'(t_0) = 0$$

The fuzzy set $F'(t_0)$ is called the Hukuhara derivative of F at t_0.

Recall that $U \sim_h V = W \in F(R)$ are defined on level sets, where $[U]^\alpha \sim_h [V]^\alpha = [W]^\alpha$ for all $\alpha \in I$ .By consideration of definition of the metric d_∞, all the level set mappings $[F(.)]^\alpha$ are Hukuhara differentiable at t_0 with Hukuhara derivatives $[F'(t_0)]^\alpha$ for each $,\alpha \in I$ when $F: T \to F(R)$ is Hukuhara differentiable at t_0 with Hukuhara derivative $F'(t_0)$.

Definition 2.10. The fuzzy integral

$$\int_a^b x(t)dt, \qquad 0 \le a \le b \le 1,$$

is defined by

$$\left[\int_a^b x(t)dt\right]^\alpha = \left[\int_a^b \underline{x}^\alpha(t)dt, \int_a^b \overline{x}^\alpha(t)dt\right],$$

provided the Lebesgue integrals on the right exist.

Remarks 2.1. If $F: T \to F(R)$ is Hukuhara differentiable and its Hukuhara derivative F' is integrable over [0,1], then

$$F(t) = F(t_0) + \int_{t_0}^t F'(s)ds,$$

for all values of t_0, t where $0 \le t_0 \le t \le 1$.

Remarks 2.2. If $x: I \to F(R)$ is Seikkala differentiable and its Seikkala derivative x' is integrable over [0,1], then

$$x(t) = x(t_0) + \int_{t_0}^t x'(s)ds,$$

for all values of t_0, t where $t_0, t \in I$.

3 Interpolation of fuzzy number

The problem of interpolation for fuzzy sets is as follows:

Suppose that at various time instant t information $f(t)$ is presented as fuzzy set. The aim is to approximate the function $f(t)$, for all t in the domain of f . Let $t_0 < t_1 < \ldots < t_n$ be *n+1* distinct points in $F(R)$ and let $\tilde{u}_0, \tilde{u}_1, \ldots, \tilde{u}_n$ be *n+1* fuzzy sets in $F(R)$. A fuzzy polynomial interpolation of the data is a fuzzy-value continuous function $f: R \to f(R)$ satisfying:

- $f(t_i) = \tilde{u}_i, \quad i = 1, \ldots, n.$
- If the data is crisp, then the interpolation f is a crisp polynomial.

A function f which fulfilling these condition may be constructed as follows. Let $C_\alpha^i = [\tilde{u}_i]^\alpha$ for $\alpha \in [0,1], i = 0,1,\ldots,n$. For each $x = (x_0, x_1, \ldots, x_n) \in R^{n+1}$ the unique polynomial of degree $\leq n$ denoted by P_X such that

$$P_X(t_i) = x_i, \quad i = 0,1,\ldots,n,$$

$$P_X(t) = \sum_{i=0}^{n} x_i \left(\prod_{i \neq j} \frac{t - t_i}{t_i - t_j} \right).$$

Finally, for each $t \in R$ and all $\xi \in R$ is defined by $f(t) \in f(R)$ by

$$(f(t))(\xi) = \sup\{\alpha \in [0,1] : \exists\, X \in C_\alpha^0 \times \cdots \times C_\alpha^n \ \textit{such that}\ P_X(t) = \xi\}.$$

The interpolation polynomial can be written level set wise as

$$[f(t)]^\alpha = \{x \in R : x = P_X(t), \quad x \in [\tilde{u}_i]^\alpha, i = 1,2,\ldots,n\}, \quad \textit{for}\ 0 \leq \alpha \leq 1.$$

When the data $\tilde{u}_i$ presents as triangular fuzzy numbers, values of the interpolation polynomial are also triangular fuzzy numbers. Then $f(t)$ has a particular simple form that is well suited to computation.

Theorem 3.1. Let $(t_i, \tilde{u}_i), i = 0,1,2,\ldots,n$ be the observed data and suppose that each of the $\tilde{u}_i = (\tilde{u}_i^l, \tilde{u}_i^c, \tilde{u}_i^r)$ is an element of $f(R)$. Then for each $t \in [t_0, t_n]$, $\tilde{f}(t) = (f^l(t), f^c(t), f^r(t)) \in f(R)$,

$$f^l(t) = \sum_{l_i(t) \geq 0} l_i(t) u_i^l + \sum_{l_i(t) < 0} l_i(t) u_i^r,$$

$$f^c(t) = \sum_{i=0}^{n} l_i(t) u_i^c,$$

$$f^r(t) = \sum_{l_i(t) \geq 0} l_i(t) u_i^r + \sum_{l_i(t) < 0} l_i(t) u_i^l.$$

Proof : Refer [17].

4 Fuzzy Initial Value Problem

The FIVP can be considered as follows

$$\frac{dx(t)}{dt} = f(t, x(t)), \qquad x(0) = \tilde{X}_0, \tag{12}$$

where $f: R_+ \times R \to R$ is a continuous mapping and $\tilde{X}_0 \in F(R)$ with α-level interval

$$[\tilde{x}_0]^\alpha = [x_{01}^\alpha, x_{02}^\alpha] \qquad 0 < \alpha \leq 1. \tag{13}$$

When $x = x(t)$ is a fuzzy number, the extension principle of Zadeh leads to the following definition:

$$f(t, x)(s) = sup\{x(\tau) :\ s = f(t, \tau)\}, \qquad s \in R \tag{14}$$

It follows that

$$[f(t, x(t))]^\alpha = [f_1^\alpha(t, x(t)), f_2^\alpha(t, x(t))], \qquad 0 < \alpha \leq 1, \tag{15}$$

where

$$f_1^\alpha(t, x(t)) = min\{f(t, w) : w\, \epsilon [x_1^\alpha(t), x_2^\alpha(t)]\}, \qquad 0 < \alpha \leq 1, \tag{16}$$

$$f_2^\alpha(t, x(t)) = max\{f(t, w) : w\, \epsilon [x_1^\alpha(t), x_2^\alpha(t)]\}, \qquad 0 < \alpha \leq 1. \tag{17}$$

Theorem 3.1. Let f satisfy

$$|f(t, x) - f(t, x^*)| \leq g(t, |x - x^*|), \qquad t \geq 0 \qquad x, x^* \in R \tag{18}$$

where $g: R_+ \times R_+ \to R_+$ is a continuous mapping such that $r \to g(t, r)$ is nondecreasing, the IVP

$$z'(t) = g(t, z(t)), \qquad z(0) = z_0, \tag{19}$$

has a solution on R_+ for $z_0 > 0$ and that $z(t) \equiv 0$ is the only solution of equation (19) for $z_0 = 0$. Then the FIVP (12) has a unique fuzzy solution.

Proof. See [21].

In the fuzzy computation, the dependency problem arises when we apply the straightforward fuzzy interval arithmetic and Zadeh's extension principle by computing the interval separately. For the dependency problem we refer [7].

5 The predictor-corrector method in dependency problem

We consider the IVP (12) but with crisp initial condition $x(t_0)=x_0 \in R$ and $t \in [t_0, T]$. The formula for predictor-corrector is obtained by combining the Adams-Bashforth three-step method and Adams-Moulton two-step method as follows:

$$\begin{aligned} x_{i+2,P} &= x(t_{i+1}) + \frac{h}{12}[5f(t_{i-1}, x(t_{i-1})) - 16f(t_i, x(t_i)) + 23f(t_{i+1}, x(t_{i+1}))], \\ x(t_{i-1}) &= x_{i-1}, \quad x(t_i) = x_i, \quad x(t_{i+1}) = x_{i+1}, \end{aligned} \tag{20}$$

$$\begin{aligned} x_{i+2,C} &= x(t_{i+1}) + \frac{h}{12}[-f(t_i, x(t_i)) + 8f(t_{i+1}, x(t_{i+1})) + 5f(t_{i+2}, x_{i+2,P}(t_{i+2}))], \\ x(t_{i-1}) &= x_{i-1}, \quad x(t_i) = x_i. \end{aligned} \tag{21}$$

We consider the right-hand side of equation (21), we modify the predictor-corrector method by using dependency problem in fuzzy computation as one function

$$W(t_{r+2}, h, x_{i+2}) = x(t_{i+1}) + \frac{h}{12}[-f(t_i, x(t_i)) + 8f(t_{i+1}, x(t_{i+1})) + 5f(t_{i+2}, x_P(t_{i+2}))], \tag{22}$$

by the equation (22) same as

$$\begin{aligned} W(t_{r+2}, h, x_{i+2}) = x(t_{i+1}) + \frac{h}{12}\Big\{ &- f(t_i, x_i) + 8f(t_{i+1}, x_{i+1}) + 5f\big(t_{i+2}, x(t_{i+1}) \\ &+ \frac{h}{12}[5f(t_{i-1}, x(t_{i-1})) - 16f(t_i, x(t_i)) + 23f(t_{i+1}, x(t_{i+1}))]\Big\}. \end{aligned} \tag{23}$$

Now, let $\tilde{X} \in F(R)$, the formula

$$W(t_r, h, X_i)(w_i) = \begin{cases} \sup\limits_{x_i \in W^{-1}(t_i, h, w_i)} \tilde{X}_i(x_i), & if \quad w_i \in range(W); \\ 0, & if \quad w_i \notin range(W), \end{cases} \tag{24}$$

can extend equation (23) in the fuzzy setting.

Let $[\tilde{X}_i]^\alpha = [x_{i,1}^\alpha, x_{i,2}^\alpha]$ represent the α -level of the fuzzy number defined in equation (24). We rewrite equation (24) using the α -level as follows:

$$W(t_r, h, [\tilde{X}_i]^\alpha) = [\min\{W(t_i, h, x) \mid x \in [x_{i,1}^\alpha, x_{i,2}^\alpha]\}, \max\{W(t_i, h, x) \mid x \in [x_{i,1}^\alpha, x_{i,2}^\alpha]\}] \tag{25}$$

By applying equation (25) in (21) we get

$$[\tilde{X}_{i+1}]^\alpha = [x_{i+1,1}^\alpha, x_{i+1,2}^\alpha], \tag{26}$$

where

$$x_{i+1,1}^\alpha = \min\{W(t, h, x) \mid x \in [x_{i,1}^\alpha, x_{i,2}^\alpha]\}, \tag{27}$$

$$x_{i+1,2}^\alpha = \max\{W(t, h, x) \mid x \in [x_{i,1}^\alpha, x_{i,2}^\alpha]\}, \tag{28}$$

or

$$x_{i+1,1}^\alpha = \min\left\{ x(t_i) + \frac{h}{12}[-f(t_{i-1}, x_{i-1}(t_{i-1})) + 8f(t_i, x_i(t_i)) + 5f(t_{i+1}, x_P(t_{i+2}))] \mid x \in [x_{i,1}^\alpha, x_{i,2}^\alpha] \right\}, \tag{29}$$

$$x_{i+1,2}^{\alpha} = \max\left\{ x(t_i) + \frac{h}{12}[-f(t_{i-1}, x_{i-1}(t_{i-1})) + 8f(t_i, x_i(t_i)) + 5f(t_{i+1}, x_P(t_{i+2}))] \mid x \in [x_{i,1}^{\alpha}, x_{i,2}^{\alpha}] \right\}. \quad (30)$$

By using the computational method proposed in [5], we compute the minimum and maximum in equations (29), (30) as follows:

$$x_{i+1,1}^{\alpha_i} = \min\left[\min_{x\in[x_{i,1}^{\alpha_i}, x_{i,1}^{\alpha_{i+1}}]} W(t,h,x), \ldots, \min_{x\in[x_{i,1}^{\alpha_n}, x_{i,2}^{\alpha_n}]} W(t,h,x), \ldots, \min_{x\in[x_{i,2}^{\alpha_{i+1}}, x_{i,2}^{\alpha_i}]} W(t,h,x) \right], \quad (31)$$

$$x_{i+1,2}^{\alpha_i} = \max\left[\max_{x\in[x_{i,1}^{\alpha_i}, x_{i,1}^{\alpha_{i+1}}]} W(t,h,x), \ldots, \max_{x\in[x_{i,1}^{\alpha_n}, x_{i,2}^{\alpha_n}]} W(t,h,x), \ldots, \max_{x\in[x_{i,2}^{\alpha_{i+1}}, x_{i,2}^{\alpha_i}]} W(t,h,x) \right]. \quad (32)$$

7 Numerical Examples

In this section, we consider the two examples to solve FDEs.

Example 6.1. Consider the following FIVP

$$\begin{cases} x'(t) = x(1-2t), & t \in [0,2]; \\ \tilde{X}^0(v) = \begin{cases} 0, & if\ v < -0.5; \\ 1-4v^2, & if\ -0.5 \le v \le 0.5; \\ 0, & if\ v > -0.5; \end{cases} \end{cases} \quad (33)$$

The exact solution of equation (33) is given by

$$[X(t)]^{\alpha} = \left[\left(-\frac{\sqrt{1-\alpha}}{2} \right) e^{t-t^2}, \left(\frac{\sqrt{1-\alpha}}{2} \right) e^{t-t^2} \right]. \quad (34)$$

The approximate fuzzy solutions by proposed Adams-Bashforth and predictor-corrector methods at $t_{20} = 2$ are given in Table 1 and Figure 1 and 2.

TABLE 1.

The error of the obtained results with the exact solution at t=2.

α	AB3 Appr.		PC Appr.		Exact		Error AB3		Error PC	
	$x_1(t;\alpha)$	$x_2(t;\alpha)$	$x_1(t;\alpha)$	$x_2(t;\alpha)$	$x_1(t;\alpha)$	$x_2(t;\alpha)$	$x_1(t;\alpha)$	$x_2(t;\alpha)$	$x_1(t;\alpha)$	$x_2(t;\alpha)$
0.0	-0.0679	0.0679	-0.0676	0.0676	-0.0677	0.0677	-2.38e-4	2.38e-4	-8.05e-5	8.05e-5
0.1	-0.0644	0.0644	-0.0641	0.0641	-0.0642	0.0642	-2.26e-4	2.26e-4	-7.63e-5	7.63e-5
0.2	-0.0607	0.0607	-0.0605	0.0605	-0.0605	0.0605	-2.13e-4	2.13e-4	-7.20e-5	7.20e-5
0.3	-0.0568	0.0568	-0.0565	0.0565	-0.0566	0.0566	-1.99e-4	1.99e-4	-6.73e-5	6.73e-5
0.4	-0.0526	0.0526	-0.0524	0.0524	-0.0524	0.0524	-1.84e-4	1.84e-4	-6.23e-5	6.23e-5
0.5	-0.0480	0.0480	-0.0478	0.0478	-0.0478	0.0478	-1.68e-4	1.68e-4	-5.69e-5	5.69e-5
0.6	-0.0429	0.0429	-0.0427	0.0427	-0.0428	0.0428	-1.50e-4	1.50e-4	-5.09e-5	5.09e-5
0.7	-0.0372	0.0372	-0.0370	0.0370	-0.0371	0.0371	-1.30e-4	1.30e-4	-4.41e-5	4.41e-5
0.8	-0.0304	0.0304	-0.0302	0.0302	-0.0303	0.0303	-1.06e-4	1.06e-4	-3.60e-5	3.60e-5
0.9	-0.0215	0.0215	-0.0214	0.0214	-0.0214	0.0214	-7.53e-5	7.53e-5	-2.54e-5	2.54e-5
1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

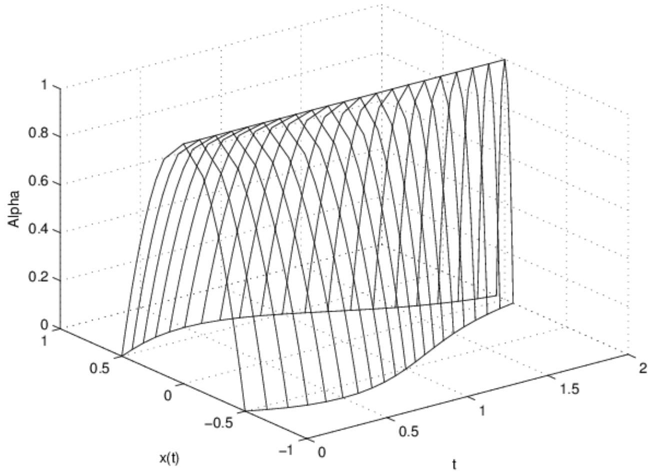

Fig 1 The approximation of fuzzy solution by predictor-corrector method (h=0.1)

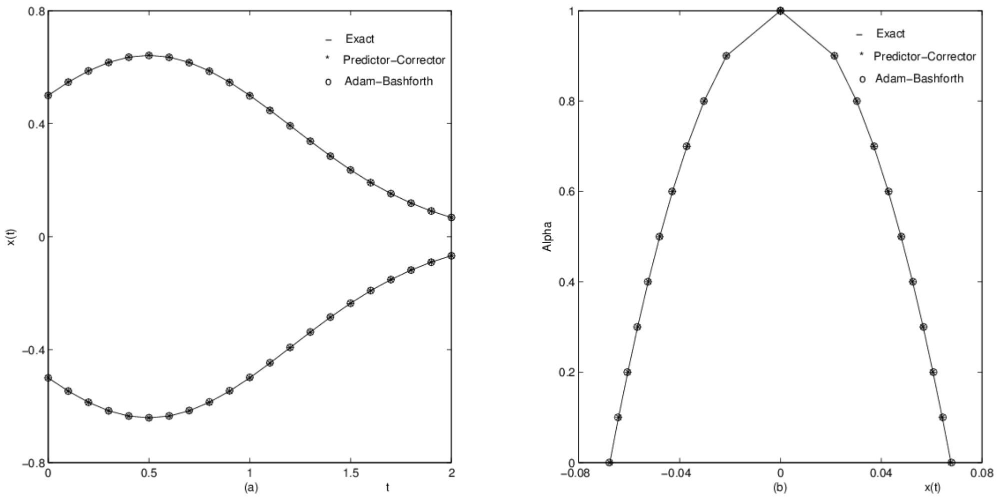

Fig 2 Comparison between the exact, Adams-Bashforth and predictor-corrector

For this example, the comparison of the absolute local error between Adams-Bashforth three-step method and Adams-Moulton two step method with the fuzzy exact solution is given in Table 1 for various values of α -level $(\alpha = 0, 0.1, \ldots, 0.9, 1)$ and fixed value of $h(t_{20} = 2)$. The graphical comparison between Adams-Bashforth three-step method and Adams-Moulton two step method with exact solutions at fixed $h(t_{10} = 1)$ are shown in Figure 2. This clearly shows that Adams-Moulton two step method gives more accurate result than Adams-Bashforth three step method. On the other hand, the result of the comparison between the fuzzy solution of Adams-Moulton two step method and the Adams-Bashforth three-step method shows decreasing of the fuzzy intervals of Adams-Moulton method.

Example 6.2. Consider the following FIVP.

$$\begin{cases} x'(t) = x(t^2 - 4t + 3), & t \in [0,2]; \\ \tilde{X}^0(v) = \begin{cases} 0, & if \ v < -0.5; \\ 1 - 4v^2, & if \ -0.5 \le v \le 0.5; \\ 0, & if \ v > -0.5; \end{cases} \end{cases} \tag{35}$$

The exact solution of equation (35) is given by

$$[X(t)]^{\alpha} = \left[\left(-\frac{\sqrt{1-\alpha}}{2} \right) e^{\frac{t^3}{3} - 2t^2 + 3}, \left(\frac{\sqrt{1-\alpha}}{2} \right) e^{\frac{t^3}{3} - 2t^2 + 3} \right] \tag{36}$$

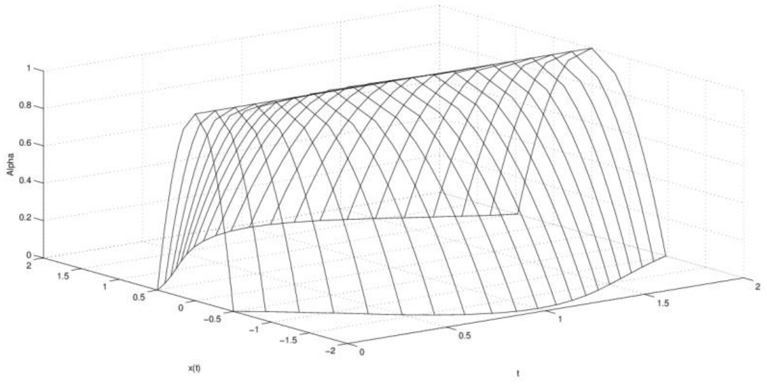

Fig 3 The approximation of fuzzy solution by predictor-corrector (h=0.1)

TABLE 2.

The error of the obtained results with the exact solution at t=2.

α	AB3 Appr		PC Appr.		Exact		Error AB3		Error PC	
	$x_1(t;\alpha)$	$x_2(t;\alpha)$	$x_1(t;\alpha)$	$x_2(t;\alpha)$	$x_1(t;\alpha)$	$x_2(t;\alpha)$	$x_1(t;\alpha)$	$x_2(t;\alpha)$	$x_1(t;\alpha)$	$x_2(t;\alpha)$
0.0	-0.9734	0.9734	-0.9740	0.9740	-0.9738	0.9738	-4.68e-4	4.68e-4	-1.33e-4	1.33e-4
0.1	-0.9234	0.9234	-0.9240	0.9240	-0.9238	0.9238	-4.44e-4	4.44e-4	-1.26e-4	1.26e-4
0.2	-0.8706	0.8706	-0.8712	0.8712	-0.8710	0.8710	-4.18e-4	4.18e-4	-1.19e-4	1.19e-4
0.3	-0.8144	0.8144	-0.8149	0.8149	-0.8147	0.8147	-3.91e-4	3.91e-4	-1.11e-4	1.11e-4
0.4	-0.7540	0.7540	-0.7545	0.7545	-0.7543	0.7543	-3.62e-4	3.62e-4	-1.03e-4	1.03e-4
0.5	-0.6883	0.6883	-0.6887	0.6887	-0.6886	0.6886	-3.31e-4	3.31e-4	-9.41e-5	9.41e-5
0.6	-0.6156	0.6156	-0.6160	0.6160	-0.6159	0.6159	-2.96e-4	2.96e-4	-8.42e-5	8.42e-5
0.7	-0.5332	0.5332	-0.5335	0.5335	-0.5334	0.5334	-2.56e-4	2.56e-4	-7.29e-5	7.29e-5
0.8	-0.4353	0.4353	-0.4356	0.4356	-0.4355	0.4355	-2.09e-4	2.09e-4	-5.95e-5	5.95e-5
0.9	-0.3078	0.3078	-0.3080	0.3080	-0.3079	0.3079	-1.48e-4	1.48e-4	-4.21e-5	4.21e-5
1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

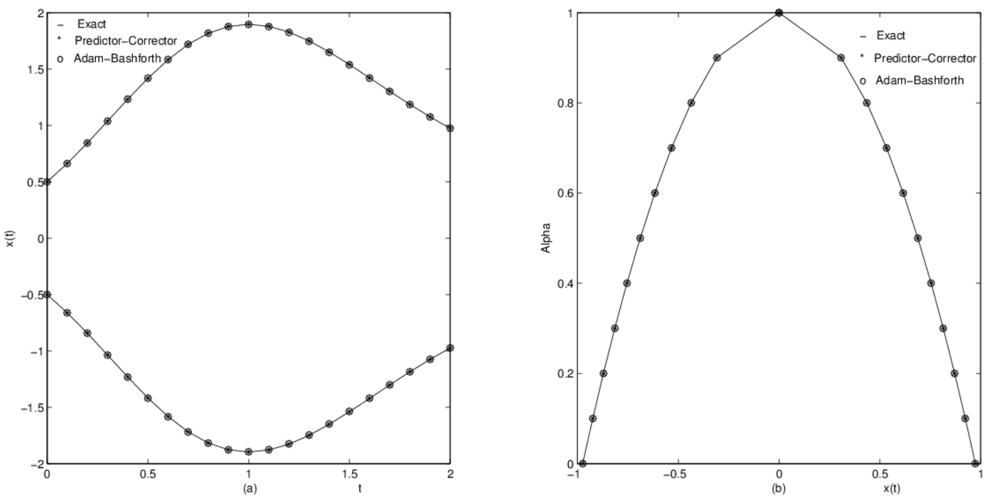

Fig 4 Comparison between the exact, Adams-Bashforth and Predictor-corrector

In this example, we compare the solution obtained by predictor-corrector method with the exact solution and the solution obtained by Adams-Bashforth. We have given the numerical values in Table 2 for fixed value of $t_{20} = 2$ and for different values of α .

8 Conclusion

In this paper we used the predictor-corrector method for solving FIVP by considering the dependency problem in fuzzy computation. We compared the solutions obtained in two numerical examples and found that predictor-corrector method gives better results than Adams-Bashforth method of order three.

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