[Volume 3 issue 10 October 2015]
PageNo.1182-1187 ISSN : 2320-7167

INTERNATIONAL JOURNAL OF MATHEMATICS AND COMPUTER RESEARCH

Single Vendor – Multi Buyer's Integrated Inventory Model with Rejection of Defective Supply Batches

[1]W.Ritha, [2]I.FrancinaNishandhi

[1,2]Department of Mathematics, Holy cross college (Autonomous) Tiruchirappalli –620-002, India

Abstract:

In this paper we consider a two echelon inventory model installation under the classical EOQ to study the effects of supply quality on cost performance. Usually that the vendor supplies their product to the buyers and the products are accepted and then used after screening any defects. K.Shouri,I.Konstantaras develops a model with a new concept and considered a single echelon inventory installation and rejection of defective supply batches in their model paper "An EOQ model with backorders and rejection of defective supply batches"[International Journal of production economics 155(2014) 148-154] Now we extend this model with two echelon inventory system with single vendor and multi buyer's model. We first present an exact model for the system expected cost and show its convexity. Optimal cost and respective values of the decision variable (i.e) planned order quantity and backorders are then obtained in a closed form. Numerical experiments were performed to illustrate the effects of parameter on the decision and the total system cost.

Keywords: Integrated model, imperfect quality, shortages, backorder, single vendor, multi buyer.

Introduction:

In recent year most inventory problems have their focus on the integration between the vendor and the buyer. For supply chain management, establishing long term strategic partnership between the buyer and the vendor is advantageous for the two parties regarding costs and therefore profits since both parties to achieve improved benefits, cooperate and share information with each other. One assumption of the classical economic order quantity model (Harris 1913) is that the supply process is of perfect quality. Real supply processes however often tail to conform to specifications, so deliveries may deviate from agreed quantities or times (or both). In this assumption Salameh and Jaber (2000) studied a joint lot sizing and inspection policy. They assumed 100% inspection of all supplies and that defective items are withdrawn from inventory by the end of inspection/screening period. This batch is sold to customers in secondary markets. Directly related is the study by Cardenas Barron (2000) corrects an error in Salameh and Jaber (2000). Goyal and Cardenas Barron (2002) revisited the work of Salameh and Jaber (2000) and presented similar approach to determine the optimal lot size quantity. Chen et al (2003) proposed an extension to the Salameh and Jaber (2000) model assuming that defective could be either rejected at a cost, or sold at a lower price or reworked instantaneously. Hayek and Salameh (2001) studied a model with imperfect quality items, where z is a random variable the production rate is finite and shortages are fully backordered. Wee et al (2007) independently extended work of salameh and Jaber (2000) to account for back orders. Khan et al(2011a) investigated the effects of both imperfect quality items and inspection errors in the eoq model Hus and Hus(2013) studied an EOQ model with imperfect quality items inspection errors, planned backorders and sales returns and they obtained closed form and the maximum planned backorders. Recently Nasr et al (2013) studied a variant of the EOQ model under random supply where every item received of imperfect quality with the same probability. Based on the survey above we extend a base model by considering two echelon inventory models.

Single vendor Multi buyer models:

The integrated inventory models for the one vendor, multi buyer's case have been discussed by a number of authors. Lal and Stalin (1984) worked on the development of a quantity discount schedule for a vendor facing several groups of homogeneous purchases, their model presents some shortcoming. Joglekar and Tharthare (1990) proposed an individually responsible and rational decision approach to the economic lot sizes for one vendor and many purchases. Siajadi et al (2006) proposed a multi shipment policy for joint economic lot size. Under this background, we consider the following imperfect quality problem with in the EOQ. A vendor ships simultaneously a lot of size Q to meet the average demand of all the buyers. Therefore if the average demand rate at buyer i is D_i units per unit time a replenishment lot containing $Qi=D_iQ/D$ units of the item is delivered to buyer i every Q_i/D_i ($\sum_{i=1}^{N} Q_i / \sum_{i=1}^{N} D_i$ =Q/D) time units. The shipment lot size Q is sum of order quantity of all the buyers (i.e) $Q=\sum_{i=1}^{N} Q_i$.

Based on this discussion the contribution of this paper summarized as follows. (i) We present an exact expected cost model of the system which is shown jointly convex in both decision variables. (ii) We show that the model optimal solution and the respective optimizers can be obtained in closed form and they all reduce to those of the classical EOQ model under perfect supply quality. (iii) We provide numerical results demonstrating the optimal expected cost on changes in supply quality.

The remainder of this paper is organized as follows. Section 2 gives the model assumptions and notation section 3 presents the exact expected cost model and the analyses leading to the optimal solution which is obtained in closed form. Section 4 provides a numerical results and a discussion of their implications in terms of system cost performance. Finally we conclude the paper in section5.

Assumptions and Notations:

To establish the mathematical model, the following notations and assumptions are used:

Notation:

Q_i : planned order quantity (in units) [derision variable]

J : planned backorders [derision variable]

T : planned between successive supply deliveries delivery interval

$T = Q_i/D_i$

T' : time between two successive acceptable deliveries . $T'=(x+1)T$

x : number of successive defective supply deliveries

p : probability that a supply batch is defective (rejected)

D_i : average demand per unit time

K_{Bi} : fixed cost per delivery

h_{Bi} : holding cost per unit time

b_i : shortage cost per unit time for the vendors:

P : production rate P>D ($D = \sum_{i=1}^{N} D_i$)

A_v : setup cost per setup

h_v : holding cost per unit time

m : number of lots delivered from the vendor to each buyer.

Q : shipment lot size in each delivery meet the demand of all the buyers

$Q = \sum_{i=1}^{N} Q_i$.

Assumptions:

1) The planning horizon is infinite
2) Demand rate is known and constant
3) Shortages are allowed and fully backordered
4) There is a fixed delivery schedule where supply batches are delivered at equally spared delivery intervals T.
5) The supplies is not totally reliable having a fixed probability to deliver a supply batch that is bellow quality standards. Defective delivery occurrences are independent of each other.
6) Each supply batch is inspected upon arrival and if found defective, the defective batch is rejected.
7) There are no emergency deliveries and the total quantity of any rejected batch routinely added to the batch quantity of next planned delivery.
8) In order to conform with the EOQ paradigm the planned backorders level satisfies the inequality J ≥ 0.
9) The system consists of multiple buyers who are supplied with a single-item by a single-vendo
10) The demands among the buyers are independent in time.

Mathematical model:

In this section we first derive an exact model for the system expected cost and then proceed in optimizing the system and determine optimal variable and cost . Although an alternative is also discussed modeling and optimization follow the classical EOQ(with backorders) paradigm using planned order quantity Q and backorders J as the problem decision variables.

Consider now the case of imperfect supply quality (p > 0) where some batches are rejected the length of inventory cycle is now a random variable depending on the respective history of consecutive previous defective deliveries and the probability of a defective batch p. Specifically let X be a random variable representing the number of consecutive defective deliveries. Using this the length of any inventory cycle can be directly expressed as T'=(X+1)T. Since the defective deliveries occurrences are independent. On the basis of the base model we derive a mathematical model under the two level supply chains with single vendor and multi buyers.

Total Expected cost per unit time for the ith buyers , $TEC_B(Q,J)$ =

$$(X+1)K_{Bi} + \frac{h(Q_i - J)^2}{2D_i} + \frac{b_i J^2}{2D_i} + b_i JXT + \frac{b_i X^2 T^2 D_i}{2}$$

$$= (X+1)K_{Bi} + \frac{h_{Bi}(Q_i - J)^2}{2D_i} + \frac{b_i J^2}{2D_i} + \frac{b_i J Q_i X}{D_i} + \frac{b_i Q_i^2 X^2}{2D_i} \qquad (1)$$

Since now X is a geometric random variable with parameter p, its expected value and second moment are respectively given by.

$$E(X) = \frac{p}{1-p} \text{ and}$$

$$E\left(X^2\right) = \text{var}(X) + \left[E(X)\right]^2 = \frac{p}{(1-p)^2} + \frac{p^2}{(1-p)^2} = \frac{p(1+p)}{(1-p)^2}$$

Therefore, the expected value of Eq(1) is given as,

$$\frac{K_{Bi}}{(1-p)} + \frac{h_{Bi}(Q_i - J)^2}{2D_i} + \frac{b_i J^2}{2D_i} + \frac{b_i J Q_i p}{D_i(1-p)} + \frac{b_i Q_i^2 p(1+p)}{2D_i(1-p)^2} \qquad (2)$$

The expected length of each inventory cycle is E(T')= $\frac{Q_i}{(1-p)D_i}$ (3)

Therefore the Total Expected cost per unit time for the i[th] buyer's,

$$TEC_B(Q,J) = \frac{K_{Bi}D_i}{Q_i} + \frac{h_{Bi}\left(Q_i - J\right)^2(1-p)}{2Q_i} + \frac{b_i J^2(1-p)}{2Q_i} + b_i Jp + \frac{b_i Q_i p(1+p)}{2D_i(1-p)} \quad (4)$$

Further the order lot size Q_i of buyer i should be in proportion of their demand of shipment lot size Q (i.e) $Q_i = \frac{D_i Q}{D}$. Therefore the substitution of Q_i in the above equation as, $TEC_B(Q,J)$ =

$$\frac{K_{Bi}D}{Q} + \frac{h_{Bi}QD_i(1-p)}{2D} - J h_{Bi}(1-p) + \frac{\left(h_{Bi}+b_i\right)(1-p)J^2 D}{2QD_i} + b_i pJ + \frac{b_i p(1+p)Q}{2D(1-p)} \quad (5)$$

The integrated production inventory model is designed for the vendor's production situation in which once the orders are placed by all the buyers the production begins and a constant number of units are added to the inventory until the production run has been completed. The vendor produces the item in the quantity of mQ in one production cycle and each will receive it in m lots each of size Q_i's such that $Q_i = \frac{D_i Q}{D}$ The total Expected cost per unit time for the vendor is given by:

$$TEC_v(Q,m) = \frac{A_v D(1-p)}{mQ} + \frac{h_v(1-p)Q}{2}\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] \quad (6)$$

Consequently the Joint total expected cost per unit time for the vendor-buyer integrated system is sum of the total expected cost of the vendor which is given by.

$$JETC(Q,J,m) = \sum_{i=1}^{N} TEC_{Bi}(Q,J) + TEC_v(Q,m)$$

$$= \frac{D}{Q}\left[\frac{A_v(1-p)}{m} + \sum_{i=1}^{N} K_{Bi}\right] + \sum_{i=1}^{N}\frac{h_{Bi}QD_i(1-p)}{2D} - \sum_{i=1}^{N} J h_{Bi}(1-p) + \sum_{i=1}^{N}\frac{\left(h_{Bi}+b_i\right)(1-p)J^2 D}{2QD_i} + \sum_{i=1}^{N} b_i pJ + \sum_{i=1}^{N}\frac{b_i p(1+p)Q}{2D(1-p)} + \frac{Q(1-p)h_v}{2}\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] \quad (7)$$

Differentiating w.r.to 'Q' the optimal order quantity of Q is expressed as.

$$Q^* = \sqrt{\frac{2D\left\{\left[\frac{A_v(1-p)}{m} + \sum_{i=1}^{N} K_{Bi}\right] + \frac{\left(h_{Bi}+b_i\right)J^2(1-p)}{2D_i}\right\}}{\frac{h_{Bi}D_i(1-p)}{D} + \frac{b_i p(1+p)}{D(1-p)} + h_v(1-p)\left[m(1-\frac{D}{P}) - 1 + \frac{2D}{P}\right]}} \quad (8)$$

Similarly differentiating w.r.to 'J' the planned backorder quantity is expressed as,

$$J^* = \frac{\sum_{i=1}^{N}\left[h_{Bi} - (h_{Bi}+b_i)p\right]QD_i}{\sum_{i=1}^{N}\left(h_{Bi}+b_i\right)(1-p)D} \quad (9)$$

Numerical Example:

In this section we consider a system consisting of three buyers and one vendor with a following data for the vendor P=25000units/year A_v=400 per setup h_v=0.2, the data for the buyer are given in table1

Buyer i	D_i units/year	K_{Bi}	h_{Bi}	b_i	P
1	1000	250	2	8	0.10
2	5000	300	4	10	0.10
3	800	350	4	14	0.10

According to the numerical data the results of Q*,J*,TEC(Q*,J*) are summarized below:

Buyer i	Q*	J*	TEC(Q*,J*)
1	5427	88.67	1571
2	2169	329	4237
3	4896	78	2059

Conclusion:

In this paper, we have considered an EOQ model with single vendor and multi buyer's to study the effects of imperfect supply quality on system operation and cost. According to the numerical result, we understand that the total expected cost is varied from the buyer to buyer.

References:

1. Atan,Z,Snyder,L.V.,2012.Inventory strategies to manage supply disruptions. In: Gurnani , H., Mehrtra,A., Ray,s.(Eds), Managing supply Disruptions.Springer-Verlag,Berlin.
2. Berk,E., Arreola. Risa,A., 1994 Note on "future supply uncertainty in EOQ model" Nav .Res.Logist 41(1),129-132
3. Cardenas . Barron. L.E ., 2000. Observation on:" Economic production quantity model for items with imperfect quality". International Journal of production Economics.67(2).(pp.201-201)
4. Cardenas-Barron. L.E,2009 Economic production quantity with rework process at a single-stage manufatuing system with planned backorders. Computer and Industrial Engineering. 57(3)1105-1113.
5. Chang.H.C,Ho,C.H.,2009.Exact closed- form solution for "Optimal inventory model for items with imperfect quality and shortage backordering"Omega 38(3-4)233-237.
6. Erolu.A, Ozdemir, G,2007. An Economic order quantity model with defective items and shortages. International Journal of production economics. 106(2)544-549.

7. Goyal, S.K., 1976. An integrated inventory model for a single supplier–single customer problem. International Journal of Production Research 15 (1), 107–111.
8. Hill, R.M., 1999. The optimal production and shipment policy for the single-vendor single-buyer integrated production-inventory model. International Journal of Production Research 37, 2463–2475
9. Hsu,J.T,Hsu,L.F,2013 An EOQ model with imperfect quality items, inspertion errors, shortages backordering and sales return. International Journal of production economics.143(1),162-170
10. Jha,J.K.,& Kripa Shanker.2013 Single vendor multi buyer integrated production-inventory model with controllable lead time and service level constraints.Applied Mathematical Modeling 37,1753-1767.
11. Joglekar, P.N., 1988. Comments on a quantity discount pricing model to increase vendor profits. Management Science 34, 1391–1398.
12. Joglekar, P.N., Tharthare, S., 1990. The individually responsible andrational decision approach to economic lot sizes for one vendor and many purchasers. Decision Sciences 21 (3), 492–506.
13. Khan .M., Jaber.M.Y, Bonney,M.,2011aAn Economic order quantity for items with imperfect quality and Inspertion errors. International Journal of production economics. 133(1).133-118.
14. Khan, M, M.Y, Guiffrida, A.L.,Zolfaghari, S,2011b.A review of the extension of a modified EOQ model for imperfect quality items. International Journal of production economics.132(1).1-12.
15. Kim, T., Hong, Y., Chang, S.Y., 2006. Joint economic procurement–production–delivery policy for multiple items in a single-manufacturer, multiple-retailer system. International Journal of Production Economics 103, 199–208
16. Lal, R., Staelin, R., 1984. An approach for developing an optimal discount pricing policy. Management Science 30, 1524–1539.
17. Lu, L., 1995. A one-vendor multi-buyer integrated inventory model. European Journal of Operational Research 81, 312–323.
18. Papachristos,S., Konstantaras. I. 2006. Economic ordering quantity model for items imperfect quality. International Journal of production economics.(100)(1) 148-154.
19. Siajadi, H., Ibrahim, R.N., Lochert, P.B., 2006. Joint economic lot size in distribution system with multiple shipment policy. International Journal of Production Economics 102, 302–316.
20. Salameh,M.K.,Jaber,M.Y., 2000 Economic production Quantity model for items with imperfect quality. International Journal of production economics.64(1-3),59-64
21. Skouri,K., Konstantaras,I.,Lagodimos,A.G.,Papachristos,S., 2014.An EOQ model with backorders and rejection of defective supply batches. International Journal of production economics.155,148-154
22. Zipkin,H.P.,2000. Foundation of Inventory management.MC Graw-Hill Higher Education new York.